Chapter 1

$$dq = dE + dW \qquad dq = de + p\,d\left(\frac{1}{\rho}\right) = de - \frac{p}{\rho^2}\,d\rho$$

$$de = \frac{\partial e}{\partial T}\Big|_p dT + \frac{\partial e}{\partial p}\Big|_T dp$$

$$d\rho = \frac{\partial \rho}{\partial T}\Big|_p dT + \left(\frac{\partial \rho}{\partial p}\right)_T dp$$

2.25
2.27

$$dq = \frac{\partial e}{\partial T}\Big|_p dT + \frac{\partial e}{\partial p}\Big|_T dp - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial T}\Big|_p dT + \frac{\partial \rho}{\partial p}\Big|_T dp\right)$$

$$= \left\{\frac{\partial e}{\partial T}\Big|_p - \frac{p}{\rho^2}\frac{\partial \rho}{\partial T}\Big|_p\right\} dT + \left\{\frac{\partial e}{\partial p}\Big|_T - \frac{p}{\rho^2}\frac{\partial \rho}{\partial p}\Big|_T\right\} dp$$

$$\frac{C_v - C_p}{(\partial p/\partial T)_\rho} = \frac{C_p - C_v}{1\Big/\frac{\partial T}{\partial \rho}\Big|_p \cdot \frac{\partial \rho}{\partial p}\Big|_T} = \frac{C_p - C_v}{\underbrace{\left(\frac{\partial \ln\rho}{\partial p}\Big|_p\right)}_{(-\beta)}\Big/\underbrace{\left(\frac{\partial \ln\rho}{\partial T}\Big|_{\cancel{p}}\right)}_{(+\cancel{\kappa_T})}}$$

$$-\frac{\kappa_T}{\beta}(C_p - C_v) = \frac{\partial e}{\partial p}\Big|_T - \frac{p}{\rho}\frac{\partial \ln\rho}{\partial p}\Big|_T$$

$$\frac{\partial e}{\partial p}\Big|_T = \frac{p}{\rho}\frac{\partial \ln\rho}{\partial p}\Big|_T - \frac{\kappa_T}{\beta}(C_p - C_v)$$

$$= \frac{p}{\rho}\kappa_T - \frac{\kappa_T(C_p - C_v)}{\beta} = \kappa_T\left(\frac{p}{\rho} - \frac{(C_p - C_v)}{\beta}\right)$$

2.28-2.30

$$dq = \frac{\partial e}{\partial p}\Big|_\rho dp + \left(\frac{\partial e}{\partial \rho}\Big|_p - \frac{p}{\rho^2}\right)d\rho$$

$$= \frac{\partial e}{\partial p}\Big|_\rho dp + \left(\frac{\partial e}{\partial \rho}\Big|_p - \frac{p}{\rho^2}\right)\left(\frac{\partial \rho}{\partial p}\Big|_T dp + \frac{\partial \rho}{\partial T}\Big|_p dT\right)$$

$$c_p\, dT = \left(\frac{\partial e}{\partial \rho}\Big|_p - \frac{p}{\rho^2}\right)\left(\frac{\partial \rho}{\partial T}\Big|_p\right)dT$$

$$= \left(\frac{\partial e}{\partial \rho}\Big|_p - \frac{p}{\rho^2}\right)\rho(-\beta)\, dT$$

$$-\frac{c_p}{\beta\rho} = \frac{\partial e}{\partial \rho}\Big|_p - \frac{p}{\rho^2}$$

$$\frac{p}{\rho^2} - \frac{c_p}{\beta\rho} = \frac{\partial e}{\partial \rho}\Big|_p$$

$$\frac{dp}{d\rho} = -\frac{\left(\frac{\partial e}{\partial \rho}\Big|_p - \frac{p}{\rho^2}\right)}{\left(\frac{\partial e}{\partial p}\Big|_\rho\right)}$$

$$= \frac{+\,c_p/\cancel{\beta}\rho}{k_T c_v/\cancel{\beta}}$$

$$= \frac{c_p}{\rho k_T c_v}$$

5.5

$$\frac{1}{T}\cancel{\frac{\partial^2 e}{\partial\rho\,\partial T}} = -\frac{1}{T}\left[\frac{\partial e}{\partial\rho}\bigg|_T - \frac{p}{\rho^2}\right] + \cancel{\frac{1}{T}\frac{\partial^2 e}{\partial T\,\partial\rho}} - \frac{1}{\rho^2}\frac{\partial p}{\partial T}\bigg|_\rho$$

$$-\frac{1}{T}\frac{\partial e}{\partial\rho}\bigg|_T + \frac{p}{\rho^2 T} - \frac{1}{\rho^2}\frac{\partial p}{\partial T}\bigg|_\rho = 0$$

$$\frac{1}{T}\frac{\partial e}{\partial\rho}\bigg|_T = \frac{p}{\rho^2 T} - \frac{1}{\rho^2}\frac{\partial p}{\partial T}\bigg|_\rho$$

$$\frac{\partial e}{\partial\rho}\bigg|_T = \frac{p}{\rho^2} - \frac{T}{\rho^2}\frac{\partial p}{\partial T}\bigg|_\rho$$

$p = \rho R T$

$\frac{\partial p}{\partial T}\Big| = \rho R$

$p - T\cdot\rho R \equiv 0$ for perfect gas

$$\frac{\partial p}{\partial T}\bigg|_\rho \cdot \frac{\partial T}{\partial\rho}\bigg|_p \cdot \frac{\partial\rho}{\partial p}\bigg|_T = -1$$

$$\frac{\partial p}{\partial T}\bigg|_\rho \left(\frac{\partial \ln\rho}{\partial p}\bigg|_T \bigg/ \frac{\partial \ln\rho}{\partial T}\bigg|_p\right) = -1$$

$$\frac{\partial p}{\partial T}\bigg|_\rho \left(K_T \big/ (-\beta)\right) = -1$$

$$\frac{\partial p}{\partial T}\bigg|_\rho = \frac{\beta}{K_T}$$

5.7

$$\frac{\partial e}{\partial \rho}\bigg|_T = \frac{1}{\rho^2}\left[p - \frac{\beta T}{K_T}\right]$$

$$c_p - c_v = \left[\frac{\partial e}{\partial \rho}\bigg|_T - \frac{p}{\rho^2}\right]\frac{\partial \rho}{\partial T}\bigg|_T$$

$$= \left\{\frac{1}{\rho^2}\left(p - \frac{\beta T}{K_T}\right) - \frac{p}{\rho^2}\right\}\frac{\partial \rho}{\partial T}\bigg|_T$$

$$c_p - c_v = -\frac{\beta T}{\rho K_T}\,\frac{\partial \ln\rho}{\partial T}\bigg|_T$$

$$c_p - c_v = \left(\frac{\partial e}{\partial \rho}\bigg|_T - \frac{p}{\rho^2}\right)\frac{\partial \rho}{\partial T}\bigg|_p$$

$$\frac{\partial e}{\partial \rho}\bigg|_T = \frac{1}{\rho^2}\left(p - \frac{\beta T}{K_T}\right)$$

$$c_p - c_v = \left(\cancel{\frac{p}{\rho}} - \frac{\beta T}{\rho^2 K_T} - \cancel{\frac{p}{\rho^2}}\right)\frac{\partial \rho}{\partial T}\bigg|_p$$

$$c_p - c_v = -\frac{\beta T}{\rho^2 K_T}\,\frac{\partial \rho}{\partial T}\bigg|_p \equiv -\beta$$

$$= +\frac{\beta^2 T}{\rho K_T}$$

$\frac{\partial s}{\partial T}\Big|_\rho$

5.8 - 5.10

$$\frac{1}{T}\frac{\partial e}{\partial T}\Big|_\rho = \frac{\partial s}{\partial T}\Big|_\rho$$

c_V

$$d\rho = \frac{\partial \rho}{\partial T}\Big|_p dp + \frac{\partial \rho}{\partial p}\Big|_T$$

$$\cancel{c_p dT = \frac{\partial e}{\partial p}\Big|}$$

$$c_p dT = \frac{\partial e}{\partial T}\Big|_p dT + \cancel{\frac{\partial e}{\partial T}\Big|_p} dp - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial T}\Big|_T dp + \frac{\partial \rho}{\partial p}\Big|_T dT\right)$$

$$c_p = \frac{\partial e}{\partial T}\Big|_p - \frac{p}{\rho^2}\cdot\frac{\partial \rho}{\partial p}\Big|_T$$

$$\frac{\partial \rho}{\partial T}\Big|_p dT + \frac{\partial \rho}{\partial p}\Big|_T dp$$

$$\frac{\partial s}{\partial T}\Big|_\rho = \frac{1}{T}\frac{\partial e}{\partial T}\Big|_\rho$$

$$\frac{\partial e}{\partial T}\Big|_\rho \cdot \frac{\partial T}{\partial \rho}\Big|_e \cdot \frac{\partial \rho}{\partial e}\Big|_T$$

$$ds = \frac{\partial s}{\partial T}\Big|_\rho dT + \frac{\partial s}{\partial \rho}\Big|_T d\rho$$

$$= \frac{de}{\partial \rho}\Big|_T d\rho + \frac{de}{\partial T}\Big|_\rho dT - \frac{p}{\rho^2} d\rho$$

5.9

$$\left.\frac{\partial s}{\partial \rho}\right|_T = \frac{1}{T}\left\{\frac{1}{\rho^2}\left(p - \frac{\beta T}{K_T}\right) - \frac{p}{\rho^2}\right\}$$

$$= \frac{1}{T}\left\{-\frac{\beta T}{\rho^2 K_T}\right\}$$

$$= -\frac{\beta}{\rho^2 K_T}$$

$\frac{K_T}{\beta} R = \frac{\beta T}{\rho}$

$R = \frac{\beta^2 T}{K_T \rho}$

$(c_p - c_v) K_T$

~~5.11~~ 5.13

$$ds = \left.\frac{\partial s}{\partial T}\right|_p dT + \left.\frac{\partial s}{\partial p}\right|_T dp$$

$$T\,ds = \left.\frac{\partial e}{\partial p}\right|_T dp + \left.\frac{\partial e}{\partial T}\right|_p dT - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial T} dT + \frac{\partial \rho}{\partial p} dp\right)$$

$$\left.\frac{\partial s}{\partial p}\right|_T = \frac{1}{T}\left.\frac{\partial e}{\partial p}\right|_T - \frac{p}{\rho^2 T} \cdot \left.\frac{\partial \rho}{\partial p}\right|_T \quad \left(\left.\frac{\partial \rho}{\partial p}\right|_T = K_T\right)$$

5.11

$$\left.\frac{\partial e}{\partial p}\right|_T = K_T\left(\frac{p}{\rho} - \frac{(c_p - c_v)}{\beta}\right) = K_T\left(\frac{p}{\rho} - \frac{\beta^2 T}{K_T \rho \beta}\right) = \left(\frac{K_T p}{\rho} - \frac{\beta T}{\rho}\right)$$

$$\left.\frac{\partial s}{\partial p}\right|_T = \frac{K_T}{T}\left(\frac{p}{\rho} - \frac{R}{\beta}\right) - \frac{p}{\rho T} K_T = -\frac{K_T (c_p - c_v)}{\beta T}$$

$c_p - c_v = \frac{\beta^2 T}{K_T \rho}$

$$\left.\frac{\partial s}{\partial p}\right|_T = -\frac{\cancel{K_T}}{\beta \cancel{T}}\left(\frac{\beta^{\cancel{2}} \cancel{T}}{\cancel{K_T} \rho}\right) = -\frac{\beta}{\rho}$$

5.13

$$\left.\frac{\partial p}{\partial T}\right|_\rho = \frac{\beta}{K_T}$$

5.15-5.16

$$ds = \frac{1}{T}\left.\frac{\partial e}{\partial T}\right|_\rho dT + \frac{1}{T}\left(\left.\frac{\partial e}{\partial \rho}\right|_T - \frac{P}{\rho^2}\right) d\rho$$

$$\left.\frac{\partial s}{\partial p}\right|_\rho dp + \left.\frac{\partial s}{\partial \rho}\right|_p d\rho = \left(\frac{1}{T}\left.\frac{\partial e}{\partial T}\right|_\rho\right)\left(\left.\frac{\partial T}{\partial p}\right|_\rho dp + \left.\frac{\partial T}{\partial \rho}\right|_p d\rho\right) + \frac{1}{T}\left(\left.\frac{\partial e}{\partial \rho}\right|_T - \frac{P}{\rho^2}\right) d\rho$$

5.15

$$\left.\frac{\partial s}{\partial p}\right|_\rho = \frac{1}{T}\left.\frac{\partial e}{\partial T}\right|_\rho \cdot \left.\frac{\partial T}{\partial p}\right|_\rho$$

$$\left.\frac{\partial s}{\partial \rho}\right|_p = \frac{1}{T}\left.\frac{\partial e}{\partial T}\right|_\rho \cdot \left.\frac{\partial T}{\partial \rho}\right|_p + \frac{1}{T}\left(\left.\frac{\partial e}{\partial \rho}\right|_T - \frac{P}{\rho^2}\right)$$

$$\left.\frac{\partial e}{\partial T}\right|_\rho = c_v \qquad \left.\frac{\partial e}{\partial \rho}\right|_T = \frac{1}{\rho^2}\left(P - \frac{\beta T}{K_T}\right)$$

hence $\left.\frac{\partial s}{\partial p}\right|_\rho = \frac{1}{T} c_v \left(\frac{K_T}{\beta}\right) = \frac{c_v K_T}{\beta T}$

5.16

$$\left.\frac{\partial s}{\partial \rho}\right|_p = \frac{1}{T} c_v \left(-\frac{\rho}{\rho\beta}\right) + \frac{1}{T\rho^2}\left(P - \frac{\beta T}{K_T} - P\right)$$

$$= -\frac{c_v \cancel{\rho}}{\rho \beta T} - \frac{\beta \cancel{T}}{\rho^2 K_T \cancel{T}} = -\frac{c_v \rho}{\beta T} - \frac{\beta}{\rho^2 K_T}$$

$$\frac{\beta}{\rho^2 K_T} = \frac{1}{\rho}\cdot\frac{\beta}{\rho K_T} = \frac{1}{\rho}\cdot\frac{(c_p - c_v)}{\beta T}$$

$$-\frac{c_v \cancel{\rho}}{\cancel{\rho}\beta T} - \frac{1}{\rho}\cdot\frac{1}{\beta T}(c_p - c_v) = -\frac{c_p}{\rho\beta T}$$

$(4N)^{1/3} \approx$

$$(4\times10^{16})^{-1/3}$$

$$= \cancel{\left(\frac{40}{10^{5}}\right)} \quad (40\times10^{15})^{-1/3}$$

$$= \frac{1}{(40\times10^{15})^{1/3}} = \frac{10^{-5}}{\sqrt[3]{40}} \qquad \frac{10^{-5}}{3.3}$$

3×3

$4\times4\times4=64$

$3\times3\times3=27 \quad 3.3$

$\ddot{r} - r\dot{\theta}^2 = F/\tilde{m}$

$\ddot{r}$ $\frac{\dot{\theta}}{r}$

$\frac{1}{2}\dot{r}^2 + \frac{g^2b^2}{2r^2} = \frac{Fr}{\tilde{m}} + c$

$\frac{g^2b^2}{r^2} = \dot{\theta}$

$g^2b^2 = r^4\dot{\theta}^2$

$\frac{\dot{r}^2}{2} + \frac{r^2\dot{\theta}^2}{2}$

$F\dot{r}\,dt = \int F\,dr$

$F = \frac{\partial\phi}{\partial r}$

$\frac{dr}{dt}$

$$r^2 d\theta = gb\,dt$$

$$r^2 \frac{d}{dt} = gb \frac{d}{d\theta}$$

$$\dot{\theta} = \frac{gb}{r^2}$$

$$\frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{C_\alpha}{r^\alpha \tilde{m}} = \frac{1}{2} g^2$$

$$r^2\dot{\theta}$$

$$\frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) = \frac{g^2}{2} - \frac{C_\alpha}{r^\alpha \tilde{m}}$$

$$r^2\dot{\theta}^2 = \left(g^2 - \frac{2C_\alpha}{r^\alpha \tilde{m}}\right) - \dot{r}^2$$

$$\dot{\theta}^2 = \left(\frac{g^2}{r^2} - \frac{2C_\alpha}{r^{\alpha+2}\tilde{m}}\right) - \frac{\dot{r}^2}{r^2}$$

$$\cancel{\frac{gb}{r^2} = \dot{\theta} =}$$

$$\frac{1}{2}\left(\dot{r}^2 + \frac{g^2 b^2}{r^2}\right) + \frac{C_\alpha}{r^\alpha \tilde{m}} = \frac{1}{2} g^2$$

$$\left(\dot{r}^2 + \frac{g^2 b^2}{r^2}\right) = g^2 - \frac{2C_\alpha}{r^\alpha \tilde{m}}$$

$$\dot{r}^2 = g^2 - \frac{2C_\alpha}{r^\alpha \tilde{m}} - \frac{g^2 b^2}{r^2}$$

$$\dot{r} = \left(g^2 - \frac{2C_\alpha}{r^\alpha \tilde{m}} - \frac{g^2 b^2}{r^2}\right)^{1/2}$$

$$\dot{\theta}^2 = \frac{g^2}{r^2}\left(1 - \frac{2C_\alpha}{g^2 r^\alpha \tilde{m}} - \frac{\dot{r}^2}{g^2}\right)$$

$$\dot{\theta} = \frac{g}{r}$$

$\frac{dr}{d\theta} = 0$ = apse A, $R_0 = b/\eta_0$ $\eta = b/r$

$$1 - \eta_0^2 - (2/\alpha)(\eta_0/\beta)^\alpha = 0$$

$$\theta_0 = \int_0^{(b/R_0)} \left[1 - \eta_0^2 - \frac{2}{\alpha}\left(\frac{\eta_0}{\beta}\right)^\alpha\right]^{-1/2} d\eta$$

$$\theta = \int_R^\infty \left(1 - \frac{2C_\alpha}{\tilde{m} g^2 r^\alpha} - \frac{b^2}{r^2}\right)^{1/2} \frac{b\,dr}{r^2}$$

$r = b/\eta$

$dr = -\frac{b}{\eta^2}\,d\eta$

$\cancel{dr = \frac{b}{d\eta}}$

$d\eta = -\frac{b\,dr}{r^2}$ $\eta = \frac{b}{r}$

$$= \int_{r=R}^{\infty}\left(1 - \frac{2C_\alpha}{\tilde{m} g^2 (r/b)^\alpha b^\alpha} - \eta^2\right) \cancel{\frac{\eta}{r}\left(-\frac{b}{\eta^2}d\eta\right)} (-d\eta)$$

$r = R$

$\eta = b/R$

$r = \infty, \eta = 0$

$$= \int_0^{b/R}\left(1 - \frac{2C_\alpha \eta^\alpha}{\tilde{m} g^2 b^\alpha} - \eta^2\right)(+d\eta) = \int_0 \left[1 - \eta^2 - \frac{2\eta^\alpha}{\alpha}\left(\frac{\alpha C_\alpha}{\tilde{m} g^2 b^\alpha}\right)\right]$$

$$\frac{1}{\beta^\alpha} = \frac{\alpha C_\alpha}{\tilde{m} g^2 b^\alpha} \qquad \beta^\alpha = b^\alpha\left(\frac{\tilde{m} g^2}{\alpha C_\alpha}\right)$$

$$\sin\tfrac{1}{2}\psi \cong \frac{\psi}{2} \approx \frac{1}{[1+(1/y^2)]^{1/2}} \approx \frac{1}{(1+x^2)^{1/2}}$$

$$(1+x^2)^{-1/2} \approx \frac{1}{x} \approx y$$

$x >> 1$

$$2\pi n_i g t \int_{b_{min}}^{b_{max}} \left(\frac{2 Z_1 Z_2 e^2}{m g^2 b}\right)^2 b\,db$$

$$= 8\pi n_i g t \frac{Z_1^2 Z_2^2 e^4}{m_e^2 g^3} \int \frac{b\,db}{b^2}$$

$$E = \sum \nu_i \varepsilon_i \qquad \nu_i = \frac{N}{Z} g_i e^{-\beta\varepsilon_i}$$

$$E = \frac{N}{Z} \sum g_i \varepsilon_i e^{-\beta\varepsilon_i}$$

$$\ln \nu_i! \cong \tfrac{1}{2}\ln\nu_i + \nu_i \ln\nu_i - \nu_i$$

$$\ln W = \sum_i \left\{ \nu_i \ln g_i - \tfrac{1}{2}\ln\nu_i + \nu_i - \nu_i \ln\nu_i \right\}$$

$$= \sum_i \left\{ \nu_i \ln\left(\frac{g_i}{\nu_i}\right) + \nu_i - \tfrac{1}{2}\ln\nu_i \right\}$$

$$= N + \sum \left\{ \nu_i \ln\left(\frac{g_i}{\nu_i}\right) - \tfrac{1}{2}\ln\nu_i \right\}$$

$$= N - \sum \left[\nu_i \ln\left(\frac{\nu_i}{g_i}\right) - \cancel{\ln \nu_i^{1/2}} \right]$$

$$\delta\left[\nu_i \ln\left(\frac{\nu_i}{g_i}\right)\right] = \delta\nu_i \ln\left(\frac{\nu_i}{g_i}\right) + \frac{\nu_i}{(\nu_i/g_i)} \frac{\delta\nu_i}{g_i}$$

$$\ln\left(\frac{\nu_i}{g_i}\right) = \ln\alpha + \beta\varepsilon_0 \qquad = \delta\nu_i \ln\left(\frac{\nu_i}{g_i}\right) + \delta\nu_i$$

$$\ln\left(\frac{\nu_i}{\alpha g_i}\right) = -\beta\varepsilon_i$$

$$\frac{\nu_i}{\alpha g_i} = e^{-\beta\varepsilon_i}$$

$$\nu_i = \alpha g_i e^{-\beta\varepsilon_i}$$

12.17 $\ln W = N - \sum \nu_i \ln\left(\frac{\nu_i}{g_i}\right)$

$$\frac{\nu_i}{g_i} = \frac{N}{Z} e^{-\beta \varepsilon_i}$$

$$\ln\left(\frac{\nu_i}{g_i}\right) = \ln\left(\frac{N}{Z}\right) - \beta \varepsilon_i$$

$$\nu_i \ln\left(\frac{\nu_i}{g_i}\right) = \nu_i \ln\left(\frac{N}{Z}\right) - \beta \nu_i \varepsilon_i$$

$$\sum \nu_i \ln\left(\frac{\nu_i}{g_i}\right) = N \ln\left(\frac{N}{Z}\right) - \beta E$$

$$\ln W = N - \left\{ N \ln \frac{N}{Z} - \beta E \right\} = N\left(1 + \ln\left(\frac{Z}{N}\right) + \beta E\right)$$

$$S = k \ln W$$

$$\left(\frac{\partial S}{\partial E}\right)_V = \frac{\partial}{\partial E}\left\{ Nk\left[1 + \ln Z/N\right]\right\} + \beta k$$

$$T dS = dE - p dV$$

$$\frac{\partial}{\partial E}\left[Nk \ln \frac{Z}{N}\right] = \frac{Nk}{Z/N} \cdot \frac{1}{N} \frac{\partial Z}{\partial \beta} \cdot \frac{\partial \beta}{\partial E}$$

$\frac{N}{(\cancel{Z/N})}$ $\quad \cancel{Nk \frac{\partial}{\partial E}\left(\ln \frac{N}{Z}\right) = \frac{Nk}{(N/Z)}\left(-\frac{N}{Z^2}\right)\frac{\partial Z}{\partial \beta} \cdot \frac{\partial \beta}{\partial E}}$

$$Nk(\ln Z - \ln N) = \frac{Nk}{Z} \frac{\partial Z}{\partial \beta} \cdot \frac{\partial \beta}{\partial E}$$

$$\sum \varepsilon_i g_i e^{-\varepsilon_i/kT} = kT^2 \frac{\partial z}{\partial T}$$

$$\frac{N}{z} \sum \varepsilon_i g_i e^{-\varepsilon_i/kT} = \frac{N}{z} kT^2 \frac{\partial z}{\partial T}$$

$$E = NkT^2 \frac{\partial \ln z}{\partial T}$$

$$S = Nk\left[1 + \ln(z/N)\right] + E/T$$

$$= Nk\left(1 + \ln\frac{z}{N}\right) + \frac{NkT^2}{T}\frac{\partial \ln z}{\partial T}$$

$$= Nk\left\{1 + \ln\frac{z}{N} + T\frac{\partial \ln z}{\partial T}\right\}$$

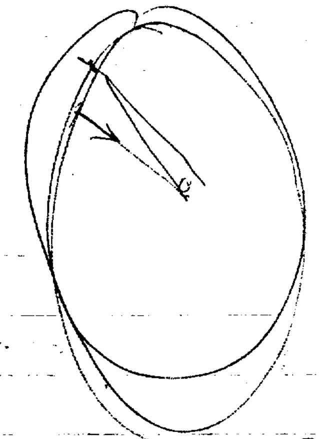
$S =$

$$TdS = dE + pdV$$

$$T\left(\frac{\partial S}{\partial V}dV + \frac{\partial S}{\partial T}dT\right) = \left(\frac{\partial E}{\partial V}dV + \frac{\partial E}{\partial T}dT\right) + pdV$$

$$pdV = T\frac{\partial S}{\partial V}dV - \frac{\partial E}{\partial V}dV$$

$$T\frac{\partial S}{\partial V} = T\,Nk\frac{\partial \ln(z/N)}{\partial V} + NkT^2\frac{\partial^2 \ln z}{\partial T\,\partial V}$$



$$\frac{m_{(1)}^2}{e^4 z_1^2 z_2^2 n_e} = \frac{1}{m_1^2 \cdot N g^2}$$

$$\frac{1}{8\pi n w p_0^2} = \frac{1}{8\pi n w} \cdot \frac{(m w^2)^2}{(z_1 z_2 e^2)^2}$$

$$= \frac{m^2 w^3}{8\pi z_1^2 z_2^2 e^4 n}$$

$$\frac{z_1 z_2 e^2}{m w^2}$$

$$b_{max} = D = \left[\frac{kT}{4\pi e^2 (2 n_e)}\right]^{1/2}$$

$$b_{min} = \frac{z_i e^2}{m_e g^2}$$

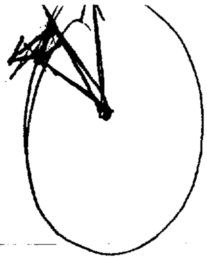
$$\Lambda = \frac{b_{max}}{b_{min}} = \left[\frac{kT}{8\pi e^2 n_e}\right]^{1/2} \cdot \frac{m_e g^2}{e^2}$$

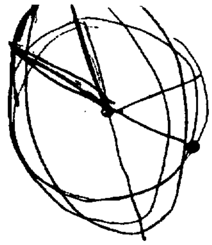
$$g^2 = \frac{3kT}{m_e} =$$

$$\Lambda = \left[\frac{kT}{8\pi \cancel{e}^2 n_e}\right]^{1/2} \frac{\cancel{m_e}(3kT/\cancel{m_e})}{e^2}$$

$$= \left[\frac{kT}{8\pi n_e}\right]^{1/2} \frac{1}{e} \frac{(3kT)}{e^2}$$

$$= \frac{3}{e^3}\left[\frac{k^3 T^3}{8\pi n_e}\right]^{1/2}$$





$v = r \cdot \theta$

$dx = r\,d\theta \cdot \cancel{dr}$

p. 22 Lagrangean? spelling?

p. 22-23 introduce Liouville eqn + BBGKY hierarchy. Are you going to develop these or drop them. Need statement of intent.

$$m_1 m_2 \ddot{\tilde{x}} = (m_1 + m_2) \tilde{F}$$

$$x = r$$

$$\sum x^2 = \frac{8\pi e^4 n_i t}{m_e^2 g^3} \ln \Lambda$$

$$\frac{\partial x^2}{\partial t} = \frac{8\pi e^4 n_i}{m_e^2 g^3} \ln \Lambda$$

$$t_D = \frac{m_e^2 g^3}{8\pi e^4 n_i \ln \Lambda}$$

$$\frac{e^2}{m_e g^2} \qquad t_s = \frac{1}{\pi b^2 n g} = \frac{1}{\pi n_p g \left(\frac{e^2}{m_e g^2}\right)^2}$$

$$= \frac{m_e^2 g^3}{\pi n_p e^4}$$

$$b_{max} = D$$
$$b_{min} = \frac{Z_i e^2}{m_e g^2}$$

$$\tan\frac{\chi}{2} = \frac{Z_1 Z_2 e^2}{\tilde{m} g^2 b}$$

$$\Lambda = \frac{b_{max}}{b_{min}} \simeq \frac{4.8(T/n_e)^{1/2}}{Z_i e^2/m_e g^2}$$

$$\chi =$$

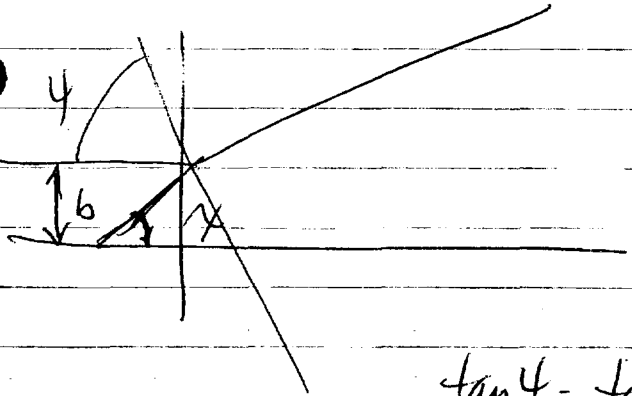
$$= \frac{4.8\, m_e g^2 (T/n_e)^{1/2}}{Z_i e^2}$$

$$\sin\frac{\chi}{2} = \left[1 + \left(\frac{\tilde{m} g^2 b}{Z_1 Z_2 e^2}\right)^2\right]^{1/2}$$

$$= \frac{4.8\, m_e g^2}{Z_i e^2}\left(\frac{T}{n_e}\right)^{1/2}$$

$$\sin\chi = 2\sin\frac{\chi}{2}\cos\frac{\chi}{2}$$

$$n_e + n_i = 2n_e \qquad D = \left[\frac{kT}{8\pi e^2 n_e}\right]^{1/2}$$



$$\psi = \frac{\pi - \chi}{2} = \frac{\pi}{2} - \frac{\chi}{2}$$

$$\tan\psi = \tan\left(\frac{\pi}{2} - \frac{\chi}{2}\right) = \frac{\sin(\pi/2 - \chi/2)}{\cos(\pi/2 - \chi/2)}$$

$$= \frac{\sin\pi/2\cos(-\chi/2) + \cos\pi/2\sin(-\chi/2)}{\cos\pi/2\cos(-\chi/2) - \sin(\pi/2)\sin(-\chi/2)}$$

$$= \frac{\cos(-\chi/2)}{-\sin(-\chi/2)} = \frac{\cos\chi/2}{\sin\chi/2} = \cot\chi/2$$

$$\frac{(1-\sin^2\chi/2)^{1/2}}{\sin\chi/2} = x$$

$$(1-\sin^2\chi/2)^{1/2} = \cancel{\psi}\, x\sin\chi/2$$
$$1-\sin^2\chi/2 = \cancel{\psi}^2\, x^2 \sin^2\chi/2$$

$$\Pi_e \cong (1+\psi)$$

$$\Pi_i \cong 1 - Z_i\psi$$

$$\rho = -en_e\Pi_e + e\sum n_i Z_i \Pi_i$$

$$\cong -en_e(1+\psi) + e\sum n_i Z_i(1 - Z_i\psi)$$

$$= -en_e - en_e\psi + e\sum n_i Z_i - e\sum n_i Z_i^2\psi$$

$$= -en_e\psi - e\sum n_i Z_i^2\psi$$

$$\psi = \frac{Ze\phi}{kT} \qquad \phi = \frac{kT\psi}{Ze}$$

$$4\pi\rho = -\frac{\phi}{D^2} = \frac{kT\cancel{\psi}}{ZeD^2} = 4\pi\left[-e\cancel{\psi}\left(n_e + \sum Z_i^2 n_i\right)\right]$$

$$\frac{kT}{Ze} = 4\pi e\left(n_e + \sum Z_i^2 n_i\right)D^2$$

$$D^2 = \frac{kT}{Ze \cdot 4\pi e\left(n_e + \sum Z_i^2 n_i\right)}$$

$$D^2 = \frac{kT}{4\pi Ze^2\left(n_e + \sum Z_i^2 n_i\right)}$$

$t_D = 1/\sum\chi^2$

$w^2 = \frac{Z Z_1 e^2}{mb}$

$\sum (\Delta E)^2$

$\sum\chi^2 = 2\pi n_2 g t \int_{b_{min}}^{b_{max}} \chi^2(g,b)\, b\, db$

$= \left(8\pi Z_i^2 e^4 n_i t / m_e^2 g^3\right) \ln\Lambda$

$1/\left(8\pi Z_i^2 e^4 n_i t / m_e^2 g^3\right) \ln\Lambda$

$t_D = m_e^2 g^3 / \left(8\pi Z_i^2 e^4 n_i t \ln\Lambda\right)$

$\langle(\Delta w_T)^2\rangle = 8\pi n_1 w^3 b^2 \ln\Lambda$

$w^2 = \frac{Z^2 e^2}{mb}$

$\langle(\Delta w_T)^2\rangle = \frac{8\pi n_1 wb Z^2 e^2}{m}$

$= \frac{8\pi n Z^2 e^2 (wb) \ln\Lambda}{m}$

$wb \stackrel{?}{=} \frac{e^2}{m_e g^3}$

$\sum\chi^2 (t_D) = 1$

$\sum(\Delta E)^2 (t_E) = E^2$

$t_D = 1/\left[\frac{d}{dt}\sum\chi^2\right]$

$t_E = E^2/\left[\frac{d}{dt}\sum\Delta E^2\right]$

4441-4160

$$\frac{\lambda^3 N}{V} = \left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2}\frac{N}{V} = z$$

$$P = T\frac{\partial S}{\partial V} - \frac{\partial E}{\partial V} \qquad Q_N^{(1/N)} = \frac{V}{N}\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}$$

$$\int_0^\infty e^{-mu^2/2kT}\,u^2 du$$

$$S = Nk\left(1 + \ln Z/N\right) + E/T$$

$$\frac{\partial S}{\partial V} = Nk\frac{\partial \ln(Z/N)}{\partial V} + \frac{\cancel{1}}{T}\frac{\partial E}{\partial V}$$

$$d^6N = g_i e^{-\beta\varepsilon_i}\frac{N}{Z}$$

$$\frac{d^6N}{N} = \frac{g_i e^{-\beta\varepsilon_i}}{Z}$$

$$Z = \sum g_i e^{-\beta\varepsilon_i}$$

$$P = NkT\frac{\partial \ln(Z/N)}{\partial V} + \frac{\partial E}{\partial V} - \frac{\partial E}{\partial V}$$

$$g_i = \left(\frac{m}{h}\right)^3 d^3x\, d^3u = \left(\frac{4\pi m^3}{h^3}\right) d^3x\, u^2 du$$

$$\frac{1}{kT}$$

$$\frac{\partial\beta}{\partial T} = -\frac{1}{kT^2}$$

$$S = Nk\ln\left[\frac{V}{N}\left(\frac{3}{2}kT\right)^{3/2}\right] + Nk\left(\frac{5}{2}\right) + \frac{3}{2}Nk\ln\left(\frac{4\pi m}{3h^2}\right)$$

$$\beta = \frac{1}{kT} \qquad T = \frac{1}{\beta k}$$

$$= \frac{5}{2}Nk + Nk\ln\frac{V}{N} + Nk\left[\ln\left\{\left(\frac{3}{2}kT\right)^{3/2}\right\} + \ln\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right]$$

$$= \frac{5}{2}Nk + Nk\ln\frac{V}{N} + Nk\ln\left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

$$E = -\frac{\partial \ln Z}{\partial\beta}$$

$$= -\frac{\partial \ln Z}{\partial T}\cdot\frac{\partial T}{\partial\beta}$$

$$+kT^2\frac{\partial \ln Z}{\partial T}$$

$$d^6N = f(u)\, d^3x\, d^3u$$

$$Z_{tr} = \left(\frac{2\pi m kT}{h^2}\right)^{3/2} V$$

$$\underline{d^6N} = f(u)\, dV\, d^3u = \frac{\cancel{4\pi} m^3 N}{h^3} \frac{1}{\left(\frac{2\pi m kT}{h^2}\right)^{3/2} V} e^{-mu^2/2kT}\, dV \cancel{u^2} d\cancel{u} d^3u$$

$$= \frac{m^3 N}{\cancel{h^3} V} \left(\frac{\cancel{h^2}}{2\pi m kT}\right)^{3/2} e^{-mu^2/2kT}\, dV\, d^3u$$

$$f(u)\, dV\, d^3u = \frac{N}{V}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mu^2/2kT}\, dV\, d^3u$$

$$\frac{S}{Nm} = \frac{\cancel{N}k}{\cancel{N}m}\left\{\frac{5}{2} \cancel{N} - \ln N + \frac{3}{2}\ln T + \frac{3}{2}\ln\left(\frac{2\pi m \cancel{k}}{h^2}\right) + \frac{3}{2}\ln k\right\}$$

$$= R\left\{\frac{5}{2} - \ln N + \ln(NkT) - \ln(NkT) + \frac{3}{2}\ln T + \frac{3}{2}\ln\left(\frac{2\pi m}{h^2}\right) + \frac{3}{2}\ln k\right\}$$

$$= R\left\{\frac{5}{2} - \cancel{\ln N} + \cancel{\ln N} + \ln k + \ln T - \ln p + \frac{3}{2}\ln T + \frac{3}{2}\ln\left(\frac{2\pi m}{h^2}\right) + \frac{3}{2}\ln k\right\}$$

$$= R\left\{\frac{5}{2} + \frac{5}{2}\ln T + \frac{5}{2}\ln k - \ln p + \frac{3}{2}\ln\left(\frac{2\pi m}{h^2}\right)\right\}$$

$$g_i = \frac{\cancel{4\pi} m^3}{h^3} d^3x \cancel{\ \ } d^3u$$

$$S = k \ln\left\{N - \sum \nu_i \ln(\nu_i/g_i)\right\}$$

$$\sum(\nu_i^0 + \Delta\nu_i)\left[\ln \nu_i^0 + \frac{\Delta\nu_i}{\nu^0} - \frac{1}{2}\left(\frac{\Delta\nu}{\nu^0}\right)^2\right]$$

$$= \Big\{ \sum \nu^0 \ln \nu^0 + \sum \Delta\nu \ln \nu + \sum \nu \frac{\Delta\nu}{\nu} + \sum \frac{\Delta\nu^2}{\nu}$$

$$- \frac{1}{2}\sum \frac{\Delta\nu^2}{\nu} - \frac{1}{2}\Delta\nu\left(\frac{\Delta\nu}{\nu}\right)^2 \Big\}$$

$$= \sum \nu \ln \nu + \sum \Delta\nu \ln \nu + \sum \Delta\nu + \frac{1}{2}\sum \frac{\Delta\nu^2}{\nu}$$

$$\underbrace{N - \sum \nu \ln \nu}_{\ln W_0} - \underbrace{\sum \Delta\nu \ln \nu}_{\overset{\parallel\!\!|}{0}} - \underbrace{\sum \Delta\nu}_{\overset{\parallel\!\!|}{0}} - \frac{1}{2}\sum \frac{\Delta\nu^2}{\nu}$$

$$\ln W \cong \ln W_0 - \frac{1}{2}\sum \frac{\Delta\nu^2}{\nu}$$

$$E = \frac{N}{Z}\sum \varepsilon_i g_i e^{-\varepsilon_i/kT}$$

$$\ln\left(\frac{N}{W_0}\right) =$$

$$Z = \sum g_i e^{-\varepsilon_i/kT}$$

$$\frac{\partial Z}{\partial T} = -\sum \frac{\varepsilon_i(-1)}{kT^2} g_i e^{-\varepsilon_i/kT}$$

$$= +\frac{1}{kT^2}\sum \varepsilon_i g_i e^{-\varepsilon_i/kT}$$

Chapter 3

$V_1 = V_r$
$V_2 = r V_\theta$
$V_3 = r\sin\theta\, V_\phi$

25.9

$$E_{ij} = \frac{1}{2}\left(V_{i;j} + V_{j;i}\right)$$

$$E_{11} = V_{1;1} = \frac{\partial V_r}{\partial r}$$

$$E_{21} = E_{12} = \frac{1}{2}\left(V_{1;2} + V_{2;1}\right) = \frac{1}{2}\left(V_{1,2} + V_{2,1}\right) - \frac{1}{2}\left[\left\{ {k \atop 12} \right\} V_k + \left\{ {k \atop 21} \right\} V_k\right]$$

$$= \frac{1}{2}\left[\frac{\partial V_1}{\partial \theta} + \frac{\partial V_2}{\partial r}\right] - \frac{1}{2}\left[2\left\{ {2 \atop 12} \right\} V_2\right] = \frac{1}{2}\left[\frac{\partial V_1}{\partial \theta} + \frac{\partial V_2}{\partial r}\right] - \frac{V_2}{r}$$

$$E_{31} = E_{13} = \frac{1}{2}\left[\frac{\partial V_1}{\partial \phi} + \frac{\partial V_3}{\partial r}\right] - \frac{1}{2}\left[\left\{ {k \atop 13} \right\} V_k + \left\{ {k \atop 31} \right\} V_k\right]$$

$$= \frac{1}{2}\left[\frac{\partial V_1}{\partial \phi} + \frac{\partial V_3}{\partial r}\right] - \frac{V_3}{r}$$

$$E_{22} = \frac{\partial V_2}{\partial \theta} - \left\{ {k \atop 22} \right\} V_k = \frac{\partial V_2}{\partial \theta} + r V_1$$

$$E_{23} = \frac{1}{2}\left[\frac{\partial V_2}{\partial \phi} + \frac{\partial V_3}{\partial \theta}\right] - \left\{ {k \atop 23} \right\} V_k = \frac{1}{2}\left[\frac{\partial V_2}{\partial \phi} + \frac{\partial V_3}{\partial \theta}\right] - \cot\theta\, V_3$$

$$E_{33} = \frac{\partial V_3}{\partial \phi} - \frac{1}{2}\left[\left\{ {k \atop 33} \right\} V_k\right] = \frac{\partial V_3}{\partial \phi} - \left\{ {1 \atop 33} \right\} V_1 - \left\{ {2 \atop 33} \right\} V_2$$

$$= \frac{\partial V_3}{\partial \phi} + r\sin^2\theta\, V_1 + \sin\theta\cos\theta\, V_2$$

$E_{11} = E_{rr}$ $E_{21} = E_{12} = E_{r\theta}\, r$ $E_{13} = E_{31} = E_{r\phi} \cdot r\sin\theta$

$E_{22} = E_{\theta\theta}\, r^2$ $E_{23} = E_{32} = E_{\theta\phi}\, r^3 \sin\theta$ $E_{33} = E_{\phi\phi}\, r^3 \sin^2\theta$

25.10

Thus

$$E_{rr} = \partial v_r / \partial r$$

$$r E_{r\theta} = \frac{1}{2}\left[\frac{\partial v_r}{\partial \theta} + \frac{\partial}{\partial r}(r v_\theta)\right] - \frac{r v_\theta}{r}$$

$$\Rightarrow E_{r\theta},\ E_{\theta r} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} - \frac{2 v_\theta}{r}\right] = \frac{1}{2}\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right]$$

$$r\sin\theta\, E_{r\phi} = \frac{1}{2}\left[\frac{\partial v_r}{\partial \phi} + \frac{\partial}{\partial r}(r\sin\theta\, v_\phi)\right] - \frac{r\sin\theta\, v_\phi}{r}$$

$$\Rightarrow \quad E_{r\phi} = \frac{1}{2r\sin\theta}\left[\frac{\partial v_r}{\partial \phi} + \sin\theta\, v_\phi + r\sin\theta\frac{\partial v_\phi}{\partial r} - 2\sin\theta\, v_\phi\right]$$

$$E_{\phi r} = E_{r\phi} = \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r}\right]$$

$$r^2 E_{\theta\theta} = \frac{\partial (r v_\theta)}{\partial \theta} + r v_r = r\frac{\partial v_\theta}{\partial \theta} + r v_r$$

$$\Rightarrow E_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}$$

$$E_{\theta\phi}\, r^2\sin\theta = \frac{1}{2}\left[\frac{\partial (r v_\theta)}{\partial \phi} + \frac{\partial (r\sin\theta\, v_\phi)}{\partial \theta}\right] - \cot\theta \cdot r\sin\theta\, v_\phi$$

$$= \frac{1}{2}\left[r\frac{\partial v_\theta}{\partial \phi} + r\cos\theta\, v_\phi + r\sin\theta\frac{\partial v_\phi}{\partial \theta} - 2r\,\cancel{\cot\theta\sin\theta}^{\cos\theta}\, v_\phi\right]$$

$$\Rightarrow E_{\phi\theta} = E_{\theta\phi} = \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{1}{r}\frac{\partial v_\phi}{\partial \theta} - \frac{1}{r}\cot\theta\, v_\phi\right]$$

$$r^2\sin^2\theta\, E_{\phi\phi} = \frac{\partial(r\sin\theta\, v_\phi)}{\partial\phi} + r\sin^2\theta\, v_r + \sin\theta\cos\theta \cdot r\, v_\theta$$

$$\Rightarrow E_{\phi\phi} = \frac{r\sin\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{\cot\theta}{r} v_\theta$$

$$E_{\phi\phi} = \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{\cot\theta}{r} v_\theta$$

end 25.10

25.11

$$T_{ij} = -p g_{ij} + \sigma_{ij}$$

$$= -p g_{ij} + 2\mu E_{ij} + \left(\zeta - \frac{2}{3}\mu\right) v^k_{;k}\, g_{ij}$$

$$\left[\begin{array}{lll} T_{11} = T_{rr} & T_{12} = T_{21} = r T_{r\theta} & T_{13} = T_{31} = r\sin\theta\, T_{r\phi} \\ T_{22} = r^2 T_{\theta\theta} & T_{23} = T_{32} = r^2\sin\theta\, T_{\theta\phi} & T_{33} = r^2\sin^2\theta\, E_{\phi\phi} \end{array}\right.$$

$$g_{11} = 1 \quad g_{22} = r^2 \quad g_{33} = r^2\sin^2\theta \quad g_{ij} = 0 \quad i \neq j.$$

$$T_{11} = -p g_{11} + 2\mu E_{11} + \left(\zeta - \frac{2}{3}\mu\right) \overset{\nabla\cdot v}{v^k_{;k}}\, g_{11}$$

$$T_{rr} = -p + 2\mu \frac{\partial v_r}{\partial r} + \left(\zeta - \frac{2}{3}\mu\right)\cdot \nabla\cdot v$$

$$r^2 T_{22} = \cancel{r^2} T_{\theta\theta} = -p\cancel{r^2} + 2\mu \cancel{r^2} E_{\theta\theta} + \left(\zeta - \frac{2}{3}\mu\right)\nabla\cdot v\, \cancel{r^2}$$

$$T_{\theta\theta} = -p + 2\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right) + \left(\zeta - \frac{2}{3}\mu\right)\nabla\cdot v$$

$$T_{33} = r^2\sin^2\theta\, T_{\phi\phi} = -p\, r^2\sin^2\theta + 2\mu\, r^2\sin^2\theta\, E_{\phi\phi} + \left(\zeta - \frac{2}{3}\mu\right)\nabla\cdot v \cdot r^2\sin^2\theta$$

$$T_{\phi\phi} = -p + 2\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{\cot\theta\, v_\theta}{r}\right) + \left(\zeta - \frac{2}{3}\mu\right)\nabla\cdot v$$

$$T_{12}=T_{21} = r\,T_{r\theta} = 2\mu\, r\, E_{r\theta} = 2\mu r \cdot \frac{1}{2}\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right]$$

$$T_{r\theta} = \mu\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right] = \cancel{\frac{\mu}{r}\left[\frac{\partial v_r}{\partial \theta} + \frac{\partial}{\partial r}(v_\theta)\right]}$$

$$= \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$T_{13}=T_{31} = r\sin\theta\, T_{r\phi} = 2\mu \cdot r\sin\theta\, E_{r\phi} = 2\mu\frac{r\sin\theta}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

$$T_{r\phi} = \mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

$$T_{23}=T_{32} = r^2\sin\theta\, T_{\theta\phi} = 2\mu \cdot r^2\sin\theta\, E_{\theta\phi}$$

$$T_{\theta\phi} = 2\mu \cdot \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right)\right]$$

$$= \mu\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right)\right]$$

Chapter 3

$$\rho \frac{Dv^i}{Dt} = f^i + T^{ji}_{\ ,j}$$

1-D planar

$$\rho \frac{Dv_z}{Dt} = f_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left[2\mu \frac{\partial v_z}{\partial z}\right] + \frac{\partial}{\partial z}\left(\zeta - \frac{2}{3}\mu\right)\frac{\partial v_z}{\partial z}$$

$$= f_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left\{2\mu \frac{\partial v_z}{\partial z} + \left(\zeta - \frac{2}{3}\mu\right)\frac{\partial v_z}{\partial z}\right\}$$

26.2 $$= f_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left\{\left(\zeta + \frac{4}{3}\mu\right)\frac{\partial v_z}{\partial z}\right\}$$

μ + ζ constant

$$T_{ij,j} = \mu\left(v_{i,j} + v_{j,i} - \frac{2}{3} v_{k,k}\delta_{ij}\right)_{,j} + \zeta v_{k,ki}$$

$$= \mu v_{i,jj} + \underbrace{\mu v_{j,ij}}_{\mu v_{k,ki}} - \frac{2}{3}\mu v_{k,ki} + \zeta v_{k,ki}$$

$$= \mu v_{i,jj} + \frac{1}{3}\mu v_{k,ki} + \zeta v_{k,ki}$$

$$= \mu v_{i,jj} + \left(\zeta + \frac{1}{3}\mu\right) v_{k,ki}$$

26.5 $$= \mu \nabla^2 v_i + \left(\zeta + \frac{1}{3}\mu\right)(\nabla\cdot v)_{,i}$$

So
Constant viscosity coeffs $\Rightarrow$

$$\rho \frac{Dv}{Dt} = f - \nabla p + \mu \nabla^2 v + \left(\zeta + \frac{1}{3}\mu\right)\nabla(\nabla\cdot v)$$

A3.91 $T^{1j}{}_{;j} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta T_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial T_{r\phi}}{\partial\phi} - \frac{1}{r}(T_{\theta\theta}+T_{\phi\phi})$

$T^{2j}{}_{;j} = \frac{1}{r}\left[\frac{1}{r^2}\frac{\partial(r^2 T_{r\theta})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta T_{\theta\theta}) + \frac{1}{r\sin\theta}\frac{\partial T_{\theta\phi}}{\partial\phi} + \frac{1}{r}(T_{r\theta} - \cot\theta\, T_{\phi\phi})\right]$

$T^{3j}{}_{;j} = \frac{1}{r\sin\theta}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta T_{\theta\phi}) + \frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial\phi} + \frac{1}{r}(T_{r\phi} + \cot\theta\, T_{\theta\phi})\right]$

$\rho a^i = f^i + T^{ji}{}_{;j}$

Use 25.10 and A3.91:

$$T^{1j}{}_{;j} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta T_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial T_{r\phi}}{\partial\phi} - \frac{1}{r}(T_{\theta\theta}+T_{\phi\phi})$$

$\eta \equiv (\zeta - \frac{2}{3}\mu)\nabla\cdot\vec{v}$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(-p + 2\mu\frac{\partial v_r}{\partial r} + \eta\right)\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\mu\left(r\frac{\partial}{\partial r}(v_\theta/r) + \frac{1}{r}\frac{\partial v_r}{\partial\theta}\right)\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\mu\left(r\frac{\partial}{\partial r}(v_\phi/r) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right)\right] - \frac{1}{r}\left[-p + 2\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right) + \eta\right.$$

$$\left. -p + 2\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right) + \eta\right]$$

$$= \cancel{-\frac{2p}{r}} - \frac{\partial p}{\partial r} + \cancel{\frac{2p}{r}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(2\mu r^2\frac{\partial v_r}{\partial r}\right) - \frac{2}{r}\cdot\frac{2\mu v_r}{r} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\eta) - \frac{2\eta}{r}$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\mu\sin\theta}{r}\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right)$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\mu\sin\theta\, r\frac{\partial}{\partial r}(v_\theta/r)\right) - \frac{2\mu}{r^2}v_\theta\cot\theta - \frac{2\mu}{r^2}\frac{\partial v_\theta}{\partial\theta}$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\mu r\frac{\partial}{\partial r}(v_\phi/r)\right) - \frac{2\mu}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(2\mu r^2\frac{\partial v_r}{\partial r}\right) - \frac{4\mu v_r}{r^2} = \frac{2}{r^2}\left[\frac{\partial}{\partial r}\left(\mu r^2\frac{\partial v_r}{\partial r}\right) - 2\mu v_r\right]$$

$$= \frac{2}{r^2}\left[\frac{\partial}{\partial r}\left(\mu r^2\frac{\partial v_r}{\partial r}\right) - \frac{\mu v_r}{r}\frac{\partial r^2}{\partial r}\right]$$

$$= 2\frac{\partial}{\partial r}\left(\mu\frac{\partial v_r}{\partial r}\right) + \frac{4}{r}\mu\frac{\partial v_r}{\partial r} - \frac{4\mu v_r}{r^2} = \frac{\partial}{\partial r}\left(2\mu\frac{\partial v_r}{\partial r}\right) + \frac{4\mu}{r}\left(\frac{\partial v_r}{\partial r} - \frac{v_r}{r}\right)$$

✓ $$= \frac{\partial}{\partial r}\left(2\mu\frac{\partial v_r}{\partial r}\right) + 4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

r $\quad \frac{1}{r^2}\frac{\partial}{\partial r}\left(2\mu r^2 \frac{\partial v_r}{\partial r}\right) - \frac{4\mu v_r}{r^2} = \frac{\partial}{\partial r}\left(2\mu \frac{\partial v_r}{\partial r}\right) + 4\mu \frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$

$\eta \quad \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\eta) - \frac{2\eta}{r} = \frac{2\eta}{r} + \frac{\partial\eta}{\partial r} - \frac{2\eta}{r} = \frac{\partial\eta}{\partial r}$

$\theta \quad \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\mu\sin\theta}{r}\frac{\partial v_r}{\partial\theta}\right) = \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\mu}{r}\frac{\partial v_r}{\partial\theta}\right) + \frac{\mu\cot\theta}{r^2}\frac{\partial v_r}{\partial\theta}$

leave alone (ϕ) $\quad \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right] - \frac{2\mu}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}$

$\theta \quad \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\mu r\sin\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right] = \frac{1}{r}\frac{\partial}{\partial\theta}\left[\mu r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right] + \mu\cot\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)$

$\theta \quad -\frac{2\mu}{r^2}\cot\theta\, v_\theta - \frac{2\mu}{r^2}\frac{\partial v_\theta}{\partial\theta} = -\frac{2\mu}{r^2}\left(\cot\theta\, v_\theta + \frac{\partial v_\theta}{\partial\theta}\right) = -\frac{2\mu}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta)$

$$T^{1j}_{;j} = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial r}\left(2\mu\frac{\partial v_r}{\partial r} + (\zeta - \tfrac23\mu)\nabla\cdot \underline{v}\right) + \frac1r\frac{\partial}{\partial\theta}\left(\frac{\mu}{r}\frac{\partial v_r}{\partial\theta} + \mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

$$+ \frac{\mu\cot\theta}{r^2}\frac{\partial v_r}{\partial\theta} + 4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right) - \frac{2\mu}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \mu\cot\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) - \frac{2\mu}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta)$$

$$T^{1j}_{;j} = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial r}\left[2\mu \frac{\partial v_r}{\partial r} + (\zeta-\tfrac23\mu)\underline{\nabla}\cdot\underline{v}\right] + \frac1r\frac{\partial}{\partial\theta}\left[\frac{\mu}{r}\frac{\partial v_r}{\partial\theta} + \mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{\mu}{\sin\theta}\frac{\partial v_r}{\partial\phi} + \mu r \frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

$$+ \frac{\mu}{r}\left[4r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right) - \frac{2}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) - \frac{2\cancel{\mu}}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{\cot\theta}{r}\frac{\partial v_r}{\partial\theta} + r\cot\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

$$rT^{2j}_{;j} = \cancel{r}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, T_{\theta\theta}) + \frac{1}{r\sin\theta}\frac{\partial T_{\theta\phi}}{\partial\phi} + \frac1r(T_{r\theta} - \cot\theta\, T_{\phi\phi})\right.$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac1r\frac{\partial v_r}{\partial\theta}\right)\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\left(-p + \frac{2\mu}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{2\mu v_r}{r}\right) + \eta\right]$$

$$-\frac{\cot\theta}{r}\left[-p + \frac{2\mu}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{2\mu v_r}{r} + \frac{2\mu\cot\theta}{r}v_\theta + \eta\right]$$

$$+\frac1r\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\mu}{r}\frac{\partial v_r}{\partial\theta}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$T^{2j}{}_{;j} = \frac{\partial}{\partial r}\left[\mu r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right] + \frac{2\mu r}{r}\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\partial}{\partial r}\left(\frac{\mu}{r}\frac{\partial v_r}{\partial \theta}\right) + \frac{2\mu}{r}\cdot\frac{1}{r}\frac{\partial v_r}{\partial \theta}$$

$$+ \frac{1}{r}\frac{\partial}{\partial \theta}\left[-p + \frac{2\mu}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{2\mu v_r}{r} + \eta\right] + \frac{\cot\theta}{r}\left[\cancel{-p} + \frac{2\mu}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{2\mu v_r}{r} \cancel{+\eta}\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] + \cancel{\frac{p\cot\theta}{r}} - \frac{\mu}{r}\left[\frac{2\cot\theta}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{2\cot\theta}{r}v_r + \frac{2\cot^2\theta}{r}v_\theta\right] - \cancel{\frac{\eta\cot\theta}{r}}$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\mu}{r}\frac{\partial v_r}{\partial \theta}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{2\mu}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{2\mu v_r}{r} + \eta\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\left[\frac{2}{r}\frac{\partial v_r}{\partial \theta} + 2r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \cancel{\frac{2\cot\theta}{r}v_r} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right.$$

$$\left. - \cancel{\frac{2\cot\theta}{r}v_r} - \frac{2\cot^2\theta}{r}v_\theta\right]$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\mu}{r}\frac{\partial v_r}{\partial \theta}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{2\mu}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right) + \left(\zeta - \frac{2}{3}\mu\right)\nabla\cdot v\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\left[\frac{3}{r}\frac{\partial v_r}{\partial \theta} + \cancel{\frac{2\cot\theta}{r}v_r} + 3r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{2\cot\theta}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} - \frac{2\cot^2\theta}{r}v_\theta\right]$$

$$= \frac{\mu}{r}\left[\frac{3}{r}\frac{\partial v_r}{\partial \theta} + 3r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\left(\frac{\partial v_\theta}{\partial \theta} - \cot\theta\, v_\theta - \frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)\right]$$

$$= \frac{\mu}{r}\left[\frac{3}{r}\frac{\partial v_r}{\partial \theta} + 3r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\left(\sin\theta\frac{\partial}{\partial \theta}\left(\frac{v_\theta}{\sin\theta}\right) - \frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)\right]$$

$$T^{2j}{}_{;j} = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\mu}{r}\frac{\partial v_r}{\partial \theta}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{2\mu}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right) + \left(\zeta - \frac{2}{3}\mu\right)\nabla\cdot v\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\left[\frac{3}{r}\frac{\partial v_r}{\partial \theta} + 3r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{2\cot\theta}{r}\left(\sin\theta\frac{\partial}{\partial \theta}\left(\frac{v_\theta}{\sin\theta}\right) - \frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)\right]$$

inθ $T^{3j}{}_{;j} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 T_{r\phi}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, T_{\theta\phi}\right) + \frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial\phi} + \frac{1}{r}T_{r\phi} + \frac{\cot\theta}{r}T_{\theta\phi}$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\mu\left(r\frac{\partial}{\partial r}(v_\phi/r) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right)\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\frac{v_\phi}{\sin\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right)\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[-p + \frac{2\mu}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{2\mu v_r}{r} + \frac{2\mu v_\theta\cot\theta}{r} + \eta\right]$$

$$+ \frac{\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right] + \frac{\mu}{r}\left[\cot\theta\left(\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\frac{v_\phi}{\sin\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right)\right]$$

$$= \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right] + \frac{2\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right] + \frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$+ \frac{\mu}{r}\left[\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] - \frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{2\mu}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{2\mu v_r}{r} + \frac{2\mu v_\theta\cot\theta}{r} + (\zeta - \tfrac{2}{3}\mu)\nabla\cdot\mathbf{v}\right]$$

$$+ \frac{\mu}{r}\left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$\frac{\mu}{r}$ term $= \frac{\mu}{r}\left[2\cancel{\mu} r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{2}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$

$$= \frac{\mu}{r}\left[3r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{3}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cos\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{2\cot\theta}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$T^{3j}{}_{;j} = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi} + \frac{\partial}{\partial r}\left[\mu r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right] + \frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{\mu\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{\mu}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{2\mu}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial\phi} + v_r + v_\theta\cot\theta\right) + (\zeta - \tfrac{2}{3}\mu)\nabla\cdot\mathbf{v}\right]$$

$$+ \frac{\mu}{r}\left[3r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{3}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cot\theta}{r}\left(\sin\theta\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right)\right]$$

26.10

$$\nabla\cdot \underset{\sim}{V} = V^i_{;i} = \frac{\partial v_r}{\partial r} + \frac{\partial}{\partial\theta}\left(\frac{v_\theta}{r}\right) + \frac{\partial}{\partial\phi}\left(\frac{v_\phi}{r\sin\theta}\right) + \frac{2v_r}{r} + \frac{\cot\theta\, v_\theta}{r}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{\cot\theta\, v_\theta}{r}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$T^{ij}_{;j} = \frac{\partial}{\partial r}\left[2\mu\frac{\partial v_r}{\partial r} - \frac{2\mu}{3}\left(\frac{\partial v_r}{\partial r} + \frac{2v_r}{r}\right)\right] + 4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

$$= \frac{\partial}{\partial r}\left[\frac{4}{3}\mu\frac{\partial v_r}{\partial r} - \frac{4}{3}\frac{\mu v_r}{r}\right] + 4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

$$= \frac{\partial}{\partial r}\left[\frac{4\mu r}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] + 4\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

$$= \frac{\partial}{\partial r}\left[\frac{4\mu r}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{4\mu r^3}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - \frac{r^3}{r^2}\frac{\partial}{\partial r}\left[\frac{4}{3}\mu\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$

$$= \frac{\partial}{\partial r}\left[\frac{4\mu r}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{4\mu r^3}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - r\frac{\partial}{\partial r}\left[\frac{4\mu}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$

but $$\frac{\partial}{\partial r}\left[\frac{4\mu r}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] - r\frac{\partial}{\partial r}\left[\frac{4\mu}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] = \frac{4\mu}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)$$

and $$\frac{4\mu}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{4\mu r^3}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right] = \frac{1}{r^3}\frac{\partial}{\partial r}\left[\frac{4\mu r^4}{3}\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$

27.30

$$\Phi = 2\mu\, E_{ij} E^{ij} + (\varsigma - \tfrac{2}{3}\mu)\,(\nabla\cdot \underset{\sim}{v})^2$$

$$\Phi = 2\mu\left[E_{rr}E^{rr} + E_{\theta\theta}E^{\theta\theta} + E_{\phi\phi}E^{\phi\phi} + 2E_{r\theta}E^{r\theta} + 2E_{r\phi}E^{r\phi} + 2E_{\theta\phi}E^{\theta\phi}\right]$$

$$+ (\varsigma - \tfrac{2}{3}\mu)(\nabla\cdot \underset{\sim}{v})^2$$

$$= 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right)^2 + \frac{1}{2}\left(\frac{1}{r}\frac{\partial v_r}{\partial\theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)^2\right.$$

$$\left. + \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r}\right)^2 + \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} + \frac{1}{r}\frac{\partial v_\phi}{\partial\theta} - \frac{v_\phi\cot\theta}{r}\right)^2\right\}$$

$$+ (\varsigma - \tfrac{2}{3}\mu)\left(\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{2v_r}{r} + \frac{\cot\theta}{r}v_\theta\right)^2$$

$$\Phi = 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right)^2 + \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial\theta}\right]^2\right.$$

$$\left. + \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\theta} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2 + \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} + \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)\right]^2\right\}$$

$$+ (\varsigma - \tfrac{2}{3}\mu)\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right]^2$$

1-D, $\varsigma = 0$ $\left(\frac{\partial}{\partial\theta} = \frac{\partial}{\partial\phi} = 0,\ v_\phi = v_\theta = 0 \quad \varsigma = 0\right)$

$$\Phi = 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{v_r}{r}\right)^2 + \left(\frac{v_r}{r}\right)^2\right\} - \frac{2}{3}\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right]^2$$

$$= 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{2v_r^2}{r^2}\right\} - \frac{2}{3}\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right]^2 = \{\ \} - \frac{2}{3}\mu\left[\frac{\partial v_r}{\partial r} + \frac{2v_r}{r}\right]^2$$

$$= 2\mu\left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{4\mu v_r^2}{r^2} - \frac{2}{3}\mu\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + 4\left(\frac{v_r}{r}\right)^2 + \frac{4v_r}{r}\frac{\partial v_r}{\partial r}\right]$$

$$= \frac{4}{3}\mu\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{4}{3}\mu\frac{v_r^2}{r^2}\right) - 2\cdot\frac{4}{3}\mu\frac{v_r}{r}\frac{\partial v_r}{\partial r} = \frac{4\mu}{3}\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{v_r^2}{r^2} - \frac{2v_r}{r}\frac{\partial v_r}{\partial r}\right] = \frac{4\mu}{3}\left[r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]^2$$

27.33

$$(\rho e+\tfrac{1}{2}\rho v^2)_{,t}+\left[\rho(e+v^2/2)v^j - v_i T^{ij}+q^j\right]_{;j}=v_i f^i$$

for spherical flow with $\zeta=0$:

$$q^r_{;r}=-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K\frac{\partial T}{\partial r}\right)$$

$$\left[\rho(e+v^2/2)v^r\right]_{;r}=\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 v_r\rho(e+v^2/2)\right]$$

$$v_r T^{11}=-v_r p+2\mu v_r\frac{\partial v_r}{\partial r}-\frac{2}{3}\mu v_r\left(\frac{\partial v_r}{\partial r}+2\frac{v_r}{r}\right)$$

$$=-pv_r+\frac{4}{3}\mu v_r\frac{\partial v_r}{\partial r}-\frac{4}{3}\mu v_r\cdot\frac{v_r}{r}$$

$$=-pv_r+\frac{4}{3}\mu v_r\left[\frac{\partial v_r}{\partial r}-\frac{v_r}{r}\right]=-pv_r+\frac{4}{3}\mu v_r\left[r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]$$

$$(v_r T^{11})_{;r}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r T^{11})$$

hence

$$\frac{\partial}{\partial t}(\rho e+\tfrac{1}{2}\rho v^2)+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2 v_r\rho(e+v^2/2)+r^2 v_r\left[+p-\frac{4}{3}\mu r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]-r^2K\frac{\partial T}{\partial r}\right\}=v_r f_r$$

27.36

$$F=\cancel{}-\frac{GM\rho}{r^2}=-g\rho$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\rho v_r\left[e+\frac{p}{\rho}-\frac{v^2}{2}-\frac{4}{3}\nu r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]-r^2K\frac{dT}{dr}\right\}=-v_r\frac{GM\rho}{r^2}$$

$$\frac{\partial}{\partial r}\left\{\underbrace{(4\pi\rho r^2 v_r)}_{\dot m}\left[h-\frac{v^2}{2}-\frac{4}{3}\nu r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]-4\pi r^2K\frac{dT}{dr}\right\}=-v_r g\rho\cdot 4\pi r^2$$

$$=-\dot m\,GM/r^2$$

$$\Rightarrow \dot m\left[h-\frac{v^2}{2}-\frac{4}{3}\nu r\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)\right]-4\pi r^2K\frac{dT}{dr}=-\dot m GM\int dr/r^2$$

$$=+\frac{\dot m GM}{r}+\text{constant}$$

✓

32, 32

32.29: $\mathcal{D} f_0 = \frac{\partial f_0}{\partial N}\frac{DN}{Dt} + \frac{\partial f_0}{\partial v^i}\frac{Dv^i}{Dt} + \frac{\partial f_0}{\partial T}\frac{DT}{Dt} + U^i\left[\frac{\partial f_0}{\partial N}\frac{\partial N}{\partial x^i} + \frac{\partial f_0}{\partial v^j}\frac{\partial v^j}{\partial x^i} + \frac{\partial f_0}{\partial T}\frac{\partial T}{\partial x^i}\right] + a^i\frac{\partial f_0}{\partial u^i}$

Use $\frac{\partial f_0}{\partial N} = \frac{f_0}{N}$

$\frac{\partial f_0}{\partial v^i} = \left(\frac{m}{kT}\right) U^i f_0$

$\frac{\partial f_0}{\partial u^i} = -\frac{mU^i}{kT} f_0$

$\frac{\partial f_0}{\partial T} = \left[\frac{mU^2}{2kT} - \frac{3}{2}\right]\frac{f_0}{T}$

$\frac{DN}{Dt} = -Nv^i{}_{;i}$ $\qquad \frac{Dv_i}{Dt} = a_i - \frac{1}{\rho}P_{;i}$ $\qquad \begin{cases} \Rightarrow \frac{DT}{Dt} = -\left(\frac{2m}{3k}\right)\frac{P}{\rho}v^i{}_{;i} \\ \frac{De}{Dt} = \left(\frac{3k}{2m}\right)\left(\frac{DT}{Dt}\right) = -\frac{P}{\rho}v^i{}_{;i} \end{cases}$

$\mathcal{D}f_0 = \frac{f_0}{N}\left(-Nv^i{}_{;i}\right) + \left(\frac{m}{kT}\right)U^i f_0\left(a_i - \rho^{-1}P_{;i}\right) + \left[\frac{mU^2}{2kT} - \frac{3}{2}\right]\frac{f_0}{T}\left(-\frac{2m}{3k}\frac{P}{\rho}\right)v^i{}_{;i}$

$+ U^i\left[\frac{f_0}{\cancel{N}}\left(\frac{P_{;i}}{P} - \frac{T_{;i}}{T}\right) + f_0\left(\frac{m}{kT}\right)U^j v^j{}_{;i} + \left(\frac{mU^2}{2kT} - \frac{3}{2}\right)\frac{f_0}{T}T_{;i}\right] - \frac{mU^i}{kT}f_0 a^i$

$f_0^{-1}\mathcal{D}f_0 = -v^i{}_{;i} + \cancel{\frac{mU^i}{kT}a_i} - \frac{mU^i}{kT\rho}P_{;i} - \left[\frac{mU^2}{2kT} - \frac{3}{2}\right]\frac{2}{3}\left(\frac{P}{\rho m^{-1}kT} = 1\right)v^i{}_{;i}$

$+ U^i\left[\frac{P_{;i}}{P} - \frac{T_{;i}}{T}\right] + \frac{m}{kT}U^i U^j v^j{}_{;i} + \left[\frac{mU^2}{2kT} - \frac{3}{2}\right]\frac{T_{;i}}{T}U^i - \cancel{\frac{mU^i a^i}{kT}}$

$= -\cancel{v^i{}_{;i}} - \frac{mU^i}{\rho kT}P_{;i} - \frac{mU^2}{3kT}v^i{}_{;i} + \cancel{v^i{}_{;i}} + U^i\frac{P_{;i}}{P} - U^i\frac{T_{;i}}{T} + \frac{mU^j}{kT}v^j{}_{;i} + \frac{mU^2}{2kT}\frac{T_{;i}}{T} - \frac{3}{2}\frac{T_{;i}}{T}$

$\frac{m}{\rho kT} = \frac{1}{P}$ $\quad = -\cancel{\frac{U^i P_{;i}}{P}} - \frac{mU^2}{3kT}v^i{}_{;i} + \cancel{\frac{U^i P_{;i}}{P}} - \frac{U^i T_{;i}}{T} + \frac{mU^i U^j}{kT}v^j{}_{;i} + \frac{mU^2 U^i}{2kT}\frac{T_{;i}}{T} - \frac{3}{2}\frac{T_{;i}}{T}U^i$

$= -\frac{mU^2}{3kT}v^i{}_{;i} + \frac{mU^i U^j}{kT}v^j{}_{;i} - \frac{U^i T_{;i}}{T} - \frac{3}{2}U^i\frac{T_{;i}}{T} + \frac{mU^2U^i}{2kT}\frac{T_{;i}}{T}$

$$f_0^{-1}\mathcal{D}f_0 = \left(-\frac{mU^2}{3kT}\delta^i_j + \frac{mU^iU^j}{kT}\right)V^j_{;i} + \left(-\frac{5}{2}U^i + \frac{mU^2U^i}{2kT}\right)\frac{T_{;i}}{T}$$

$$= \frac{m}{kT}\left\{-\frac{1}{3}U^2\delta^i_j + U^iU^j\right\}V^j_{;i} + \left(-\frac{5}{2} + \frac{mU^2}{2kT}\right)U^i(\ln T)_{;i}$$

32.36 & 32.37

$$\mathcal{J}(\phi_1) = -\frac{1}{N^2}\iint [\phi_1(u') + \phi_1(u_1') - \phi_1(u) - \phi_1(u_1)]\, f_0 f_0\, g\sigma\, d\Omega\, d^3u_1$$

$$J(\phi_1) = \iint [\quad] \ldots \qquad \mathcal{J}(\phi_1) = -\frac{1}{N^2} J(\phi_1) \qquad (32.22)$$

$$-N^2 \mathcal{J}(\Phi_1) = \iint [\phi_1(\underset{\sim}{u}') + \phi_1(\underset{\sim}{u}_1') - \phi_1(\underset{\sim}{u}) - \phi_1(\underset{\sim}{u}_1)]\, f_0(\underset{\sim}{u}) f_0(\underset{\sim}{u}_1)\, g\sigma(\underset{\sim}{\Omega})\, d\Omega\, d^3u_1 = \mathcal{D}f_0$$

Where $f_0 \equiv N\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-mu^2/2kT\right)$ (32.11)

(32.33)

now $$\mathcal{D}f_0 = \left\{U_i\left[\frac{mU^2}{2kT} - \frac{5}{2}\right](\ln T)_{,i} + \left(\frac{m}{kT}\right)\left(U_iU_j - \frac{1}{3}U^2\delta_{ij}\right)V_{i,j}\right\} f_0$$
$$= -N^2\mathcal{J}(\phi_1)$$

(see notes for check of derivation)

We assume trial form for $\phi_1(\underset{\sim}{u})$:

$$\Phi_1 = -N^{-1}\left[\left(\frac{2kT}{m}\right)^{1/2} A_i\, (\ln T)_{,i} + B_{ij} D_{ij} + \Psi\right] \qquad (33.35)$$

And substitute this into

$$\mathcal{D}f_0 = -N^2\mathcal{J}(\Phi_1)$$

$$-N^2\mathcal{J}(\Phi_1) = -N^{-1}\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\iint [A_i(\underset{\sim}{u}') + A_i(\underset{\sim}{u}_1') - A_i(\underset{\sim}{u}) - A_i(\underset{\sim}{u}_1)]\, f_0(\underset{\sim}{u}) f_0(\underset{\sim}{u}_1)\cdot g\sigma\, d\Omega\, d^3u_1$$
$$- N^{-1}D_{ij}\iint [B_{ij}(\underset{\sim}{u}') + B_{ij}(\underset{\sim}{u}_1') - B_{ij}(\underset{\sim}{u}) - B_{ij}(\underset{\sim}{u}_1)]\, f_0(\underset{\sim}{u}) f_0(\underset{\sim}{u}_1)\, g\sigma\, d\Omega\, d^3u_1$$
$$- N^{-1}\iint [\Psi(\underset{\sim}{u}') + \Psi(\underset{\sim}{u}_1') - \Psi(\underset{\sim}{u}) - \Psi(\underset{\sim}{u}_1)]\, f_0(\underset{\sim}{u}) f_0(\underset{\sim}{u}_1)\, g\sigma\, d\Omega\, d^3u_1$$

or $$-N^2\mathcal{J}(\Phi_1) = -N^{-1}\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\, J(A_i) - N^{-1}D_{ij}\, J(B_{ij}) - N^{-1}J(\Psi)$$

where we have defined $J(\phi_1) = \iint [\phi_1(u') + \phi_1(u_1') - \phi_1(u) - \phi_1(u_1')] f_0(u) f_0(u_1) \ldots$
$= -N^2\mathcal{J}(\phi_1)$

Then $$-N^2\mathcal{J}(\phi_1) = -N^{-1}\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\left[-N^2\mathcal{J}(A_i)\right] - N^{-1}D_{ij}\left[-N^2\mathcal{J}(B_{ij})\right] - N^{-1}\left[-N^2\mathcal{J}(\Psi)\right]$$

Thus

$$-N^2 \mathcal{J}(\Phi_1) = +N\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\, \mathcal{J}(A_i) + N D_{ij}\, \mathcal{J}(B_{ij}) + N\, \mathcal{J}(4)$$

$$= \mathcal{D} f_0 = U_i\left[\frac{mU^2}{2kT} - \frac{5}{2}\right](\ln T)_{,i} f_0 + \left(\frac{m}{kT}\right)\left(U_i U_j - \frac{1}{3}U^2\delta_{ij}\right) D_{ij} f_0 + 0 \quad (32.33)$$

$\Rightarrow$

$$(1) \quad N\left(\frac{2kT}{m}\right)^{1/2}\cancel{(\ln T)_{,i}}\, \mathcal{J}(A_i) = U_i\left[\frac{mU^2}{2kT} - \frac{5}{2}\right]\cancel{(\ln T)_{,i}} f_0$$

$$(2) \quad N \cancel{D_{ij}}\, \mathcal{J}(B_{ij}) = \left(\frac{m}{kT}\right)\left(U_i U_j - \frac{1}{3}U^2\delta_{ij}\right)\cancel{D_{ij}} f_0$$

$$(3) \quad N\mathcal{J}(4) = 0$$

or

$$(1) \quad N\, \mathcal{J}(A_i) = \left(\frac{m}{2kT}\right)^{1/2} U_i\left[\frac{mU^2}{2kT} - \frac{5}{2}\right] f_0$$

$$(2) \quad N\mathcal{J}(B_{ij}) = \left(\frac{m}{kT}\right)\left(U_i U_j - \frac{1}{3}U^2\delta_{ij}\right) f_0 = 2\left(\frac{m}{2kT}\right)\left(U_i U_j - \frac{1}{3}U^2\delta_{ij}\right) f_0$$

$$(3) \quad \mathcal{J}(4) = 0$$

Rescale the velocities; define $\mathcal{U}_i = \left(\frac{m}{2kT}\right)^{1/2} U_i$, $\mathcal{U} = \left(\frac{m}{2kT}\right)^{1/2} U$, (32.39)
then

$$(1) \quad N\, \mathcal{J}(A_i) = \mathcal{U}_i\left[\mathcal{U}^2 - \frac{5}{2}\right] f_0 \quad (32.36)$$

$$(2) \quad N\, \mathcal{J}(B_{ij}) = 2\left(\mathcal{U}_i \mathcal{U}_j - \frac{1}{3}\mathcal{U}^2\delta_{ij}\right) f_0 \quad (32.37)$$

$$(3) \quad \mathcal{J}(4) = 0 \quad (32.38)$$

$$A_i = A(\mathcal{U},T)\mathcal{U}_i \qquad (32.40)$$

$$\text{or } \underset{\sim}{A} = A(\mathcal{U},T)\,\underset{\sim}{\mathcal{U}}$$

Now use (32.19)-(32.21): $\int f_i\, d^3u = 0$; $\int \underset{\sim}{u} f_i\, d^3u = 0$; $\int U^2 f_i\, d^3u = 0$
and $f_1 = f_0\phi_1$, to obtain:

$$\int f_0\phi_1\, d^3U = 0 \qquad (32.43)$$

$$\int U_i f_0\phi_1\, d^3U = 0 \qquad (32.44)$$

$$\int U^2 f_0\phi_1\, d^3U = 0 \qquad (32.45)$$

From (32.40)-(32.41) + (32.35)

$$\phi_1 = -N^{-1}\left\{\left(\frac{2kT}{m}\right)^{1/2} A(\mathcal{U},T)\,\mathcal{U}_i\,(\ln T)_{,i} + D_{ij} : B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}) + \alpha_1 + \underset{\sim}{\alpha}_2\cdot\underset{\sim}{U} + \alpha_3 U^2\right\}$$

Integral $\int f_0\Phi_1 d^3U$ becomes:

$$0 = \cancel{-N^{-1}}\int f_0\left[\left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\, A(\mathcal{U},T)\mathcal{U}_i + D_{ij}\, B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}) + \alpha_1 + \underset{\sim}{\alpha}_2\cdot\underset{\sim}{U} + \alpha_3 U^2\right] d^3U$$

$$= \left(\frac{2kT}{m}\right)^{1/2}(\ln T)_{,i}\int f_0\, A(\mathcal{U},T)\,\mathcal{U}_i\, d^3\mathcal{U} + D_{ij}\int B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}) f_0\, d^3\mathcal{U}$$

$$+ \alpha_1\int f_0\, d^3U + \underset{\sim}{\alpha}_2\cdot\int \underset{\sim}{U} f_0\, d^3U + \alpha_3\int f_0 U^2 d^3U$$

$$= (\ln T)_{,i}\int \cancelto{0}{f_0 A(\mathcal{U},T) U_i}\, d^3U + D_{ij}\int \cancelto{0}{B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}) f_0}\, d^3U$$

$$+ \int\alpha_1 f_0\, d^3U + \int \underset{\sim}{\alpha}_2\cdot\underset{\sim}{U} f_0\, d^3U + \int\alpha_3 f_0 U^2 d^3U$$

$$0 = \int(\alpha_1 + \alpha_3 U^2) f_0\, d^3U \qquad (32.46)$$

Integral $\int U_i f_0 \Phi_1 \, d^3U = 0$ becomes:

$$0 = \cancel{N^{-1}} \Big\{ \int d^3U f_0 \left(\frac{2kT}{m}\right)^{1/2} A(\mathcal{U},T)\,\mathcal{U}_i (\ln T)_{,i}\, U_i + \int d^3U f_0 D_{ij} \cdot B(\mathcal{U},T) \overbrace{\cancel{(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}U^2\delta_{ij})U_i}}^{0}$$

$$+ \int d^3U f_0 (\alpha_1 + \underset{\sim}{\alpha_2} \cdot \underset{\sim}{U} + \alpha_3 U^2) U_i \Big\}$$

$$= \int d^3U f_0\, U_i^2 A(\mathcal{U},T)(\ln T)_{,i} + \int d^3U f_0 (\alpha_1 + \alpha_{2j}U_j + \alpha_3 U^2) U_i$$

2nd integral is non-zero only when $j = i$, hence

$$0 = (\ln T)_{,i} \int A(\mathcal{U},T)\, U_i^2 f_0 \, d^3U + \alpha_{2i} \int U_i^2 f_0 \, d^3U$$

$0 = $ ~~[illegible]~~

$$0 = \int [\alpha_{2i} + A(\mathcal{U},T)(\ln T)_{,i}] f_0 U^2 d^3U \qquad (32.47)$$

Integral $\int U_i^2 f_0 \Phi_1 \, d^3U = 0$ becomes:

$$0 = \cancel{N^{-1}} \Big\{ \int d^3U f_0 \left(\frac{2kT}{m}\right)^{1/2} A(\mathcal{U},T)\,\mathcal{U}_i (\ln T)_{,i}\, U_i^2 + \int d^3U f_0 D_{ij} \cdot B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}U^2\delta_{ij})U_i^2$$

$$+ \int d^3U f_0 (\alpha_1 + \underset{\sim}{\alpha_2} \cdot \underset{\sim}{U} + \alpha_3 U^2) U^2 \Big\}$$

$$= \int d^3U f_0 A(\mathcal{U},T) \overbrace{\cancel{(\ln T)_{,i} U_i^3}}^{0} + \int d^3U f_0 D_{ij} B(\mathcal{U},T) \overbrace{\cancel{(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}U^2\delta_{ij})U_i^2}}^{0}$$

$$+ \int [\alpha_1 f_0 U^2 + f_0 \alpha_{2i} U_i U^2 + \alpha_3 f_0 U^4] d^3U$$

$$0 = \int [\alpha_1 U^2 + \alpha_3 U^4] f_0 \, d^3U \qquad (32.48)$$

$$q_k = \frac{m}{2}\int U_k U^2 f_0 \Phi_1 d^3U$$

$$= -\frac{m}{N2}\int f_0 d^3U\, U_k U^2 \left\{ \left(\frac{2kT}{m}\right)^{1/2} A(\mathcal{U},T)\mathcal{U}_i (\ln T)_{,i} + D_{ij} B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij}) \right\}$$

$$= -\frac{m}{2N}\int f_0 d^3U\, U^2 U_k \left\{ A(\mathcal{U},T)(\ln T)_{,i} U_i + \cancel{D_{ij} B(\mathcal{U},T)(\mathcal{U}_i\mathcal{U}_j - \tfrac{1}{3}\mathcal{U}^2\delta_{ij})} \right\}$$

$$= -\frac{m}{2N}(\ln T)_{,i}\int f_0 d^3\mathcal{U}\, A(\mathcal{U},T) U_i U_k U^2$$

Now

$$f_0 = N\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(-mu^2/2kT\right) = N\left(\frac{m}{2kT}\right)^{3/2}\pi^{-3/2}\exp\left[-\mathcal{U}^2\right]$$

and $f_0\, d^3U = N\pi^{-3/2} e^{-\mathcal{U}^2} d^3\mathcal{U} \equiv N f_0\, d^3\mathcal{U}$

and thus $f_0 \equiv \pi^{-3/2} e^{-\mathcal{U}^2}$ (32.52)

Then

$$q_k = -\frac{m}{2N}\cdot N\left[\int f_0 d^3\mathcal{U}\, A(\mathcal{U},T) U^2 U_i U_k\right](\ln T)_{,i}$$

$$= -\frac{m}{2}(\ln T)_{,i}\int f_0 d^3\mathcal{U} A(\mathcal{U},T) U_i U_k U^2$$

$U^2 = \left(\frac{2kT}{m}\right)\mathcal{U}^2$

now scale U_i and U_k and U^2 by $\left(\frac{m}{2kT}\right)^{1/2}$ $\quad U_i U_k = \left(\frac{2kT}{m}\right)\left(\frac{m}{2kT}\right)U_i U_k = \left(\frac{2kT}{m}\right)\mathcal{U}_i\mathcal{U}_k$

$$q_k = -\frac{2(kT)^2}{m}(\ln T)_{,i}\int f_0\, d^3\mathcal{U}\, A(\mathcal{U},T)\mathcal{U}_i\mathcal{U}_k\mathcal{U}^2$$

$$= -\frac{2k^2T}{m} T_{,i}\int f_0 d^3\mathcal{U}\, A(\mathcal{U},T)\mathcal{U}_i\mathcal{U}_k\mathcal{U}^2$$

$$= -\frac{2k^2T}{m} T_{,i}\ \delta_{ik}\int f_0 d^3\mathcal{U} A(\mathcal{U},T)\mathcal{U}_i\mathcal{U}_k\mathcal{U}^2$$

$$q_k = -\left[\frac{2k^2T}{m}\int f_0\, d^3\mathcal{U}\, A(\mathcal{U},T)\mathcal{U}_i^2\mathcal{U}^2\right] T_{,k} \qquad (32.53)$$

U_i $\quad d(\cos\theta) = -\sin\theta\, d\theta \quad \cos^2\theta\, d(\cos\theta) = \frac{\cos^3\theta}{3}$

$$\int \mathcal{U}_\ell^{\,2}\,\mathcal{U}^2 A(\mathcal{U},T)\, f_0(\mathcal{U})\,\mathcal{U}^2 d\mathcal{U}\,\sin\theta\, d\theta\, d\phi$$

and choose $\mathcal{U}_\ell$ so that $\mathcal{U}_\ell = \mathcal{U}\cos\phi$, then

$$\int \mathcal{U}^4 A(\mathcal{U},T)\, f_0(\mathcal{U})\,\mathcal{U}^2 d\mathcal{U} \int \cos^2\theta \sin\theta\, d\theta\, d\varphi$$

$$= \left(\frac{2}{3}\right) 2\pi \int \mathcal{U}^6 A(\mathcal{U},T)\, f_0(\mathcal{U})\, d\mathcal{U} = \frac{4}{3}\pi \int \mathcal{U}^6 A(\mathcal{U},T)\, f_0(\mathcal{U})\, d\mathcal{U}$$

and the thermal conductivity K is :

$$K = \frac{2k^2T}{m}\cdot\int = \frac{8}{3}\frac{\pi k^2 T}{m}\int \mathcal{U}^6 A(\mathcal{U},T)\, f_0(\mathcal{U})\, d\mathcal{U} \;; \qquad (32.54)$$

$$q_k = -K\, T_{,k}$$

$$\sigma_{ij}' = -m \int U_i U_j f_0 \Phi_1\, d^3U$$

$$= -m \int U_i U_j \left[\cancel{N}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\mathcal{U}^2}\right]\left[-N^{-1} D_{k\ell} B(\mathcal{U},T)\left(\mathcal{U}_k\mathcal{U}_\ell - \tfrac{1}{3}\mathcal{U}^2\delta_{k\ell}\right)\right] d^3U$$

$$= +m \int U_i U_j \left(\pi^{-3/2} e^{-\mathcal{U}^2}\right) d^3\mathcal{U}\, D_{k\ell}\left(\mathcal{U}_k\mathcal{U}_\ell - \tfrac{1}{3}\mathcal{U}^2\delta_{k\ell}\right) B(\mathcal{U},T)$$

$$= +m\left(\frac{2kT}{m}\right)\int f_0\, d^3\mathcal{U}\, \mathcal{U}_i\mathcal{U}_j\left(\mathcal{U}_k\mathcal{U}_k - \tfrac{1}{3}\cancel{\mathcal{U}^2\delta_{k\ell}}^{\,0}\right) D_{k\ell}\, B(\mathcal{U},T)$$

$$= 2kT \int f_0\, d^3\mathcal{U}\, \mathcal{U}_i^{\,2}\mathcal{U}_j^{\,2}\, 2D_{ij}'\, B(\mathcal{U},T)$$

$$\mu = 2kT \int f_0\, d^3\mathcal{U}\, \mathcal{U}_i^{\,2}\mathcal{U}_j^{\,2}\, B(\mathcal{U},T)$$

$(\cancel{1+\cos^2\theta})\ \sin\theta\, d\theta\, d\phi$

$\cos^2\theta\,(1-\cos^2\theta)\sin\theta\, d\theta$

$= [\cos^2\theta - \cos^4\theta]\sin\theta\, d\theta$

$= [\cos^2 - \cos^4)\, d(\cos\theta)] = \frac{\cos^3}{3} - \frac{\cos^5}{5}$

$= \frac{2}{3} - \frac{2}{5}$

$= \frac{10-6}{15} = \frac{4}{15}$

let $U_i = U\cos\theta \quad U_j = U\sin\theta\cos\phi \quad d^3U = U^2\, dU \sin\theta\, d\theta\, d\phi$

then

$$\mu = 2kT \int f_0 B(U,T)\, U^2 \cdot U^4\, dU \int \cos^2\theta \sin^2\theta \sin\theta\, d\theta \int \cos^2\phi\, d\phi$$

$$= 2kT \int f_0(U)\, B(U,T)\, U^6\, dU \underbrace{\int \cos^2\theta \sin^3\theta\, d\theta}_{4/15} \underbrace{\int \cos^2\phi\, d\phi}_{\pi}$$

$$= \frac{8\pi kT}{15} \int f_0(U)\, B(U,T)\, U^6\, dU \qquad (32.58)$$

$$\delta(F I(F)) = \delta F\, I(F) + F\, I(\delta F) \qquad = C\left[\int d^3\mathcal{U}\, F\,(2G - \mathcal{J}(F))\right]$$

$$H(= K \text{ or } \mu) = C\left[2\int F G\, d^3\mathcal{U} - \int F\, \mathcal{J}(F)\, d^3\mathcal{U}\right]$$

F is the solution of $\mathcal{J}(F) = G$

Variation of H with small variations δF of F:

$$\delta H = 2C\int \delta F\,[G - \mathcal{J}(F)]\, d^3\mathcal{U} \qquad (33.3)$$

$$K = \frac{2k^2T}{3\cancel{n}m}\int u^2\, \mathcal{U}^2 A(\mathcal{U},T)\, f_0(\mathcal{U})\, d^3\mathcal{U}$$
$$- \frac{5}{2}\int \mathcal{U}^2 A(\mathcal{U},T)\, f_0(\mathcal{U})\, d^3\mathcal{U}\left(\frac{2k^2T}{3m}\right)$$

$$\cancel{K = \int \mathcal{U}^2 d^3\mathcal{U}\, A(\mathcal{U},T)\, f_0(\mathcal{U})\left[\frac{2k^2T}{m}u_e^2 - \frac{5}{2}\right]}$$

$$K = \left(\frac{2k^2T}{3m}\right)\int A(\mathcal{U},T)\, \mathcal{U}^2 f_0(u)\,[\mathcal{U}^2 - 5/2]\, d^3\mathcal{U}$$

$$= \left(\frac{2k^2T}{3m}\right)\int A_i\, \mathcal{U}_i\, f_0(\mathcal{U})\,[\mathcal{U}^2 - 5/2]\, d^3\mathcal{U} \qquad (33.10)$$

now (32.36) $N\mathcal{J}(A_i) = \mathcal{U}_i(\mathcal{U}^2 - 5/2) f_0 = \mathcal{U}_i(\mathcal{U}^2 - 5/2) \cdot N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\mathcal{U}^2}$

$$= \mathcal{U}_i(\mathcal{U}^2 - 5/2)\, N\left(\frac{m}{2kT}\right)^{3/2}\left(\pi^{-3/2} e^{-\mathcal{U}^2}\right)$$

$$= \mathcal{U}_i(\mathcal{U}^2 - 5/2)\, N\left(\frac{m}{2kT}\right)^{3/2} f_0(u)$$

hence

$$\mathcal{J}(A_i) = \left(\frac{m}{2kT}\right)^{3/2} f_0(u)\, \mathcal{U}_i(\mathcal{U}^2 - 5/2)$$

$$\Longrightarrow \mathcal{U}_i f_0(\mathcal{U})(\mathcal{U}^2 - 5/2) = \left(\frac{2kT}{m}\right)^{3/2} \mathcal{J}(A_i)$$

and therefore

$$K = \left(\frac{2k^2T}{3m}\right) \int A_i \left(\frac{2kT}{m}\right)^{3/2} \mathcal{I}(A_i)\, d^3\mathcal{U}$$

no! nondimensionalizing $\mathcal{I}(A_0)$ gives a factor that cancels the $\left(\frac{m}{2kT}\right)^{3/2}$, hence

$$\mathcal{I}(A_i) = \mathcal{U}_i(\mathcal{U}^2 - 5/2)\, f_0(\mathcal{U})$$

and

$$K = \frac{2k^2T}{3m}\left\{2\int A_i\, \mathcal{U}_i\, (\mathcal{U}^2 - 5/2) f_0(\mathcal{U})\, d^3\mathcal{U} - \int A_i \mathcal{I}(A_i)\, d^3\mathcal{U}\right\}$$

$$= C\left[\int F G\, d^3\mathcal{U} - \int F \mathcal{I}(F)\, d^3\mathcal{U}\right]$$

with $F = A_i$, $G = \mathcal{U}_i(\mathcal{U}^2 - 5/2) f_0$, $C = \frac{2k^2T}{3m}$

Viscosity μ:

$$\mu = (8\pi kT/15)\int B(\mathcal{U},T)\, \mathcal{U}^6 f_0(\mathcal{U})\, d\mathcal{U}$$

$$= \frac{1}{4\pi}\left(\frac{8\pi kT}{15}\right)\int B(\mathcal{U},T)\, \mathcal{U}^4 f_0(\mathcal{U})\, d^3\mathcal{U}$$

$$= (2kT/15)\int B(\mathcal{U},T)\, \mathcal{U}^4 f_0(\mathcal{U})\, d^3\mathcal{U}$$

Now $\mathcal{U}_\ell^0 \mathcal{U}_m \equiv \mathcal{U}_\ell \mathcal{U}_m - \frac{1}{3}\mathcal{U}^2 \delta_{\ell m}$ satisfies $(\mathcal{U}_\ell^0 \mathcal{U}_m)(\mathcal{U}_\ell^0 \mathcal{U}_m) = \frac{2}{3}\mathcal{U}^4$, hence

$$\mathcal{U}^4 = \frac{3}{2}(\mathcal{U}_\ell^\circ \mathcal{U}_m)(\mathcal{U}_\ell^\circ \mathcal{U}_m)$$

and

$$\mu = \frac{3}{2}\left(\frac{2kT}{15}\right)\int B(\mathcal{U},T)(\mathcal{U}_\ell^\circ \mathcal{U}_m)(\mathcal{U}_\ell^\circ \mathcal{U}_m) f_0(u)\,d^3\mathcal{U} \qquad (33.15a)$$

$$= (kT/5)\int B_{\ell m}(\mathcal{U}_\ell^\circ \mathcal{U}_m) f_0(u)\,d^3\mathcal{U} \qquad (33.15b)$$

Now $N\mathcal{I}(B_{\ell m}) = 2(\mathcal{U}_\ell^\circ \mathcal{U}_m)f_0 = N\mathcal{I}[B_{\ell m}(U)]$

$$\Rightarrow N\,\mathcal{I}(B_{\ell m}(u)) = 2(\mathcal{U}_\ell^\circ \mathcal{U}_m)Nf_0$$

hence

$$\mathcal{I}(B_{\ell m}(u)) = 2\,\mathcal{U}_\ell^\circ \mathcal{U}_m f_0$$

and

$$\mu = (kT/10)\int B_{\ell m}\,\mathcal{I}(B_{\ell m})\,d^3\mathcal{U} \qquad (33.15c)$$

Thus $\qquad (33.16)$

$$\mu = (kT/10)\left[2\int B_{\ell m}(2\mathcal{U}_\ell^\circ \mathcal{U}_m f_0)\,d^3\mathcal{U} - \int B_{\ell m}\,\mathcal{I}(B_{\ell m})\,d^3\mathcal{U}\right]$$

$$\mu = C\left[2\int F G\,d^3\mathcal{U} - \int F\,\mathcal{I}(F)\,d^3\mathcal{U}\right]$$

with $F = B_{\ell m}$, $G = 2\mathcal{U}_\ell^\circ \mathcal{U}_m f_0$, $C = kT/10$

$$H = C\left[2\alpha \int F_0 G\, d^3u - \alpha^2 \int F_0\, J(F_0)\, d^3u\right]$$

$$\alpha = \int F_0 G\, d^3u \Big/ \int F_0 J\, d^3u$$

$$\Rightarrow H = C\left[2\frac{\left(\int F_0 G\, d^3u\right)^2}{\int F_0 J\, d^3u} - \left(\frac{\int F_0 G\, d^3u}{\int F_0 J\, d^3u}\right)^2 \int F_0 J\, d^3u\right]$$

Calculate K:

$$= C\left(\int F_0 G\, d^3u\right)^2 \Big/ \int F_0 J\, d^3u$$

Trial function $F = \alpha F_0 \qquad F_0 = A_i^0 = A^0 u_i$

$$A^0 = (u^2 - 5/2) \qquad F_0 = u_i(u^2 - 5/2)$$

$$G = u_i(u^2-5/2) f_0$$

$$K_1 = C_K \frac{\left[\int u_i(u^2-5/2)\, u_i(u^2-5/2) f_0\, d^3u\right]^2}{\int u_i(u^2-5/2)\, J\left[u_i(u^2-5/2)\right] d^3u}$$

$$= \left(\frac{2k^2T}{3m}\right)\frac{\left(\int u^2(u^2-5/2)^2 f_0\, d^3u\right)^2}{\int u_i(u^2-5/2)\, J\left[u_i(u^2-5/2)\right] d^3u} = \frac{2k^2T}{3m}\cdot\frac{I_1^2}{I_2}$$

$$I_1 = \int u^2(u^2-5/2)^2 \pi^{-3/2} e^{-u^2}\cdot 4\pi u^2\, du$$

$$= (4\pi\,\pi^{-3/2})\int u^4(u^2-5/2)^2 e^{-u^2}\, du = 4\pi^{-1/2}\int\left[u^8 - 5u^6 + \tfrac{25}{4}u^4\right]e^{-u^2}\, du$$

15×
7
105

$$= (4/\sqrt{\pi})\left[\int u^8 e^{-u^2} du - 5\int u^6 e^{-u^2} du + \tfrac{25}{4}\int u^4 e^{-u^2} du\right] = \frac{4}{\sqrt{\pi}}\left\{\frac{7\cdot5\cdot3}{2(2)^4}\pi^{1/2} - 5\frac{5\cdot3}{2(2)^3}\pi^{1/2} + \frac{25}{4}\frac{3}{2(2)^2}\pi^{1/2}\right\}$$

$$= 4\left\{\frac{105}{32} - \frac{75}{16} + \frac{25}{32}\right\} = 4\left\{\frac{105}{32} - \frac{150}{32} + \frac{75}{32}\right\} = 4\left\{\frac{105-75}{32}\right\} = 4\left\{\frac{30}{32}\right\} = \frac{30}{8} = \frac{15}{4}$$

$$I_2 = \int$$

$$-N^2 \mathcal{J}(\Phi_1) = \iint \left[\phi_1(\mathbf{u}') + \phi_1(\mathbf{u}_1') - \phi_1(\mathbf{u}) - \phi_1(\mathbf{u}_1)\right]$$

$$\cdot f_0(\mathbf{u}) f_0(\mathbf{u}_1)\, g\, \sigma(\mathbf{\Omega})\, d\Omega\, d^3u_1$$

$$= \iint [\quad]\, N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-U^2} \cdot N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-U_1^2} \cdot \gamma\left(\frac{2kT}{m}\right)^{1/2} \sigma\, d\Omega \left(\frac{2kT}{m}\right)^{3/2} d^3U_1$$

$$= \iint [\quad] N^2 \left(\pi^{-3/2} e^{-U^2}\right)\left(\pi^{-3/2} e^{-U_1^2}\right) \cancel{\gamma_1 \left(\frac{2kT}{m}\right)^{1/2}} \left(\frac{m}{2kT}\right)^{1} \cancel{\left(\frac{m}{2kT}\right)^{3/2}} \cancel{\left(\frac{2kT}{m}\right)^{3/2}} \gamma\sigma\, d\Omega\, d^3U_1$$

$$-\cancel{N^2}\mathcal{J}(\Phi_1) = \cancel{N^2} \iint f_0(U) f_0(U_1) [\quad] \left(\frac{m}{2kT}\right) \gamma\sigma\, d\Omega\, d^3U_1$$

$$\mathcal{J}(\Phi_1) = \left(\frac{m}{2kT}\right) \iint \left[\phi_1(\mathbf{u}) + \phi_1(\mathbf{u}_1) - \phi_1(\mathbf{u}') - \phi_1(\mathbf{u}_1')\right] f_0(U) f_0(U_1)\, \gamma\sigma\, d\Omega\, d^3U_1$$

$$\phi_1(\mathbf{u}) = U_i(U^2 - 5/2) = \left(\frac{2kT}{m}\right)^{3/2} U_i (U^2 - 5/2)$$ (ok because 5/2 was a hokey in there anyway!)

$$\mathcal{J}(\Phi_1) = \left(\frac{2kT}{m}\right)^{1/2} \iint \left[\phi_1(\mathbf{U}) + \phi_1(\mathbf{U}_1) - \phi_1(\mathbf{U}') - \phi_1(\mathbf{U}_1')\right] f_0(U) f_0(U_1)\, \gamma\sigma\, d\Omega\, d^3U_1$$

Now

$$I_2 = \iint d^3U\, d^3U_1 \int \sigma(\mathbf{\Omega})\, d\Omega\, f_0(U) f_0(U_1)\, \gamma \,\cdot$$

$$(U^2 - 5/2)\mathbf{U} \cdot \left[\mathbf{U}(U^2 - 5/2) + \mathbf{U}_1(U_1^2 - 5/2) - \mathbf{U}'(U'^2 - 5/2) - \mathbf{U}_1'(U_1'^2 - 5/2)\right]$$

$$d^3u\, d^3u_1 = (4\pi\gamma^2 d\gamma)(4\pi\Gamma^2 d\Gamma)(d\omega_\gamma/4\pi)(d\omega_\Gamma/4\pi)$$

$$f_0(u)\, f_0(u_1) = \pi^{-3}\exp(-u^2-u_1^2) = \pi^{-3}\exp[-2\Gamma^2 - \tfrac{1}{2}\gamma^2]$$

$$\mathbf{u} = \boldsymbol{\Gamma} + \tfrac{1}{2}\boldsymbol{\gamma} \Rightarrow u^2 = \Gamma^2 + \tfrac{1}{4}\gamma^2 + \boldsymbol{\gamma}\cdot\Gamma$$

and $u^2 - 5/2 = \Gamma^2 + \gamma^2/4 - 5/2 + \boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}$

Drop $-5/2$ terms throughout square bracket and consider

$$\mathbf{u}\cdot[u^2\mathbf{u} + u_1^2\mathbf{u}_1 - u'^2\mathbf{u}' - u_1'^2\mathbf{u}_1']$$

$$= u^2\cdot u^2 + u_1^2\,\mathbf{u}\cdot\mathbf{u}_1 - u'^2\,\mathbf{u}\cdot\mathbf{u}' - u_1'^2\,\mathbf{u}\cdot\mathbf{u}_1'$$

$$= [(\boldsymbol{\Gamma}+\boldsymbol{\gamma}/2)\cdot(\boldsymbol{\Gamma}+\boldsymbol{\gamma}/2)]^2 + [(\boldsymbol{\Gamma}-\boldsymbol{\gamma}/2)\cdot(\boldsymbol{\Gamma}-\boldsymbol{\gamma}/2)(\boldsymbol{\Gamma}-\boldsymbol{\gamma}/2)\cdot(\boldsymbol{\Gamma}+\boldsymbol{\gamma}/2)$$

$$-(\boldsymbol{\Gamma}+\boldsymbol{\gamma}'/2)\cdot(\boldsymbol{\Gamma}+\boldsymbol{\gamma}'/2)(\boldsymbol{\Gamma}+\boldsymbol{\gamma}/2)\cdot(\boldsymbol{\Gamma}+\boldsymbol{\gamma}'/2)$$

$$-(\boldsymbol{\Gamma}-\boldsymbol{\gamma}'/2)\cdot(\boldsymbol{\Gamma}-\boldsymbol{\gamma}'/2)(\boldsymbol{\Gamma}+\boldsymbol{\gamma}/2)\cdot(\boldsymbol{\Gamma}-\boldsymbol{\gamma}/2)$$

$$= [\Gamma^2 + \boldsymbol{\gamma}\cdot\boldsymbol{\Gamma} + \gamma^2/4]^2 + [\Gamma^2 - \boldsymbol{\gamma}\cdot\Gamma + \gamma^2/4][\Gamma^2 - \gamma^2/4]$$

$$-[\Gamma^2 + \boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}' + \gamma'^2/4][\Gamma^2 + \boldsymbol{\Gamma}\cdot(\boldsymbol{\gamma}+\boldsymbol{\gamma}')/2 + \boldsymbol{\gamma}\cdot\boldsymbol{\gamma}'/4]$$

$$-[\Gamma^2 - \boldsymbol{\Gamma}\cdot\boldsymbol{\gamma} + \gamma'^2/4][\Gamma^2 + \boldsymbol{\Gamma}\cdot(\boldsymbol{\gamma}-\boldsymbol{\gamma}')/2 - \boldsymbol{\gamma}\cdot\boldsymbol{\gamma}'/4]$$

$$\cancel{= \Gamma^2(\Gamma^2+\gamma^2/4) + \Gamma^2(\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}) + \boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}[\Gamma^2+\gamma^2/4] + (\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma})^2} + \gamma^2/4\,(\Gamma^2 + \boldsymbol{\gamma}\cdot\Gamma + \gamma^2/4)$$

$$= [(\Gamma^2+\gamma^2/4) + \boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}][(\Gamma^2+\gamma^2/4) + \boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}] + [(\Gamma^2+\gamma^2/4) - \boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}][\Gamma^2 - \gamma^2/4]$$

$$* - [\Gamma^2 + \boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}' + \gamma'^2/4]\Gamma^2 \;*\; - [\Gamma^2 + \boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}' + \gamma'^2/4]\boldsymbol{\Gamma}\cdot(\boldsymbol{\gamma}+\boldsymbol{\gamma}') - [\Gamma^2 + \boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}' + \gamma'^2/4](\boldsymbol{\gamma}\cdot\boldsymbol{\gamma}'/4)$$

$$- [\Gamma^2 - \boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}' + \gamma'^2/4]\Gamma^2 - [\Gamma^2 + \boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}' + \gamma'^2/4]\boldsymbol{\Gamma}\cdot(\boldsymbol{\gamma}-\boldsymbol{\gamma}') + [\Gamma^2 - \boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}' + \gamma'^2/4](\boldsymbol{\gamma}\cdot\boldsymbol{\gamma}'/4)$$

$$u^2 \boldsymbol{u}^2 + u_1^2\, \boldsymbol{u}_1\cdot\boldsymbol{u} - u'^2\, \boldsymbol{u}\cdot\boldsymbol{u}' - u_1'^2\, \boldsymbol{u}\cdot\boldsymbol{u}_1'$$

$$= (\boldsymbol{\Gamma} + \tfrac{1}{2}\boldsymbol{\gamma})^2 (\boldsymbol{\Gamma} + \tfrac{1}{2}\boldsymbol{\gamma}) + (\boldsymbol{\Gamma} - \tfrac{1}{2}\boldsymbol{\gamma})^2 (\boldsymbol{\Gamma} - \tfrac{1}{2}\boldsymbol{\gamma})$$

$$- (\boldsymbol{\Gamma} + \tfrac{1}{2}\boldsymbol{\gamma}')^2 \cdot (\boldsymbol{\Gamma} + \tfrac{1}{2}\boldsymbol{\gamma}') - (\boldsymbol{\Gamma} - \tfrac{1}{2}\boldsymbol{\gamma}')^2 (\boldsymbol{\Gamma} - \tfrac{1}{2}\boldsymbol{\gamma}')$$

$$= (\cancel{\Gamma}^2 + \tfrac{1}{4}\gamma^2)\boldsymbol{\Gamma} + \cancel{(\Gamma^2 + \tfrac{1}{4}\gamma^2)(\tfrac{1}{2}\boldsymbol{\gamma})} + \cancel{(\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})\boldsymbol{\Gamma}} + (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})(\tfrac{1}{2}\boldsymbol{\gamma})$$

$$+ (\cancel{\Gamma}^2 + \tfrac{1}{4}\gamma^2)\boldsymbol{\Gamma} - \cancel{(\Gamma^2 + \tfrac{1}{4}\gamma^2)(\tfrac{1}{2}\boldsymbol{\gamma})} - \cancel{(\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})}\boldsymbol{\Gamma} + (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})(\tfrac{1}{2}\boldsymbol{\gamma})$$

$$- (\cancel{\Gamma}^2 + \tfrac{1}{4}\gamma'^2)\boldsymbol{\Gamma} - \cancel{(\Gamma^2 + \tfrac{1}{4}\gamma'^2)(\tfrac{1}{2}\boldsymbol{\gamma}')} - \cancel{(\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')\boldsymbol{\Gamma}} - (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')(\tfrac{1}{2}\boldsymbol{\gamma}')$$

$$- (\cancel{\Gamma}^2 + \tfrac{1}{4}\gamma'^2)\boldsymbol{\Gamma} + \cancel{(\Gamma^2 + \tfrac{1}{4}\gamma'^2)(\tfrac{1}{2}\boldsymbol{\gamma}')} + \cancel{(\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')}\boldsymbol{\Gamma} - (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')(\tfrac{1}{2}\boldsymbol{\gamma}')$$

$$= \boldsymbol{\Gamma}\,\cancel{(\gamma^2/2 - \gamma'^2/2)} + (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma} - (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')\boldsymbol{\gamma}'$$

$$(\boldsymbol{\Gamma} + \tfrac{1}{2}\boldsymbol{\gamma})\cdot\left\{\boldsymbol{\Gamma}\,\cancel{(\gamma^2/2 - \gamma'^2/2)} + (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma} - (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')\boldsymbol{\gamma}'\right\}$$

$$= \cancel{\Gamma^2(\gamma^2/2 - \gamma'^2/2)} + \tfrac{1}{4}\cancel{(\gamma^2 - \gamma'^2)}(\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}) + (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})^2 + \tfrac{1}{2}(\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})\gamma^2$$
$$- (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')^2 - \tfrac{1}{2}(\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')(\boldsymbol{\gamma}'\cdot\boldsymbol{\gamma})$$

$$= \cancel{\Gamma^2(\gamma^2 - \gamma'^2)/2} + (\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma})[\gamma^2/2 + \cancel{\gamma^2/4 - \gamma'^2/4}] + (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})^2 - (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')^2$$
$$- \tfrac{1}{2}\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}'(\boldsymbol{\gamma}'\cdot\boldsymbol{\gamma})$$

$$= (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma})^2 - (\boldsymbol{\Gamma}\cdot\boldsymbol{\gamma}')^2 + (\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma})\gamma^2/2 - \tfrac{1}{2}(\boldsymbol{\gamma}'\cdot\boldsymbol{\Gamma})(\boldsymbol{\gamma}'\cdot\boldsymbol{\gamma})$$

$$\cancel{+ \Gamma^2(\gamma^2 - \gamma'^2)/4 + (\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma})(\gamma^2/4 - \gamma'^2/4)}$$

Now:

$$I_2 = \left(\frac{2kT}{m}\right)^{1/2} \iint d^3u\, d^3u_1 \int \sigma\, d\Omega\, g\, f_0(u) f_0(u_1)\, (u^2 - 5/2)\, \underset{\sim}{u} \cdot \left[u^2 \underset{\sim}{u} + u_1^2 \underset{\sim}{u}_1 - u'^2 \underset{\sim}{u}' - u_1'^2 \underset{\sim}{u}_1'\right]$$

$$= \left(\frac{2kT}{m}\right)^{1/2} \iint \Gamma^2 d\Gamma\, d\omega_\Gamma \cdot \gamma^2 d\gamma\, d\omega_\gamma \int \sigma\, d\Omega\, \gamma\, \frac{e^{-(2\Gamma^2 + \frac{1}{2}\gamma^2)}}{\pi} \left[\cancelto{0}{\left(\Gamma^2 + \frac{\gamma^2}{4} - \frac{5}{2}\right)} + \underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma}\right] \Big\{ \cancelto{0}{(\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma})^2 - (\underset{\sim}{\gamma}' \cdot \underset{\sim}{\Gamma})^2}$$
$$+ \frac{\gamma^2}{2}\left[\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma} - (\underset{\sim}{\gamma}' \cdot \underset{\sim}{\Gamma}) \cos\chi\right]\Big\}$$

$$= \left(\frac{2kT}{m}\right)^{1/2} \iint \Gamma^2 d\Gamma\, d\omega_\Gamma\, \gamma^2 d\gamma\, d\omega_\gamma \int \sigma\, d\Omega\, \gamma\, \frac{e^{-(2\Gamma^2 + \frac{1}{2}\gamma^2)}}{\pi} \left\{(\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma})\right\} \frac{\gamma^2}{2} \left[\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma} - (\underset{\sim}{\gamma}' \cdot \underset{\sim}{\Gamma}) \cos\chi\right]$$

write $\underset{\sim}{\gamma}' = \cos\chi\, \underset{\sim}{\gamma} + \gamma \sin\chi\, \underset{\sim}{n}$

then $\underset{\sim}{\Gamma} \cdot \underset{\sim}{\gamma}' = \cos\chi\, \underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma} + \gamma \sin\chi\, \underset{\sim}{n} \cdot \underset{\sim}{\Gamma}$

$$(\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma}) \gamma^2 [\ \ ] = \frac{\gamma^2}{2} (\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma})^2 - \frac{\gamma^2}{2} \cos^2\chi\, (\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma})^2 - \frac{\gamma^3}{2} \sin\chi \cos\chi\, (\underset{\sim}{n} \cdot \underset{\sim}{\Gamma})(\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma})$$

$(\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma}) \gamma^2 [\ \ ]$

integral $\to 0$

$$= \frac{\gamma^2}{2} (\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma})^2 (1 - \cos^2\chi) - \cancel{\frac{\gamma^2}{2} \gamma \sin\chi \cos\chi\, (\underset{\sim}{n} \cdot \underset{\sim}{\Gamma})(\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma})}$$

$$I_2 = \left(\frac{2kT}{m}\right)^{1/2} \frac{2}{\pi^3} \iint (\Gamma^2 d\Gamma\, d\omega_\Gamma)(\gamma^2 d\gamma\, d\omega_\gamma) \int \gamma\, \sigma\, d\Omega\, e^{-(2\Gamma^2 + \frac{1}{2}\gamma^2)}\, \frac{\gamma^2}{2} (1 - \cos^2\chi) \underbrace{(\underset{\sim}{\gamma} \cdot \underset{\sim}{\Gamma})^2}_{\gamma^2 \Gamma^2 \cos^2\psi}$$

$$= \left(\frac{2kT}{m}\right)^{1/2} \frac{1}{2\pi^3} \iint \Gamma^2 d\Gamma\, d\omega_\Gamma \int d\omega_\gamma \int \sigma\, d\Omega\, e^{-(2\Gamma^2 + \frac{1}{2}\gamma^2)} \underbrace{\gamma^2 \cdot \gamma \cdot \gamma^2 \cdot \gamma^2 \Gamma^2 \cos^2\psi}\ \underbrace{(1 - \cos^2\chi)}_{\sin^2\chi}$$

$$= \frac{1}{2\pi^3} \left(\frac{2kT}{m}\right)^{1/2} \int d\gamma \left(\gamma^7 4\pi e^{-\gamma^2/2}\right) \int \sigma\, d\Omega \sin^2\chi \int \left[d\Gamma\, \Gamma^4 \exp(-2\Gamma^2)\right] \underbrace{\left(\int \frac{d\omega_\gamma}{4\pi}\right)}_{1} \int \frac{d\omega_\Gamma \cos^2\psi}{4\pi}$$

$$= \frac{1}{2\pi^3} \left(\frac{2kT}{m}\right)^{1/2} \int d\gamma \left(4\pi \gamma^7 e^{-\gamma^2/2}\right) \int \sigma(\gamma, \chi)\, d\Omega \sin^2\chi \int d\Gamma \left(4\pi \Gamma^4 e^{-2\Gamma^2}\right) \int \frac{d\omega_\Gamma \cos^2\psi}{4\pi}$$

$$d\omega_\Gamma = \sin\psi\, d\psi\, d\phi \Rightarrow \int \frac{\cos^2\psi \sin\psi\, d\psi\, d\phi}{4\pi} = \left[\frac{\cos^3\psi}{3}\right]_0^\pi \frac{(2\pi)}{4\pi} = \frac{1}{3}$$

$$\int_0^\infty x^{2n} e^{-x^2}dx = \frac{(2n-1)!!}{(2)^n\, 2}\sqrt{\pi} \qquad \int_0^\infty x^{2n} e^{-ax^2}dx = \frac{(2n-1)!!}{2(2a)^n}\sqrt{\pi/a}$$

$$\int_0^\infty x^{2n+1}e^{-x^2}dx = \frac{n!}{2}$$

$n=2$
$a=2$

$$\int 4\pi\, \Gamma^4 e^{-2\Gamma^2} d\Gamma = \frac{\cancel{4\pi}\,3}{2(4)^2}\sqrt{\pi/2} = \frac{3}{32}\sqrt{\frac{\pi}{2}}\cdot 4\pi = \frac{3}{8}\frac{\pi^{3/2}}{2^{1/2}}$$

So we have:

$$\int d\Gamma(\ )\cdot\int d\omega_\Gamma\cdot\int d\omega_\gamma = \frac{1}{8}\frac{\pi^{3/2}}{2^{1/2}}$$

$$\int d\Omega\, \sigma(\gamma,\chi)\sin^2\chi \equiv \sigma_{(2)}(\gamma)$$

hence

$$I_2 = \frac{1}{2\pi^3}\left(\frac{2kT}{m}\right)^{1/2}\int d\gamma\,(4\pi\gamma^7 e^{-\gamma^2/2})\cdot\sigma_{(2)}\gamma\;\frac{\pi^{3/2}}{8\cdot 2^{1/2}}$$

$$= \frac{\pi^{3/2}}{16\pi^3\sqrt{2}}\left(\frac{2kT}{m}\right)^{1/2}\int d\gamma\,\sigma_{(2)}(\gamma)(4\pi\gamma^7e^{-\gamma^2/2}) = \frac{1}{16\pi^{3/2}}\left(\frac{kT}{m}\right)^{1/2}\int d\gamma\,\sigma_{(2)}(\gamma)(4\pi\gamma^7 e^{-\gamma^2/2})$$

$$= \frac{4\pi}{16\pi^{3/2}}\left(\frac{kT}{m}\right)^{1/2}\int\left(\frac{2y\,dy}{\gamma}\right)\gamma^7 e^{-y^2}\sigma_{(2)}(y) = \frac{1}{4\pi^{1/2}}\left(\frac{kT}{m}\right)^{1/2}\int 2y\,dy\,(8y^6)e^{-y^2}\sigma_{(2)}(y)$$

$y^2 = \gamma^2/2$
$\gamma^2 = 2y^2$
$2\gamma\, d\gamma = 4y\, dy$
$d\gamma = \frac{2y\,dy}{\gamma}$

$$\boxed{I_2 = 4\left(\frac{kT}{m\pi}\right)^{1/2}\int y^7 e^{-y^2}\sigma_{(2)}(y)} \qquad (33.31)$$

$\gamma^6 = 2^3 y^6$

$$K = \left(\frac{2k^2T}{3m}\right)\frac{I_1^2}{I_2} = \left(\frac{2k^2T}{3m}\right)\frac{(15/4)^2}{4\left(\frac{kT}{m\pi}\right)^{1/2}\int y^7 e^{-y^2}\sigma_{(2)}(y)}$$

$$K_1 = \left(\frac{\cancel{8}k^2T}{\cancel{3}m}\right)\left(\frac{m\pi}{kT}\right)^{1/2}\left(\frac{15\cdot\cancel{15}^{5}}{\cancel{4}_{2}\cdot\cancel{4}\cdot 4}\right)\left[\int y^7 e^{-y^2}\sigma_{(2)}(y)\,dy\right]^{-1}$$

$$= k\left(\frac{kT\pi}{m}\right)^{1/2}\cdot\frac{75}{32}\left\{\int y^7 e^{-y^2}\sigma_{(2)}(y)\,dy\right\}^{-1}$$

$$= \frac{75}{32}\left(\frac{\pi k^3 T}{m}\right)^{1/2}\left\{\int dy\, y^7 e^{-y^2}\sigma_{(2)}(y)\right\}^{-1} \qquad (33.32)$$

Rigid sphere:

$$\int dy\, y^7 e^{-y^2}\sigma_{(2)}(y) = \frac{2\pi d^2}{3}\int dy\, y^7 e^{-y^2}$$

$$= \frac{2\pi}{3} d^2\,\frac{3!}{2} = 2\pi d^2$$

$$K_1 = \frac{75}{32(2\pi d^2)}\left(\frac{\pi k^3 T}{m}\right)^{1/2} = \frac{75}{64 d^2}\left(\frac{k^3 T}{\pi m}\right)^{1/2}$$

$$\mu_1 = \frac{5}{8}\frac{(\pi m k T)^{1/2}}{2\pi d^2} = \frac{5}{16 d^2}\left(\frac{mkT}{\pi}\right)^{1/2}$$

$$\boxed{g^{-4/\alpha} = \left(\frac{4kT}{m}\right)^{-2/\alpha} y^{-4/\alpha}}$$

~~$\gamma = \sqrt{2}\,y$~~

$\frac{\gamma^2}{2} = y^2$

Inverse power law:

$$\int dy\, y^7 e^{-y^2}\left\{2\pi\left(\frac{\alpha C_\alpha}{\tilde m}\right)^{2/\alpha} g^{-4/\alpha} A_2(\alpha)\right\}$$

$$g = \left(\frac{2kT}{m}\right)^{1/2}\gamma = 2\left(\frac{kT}{m}\right)^{1/2} y$$

$$2\pi A_2(\alpha)\left(\frac{\alpha C_\alpha}{\tilde m}\right)^{2/\alpha}\int dy\, y^7 e^{-y^2}\cdot\left(\frac{4kT}{m}\right)^{-2/\alpha} y^{-4/\alpha} = 4\pi\left(\frac{kT}{m}\right)^{-2/\alpha} A_2(\alpha)\left(\frac{\alpha C_\alpha}{\tilde m}\right)^{2/\alpha}\int dy\, y^7 y^{-4/\alpha} e^{-y^2}$$

$$= 4\pi A_2(\alpha)\left(\frac{4kT}{m}\right)^{-2/\alpha}\left(\frac{\alpha C_\alpha}{\tilde m}\right)^{2/\alpha}\int dy\, y^{(7-4/\alpha)} e^{-y^2} = 4\pi A_2(\alpha)\left(\frac{m}{4kT}\right)^{2/\alpha}\left(\frac{\alpha C_\alpha}{\tilde m}\right)^{2/\alpha}\int dy\, e^{-y^2} y^{(7-4/\alpha)}$$

$C_\alpha = e$
$\alpha = 1$

Inverse power law:

$$\int dy\, y^7 e^{-y^2} \left\{ 2\pi \left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha} g^{-4/\alpha} A_2(\alpha) \right\}$$

$$= 2\pi \left(\frac{\alpha C_\alpha}{\tilde{m}}\right)^{2/\alpha} \int dy\, y^7 e^{-y^2} g^{-4/\alpha} A_2(\alpha)$$

now $g = \left(\frac{2kT}{m}\right)^{1/2} \gamma$ and $\gamma = \sqrt{2}\, y$

hence

$$g = \left(\frac{4kT}{m}\right)^{1/2} y \Rightarrow g^{-4/\alpha} = \left(\frac{4kT}{m}\right)^{-2/\alpha} y^{-4/\alpha} = \left(\frac{m}{4kT}\right)^{2/\alpha} y^{-4/\alpha}$$

and

$$\int = 2\pi \left(\frac{\alpha C_\alpha}{m}\right)^{2/\alpha} \left(\frac{m}{4kT}\right)^{2/\alpha} A_2(\alpha) \int dy\, y^7 y^{-4/\alpha} e^{-y^2}$$

$$= 2\pi \left(\frac{\alpha C_\alpha}{4kT}\right)^{2/\alpha} A_2(\alpha) \int dy\, y^{(7-4/\alpha)} e^{-y^2}$$

But $\int dy\, y^{(7-4/\alpha)} e^{-y^2} = \frac{2^{2/\alpha}}{16} \Gamma(4-2/\alpha)$

hence

$$\int = \frac{\pi}{8} \left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha} A_2(\alpha)\, \Gamma(\cancel{4}-2/\alpha)$$

and

$$\mu_1 = \frac{5}{8} \left(\pi m k T^{1/2}\right) \Big/ \left\{ \frac{\pi}{8} \left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha} A_2(\alpha)\, \Gamma(4-2/\alpha) \right.$$

$$= 5 \left(\frac{mkT}{\pi}\right)^{1/2} \left(\frac{2kT}{\alpha C_\alpha}\right)^{2/\alpha} \left[A_2(\alpha)\, \Gamma(4-2/\alpha)\right]^{-1}$$

$7-4/\alpha+1=8-4/\alpha \quad \frac{\nu}{2}=4-2/\alpha$

$$= 4\pi A_2(\alpha)\left(\frac{\alpha C_\alpha}{4kT}\right)^{2/\alpha}\int dy\, e^{-y^2}\, y^{(7-4/\alpha)}$$

$$\int_0^\infty x^{\nu-1} e^{-\beta x^2-\gamma x}\,dx = (2\beta)^{-\nu/2}\,\Gamma(\nu)\exp(\gamma^2/8\beta)\,D_{-\nu}(\gamma/\sqrt{2\beta})$$

$$= (1\cdot 2)^{-(4-2/\alpha)}\,\Gamma(4-2/\alpha) = \frac{2^{2/\alpha}}{2^4} = \frac{2^{2/\alpha}}{16}\,\Gamma(4-2/\alpha)$$

Thus

$$\int = 4\pi A_2(\alpha)\left(\frac{\alpha C_\alpha}{4kT}\right)^{2/\alpha}\cdot\frac{2^{2/\alpha}}{16}\,\Gamma(4-2/\alpha)$$

$$= \frac{\cancel{4}\pi A_2(\alpha)}{\cancel{16}\,4}\left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha}\Gamma(4-2/\alpha)$$

$$= \frac{\pi A_2(\alpha)}{4}\left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha}\Gamma(4-2/\alpha)$$

$$\mu = \frac{5}{8}(\pi m kT)^{1/2} \Big/ \left\{\frac{\pi A_2(\alpha)}{4}\left(\frac{\alpha C_\alpha}{2kT}\right)^{2/\alpha}\Gamma(4-2/\alpha)\right\}$$

$$= \frac{5}{\cancel{8}\,2}(\pi m kT)^{1/2}\cdot\frac{\cancel{4}}{\pi A_2(\alpha)}\left(\frac{2kT}{\alpha C_\alpha}\right)^{2/\alpha} \Big/ \Gamma(4-2/\alpha)$$

$$= \frac{5}{2}\left(\frac{mkT}{\pi}\right)^{1/2}\left(\frac{2kT}{\alpha C_\alpha}\right)^{2/\alpha} \Big/ \left[A_2(\alpha)\,\Gamma(4-2/\alpha)\right]$$