

**TITLE:** NUCLEAR INERTIA FOR FISSION IN A GENERALIZED CRANKING MODEL

**AUTHOR(S):** J. Kunz, T-9, B279

J. R. Nix, T-9, B279

**MASTER**

**SUBMITTED TO:** to be presented at the Nuclear Dynamics Workshop III to be held at Copper Mountain, CO, on March 4-9, 1984.

### DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes.

## INERTIA FOR FISSION IN A GENERALIZED CRANKING MODEL

J. Kunz and J. R. Nix  
Theoretical Division, Los Alamos National Laboratory  
Los Alamos, New Mexico 87545

The cranking model [1] has been widely used to calculate the moments of inertia associated with collective degrees of freedom. After the inclusion of pairing correlations, theoretical results obtained with the cranking model for nuclear rotations and  $\gamma$ -vibrations were in relatively good agreement with experimental data. Calculations of  $\beta$ -vibrational moments of inertia were performed in the cranking model for fission deformations. The results were several times the irrotational values [2] and gave good agreement with experimental spontaneous-fission lifetimes [3,4], although in some cases a renormalization factor of 0.8 was required [4].

As pointed out by many authors (see ref. 5), the Inglis cranking model has two serious deficiencies. First, problems arise when the cranking potential contains momentum-dependent terms. Second, in the case of pairing strength the inertia approaches zero instead of a finite irrotational limit.

Alternative approaches to the cranking model which did not lead to such deficiencies were developed by Migdal [6], Belyaev [7] and Thouless [8].

They showed that these deficiencies of the cranking model are due to a lack of self-consistency, since the reaction of the mean field on the cranking motion is neglected in the Inglis model. In ref. 5 we have pointed out these deficiencies and developed a generalized cranking model for stationary cranking. Here we show how to develop a time-dependent formalism for cranking to  $\beta$ -vibrations and fission [9].

We start with the time-dependent equation for the generalized density

$$\rho(\mathbf{r}, t) = \rho(\mathbf{r}, \epsilon) + \delta\rho(\mathbf{r}, t) \quad (1)$$

where  $\rho(\mathbf{r}, \epsilon)$  is the stationary density and  $\delta\rho(\mathbf{r}, t)$  is the time-dependent part. We need that the Hamiltonian  $\mathcal{H}$  and consequently the generalized cranking potential depend on the collective variable  $\epsilon$ . Furthermore, we assume that the cranking motion is adiabatic, which permits the replacement

Choosing the basis so that

$$[\mathcal{H}_0, \mathcal{R}_0] = 0 \quad , \quad (2)$$

we then obtain to lowest order in the collective variable the equation

$$i\hbar\dot{\mathcal{R}}_0 = [\mathcal{H}_0, \mathcal{R}_1] + [\mathcal{H}_1, \mathcal{R}_0] \quad (3)$$

for the generalized density matrix. Here  $\mathcal{H}$  and  $\mathcal{R}$  symbolize the matrices

$$\mathcal{H} = \begin{pmatrix} h & -\Delta \\ \Delta^* & -h^* \end{pmatrix} \text{ and } \mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa & 1-\rho^* \end{pmatrix} .$$

The usual cranking-model approximation consists of neglecting the  $\mathcal{H}_1$  term in eq. (3). We obtain  $\dot{\mathcal{R}}_0$ , which appears on the left-hand side of eq. (3), by differentiating eq. (2) with respect to time.

From this point onwards we proceed analogously to ref. 5 and evaluate the first-order correction to the generalized density matrix  $\mathcal{R}_1$ . Its trace with the generalized collective momentum operator then yields the nuclear inertia  $B$ . However, in contrast to the stationary formalism, the time-dependent formalism leads to an additional pairing-vibration coupling term [3] because of the implicit dependence of the pairing gap on the collective variable.

Keeping the  $\mathcal{H}_1$  term in eq. (3) gives rise to two additional contributions to the inertia that are proportional to  $h_1$  and  $\Delta_1$ . The  $h_1$  contribution arises when the potential contains momentum-dependent terms. In the stationary case one obtains

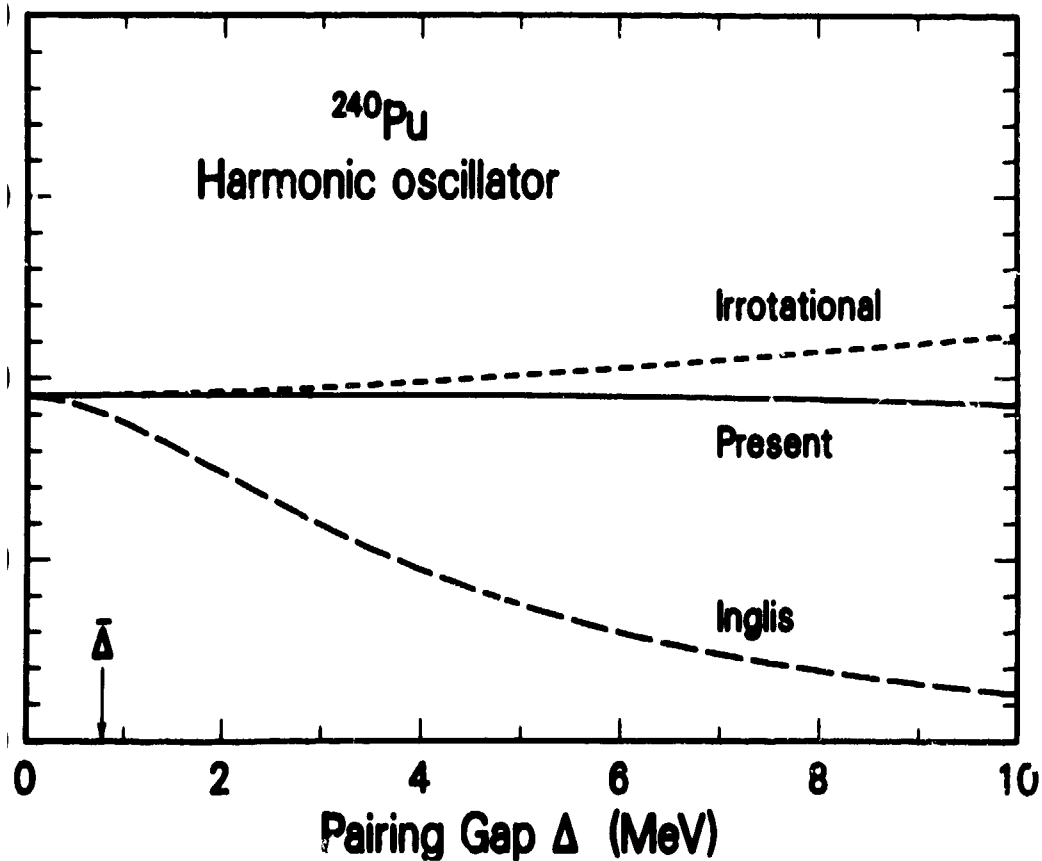
$$h_1 \propto (1 - m/m^*) \quad , \quad (4)$$

where  $m^*$  is the effective mass. This can lead to a considerable change in the inertia [5]. We expect a similar relationship to also hold in the time-dependent case [10]. The additional  $\Delta_1$  term, for which an explicit expression is obtained from the continuity equation [6], keeps the nuclear inertia finite in the limit of large pairing strength.

To demonstrate the effect of the  $\Delta_1$  contribution on the inertia, we now specialize to the harmonic-oscillator potential. In the limit of zero temperature and a constant pairing gap, we obtain for the inertia

$$\hbar^2 \sum_{p,q} |\langle p | i \frac{\partial}{\partial \varepsilon} | q \rangle|^2 \frac{E_p E_q - \hbar_p \hbar_q + \Delta^2}{2E_p E_q (E_p + E_q)} + \hbar^2 \sum_p \frac{1}{8E_p^5} (\hbar'_p \Delta - \hbar_p \Delta')^2, \quad (5)$$

time denotes differentiation with respect to  $\varepsilon$ . Note the plus sign of  $\Delta^2$  in the first term, which arises from the  $\Delta_1$  contribution. In fig. 1 we show the first term of the inertia for  $\beta$ -vibrations as a function of pairing strength, calculated with respect to Nilsson's spheroidal deformation parameter  $\varepsilon$  [2,4,5]. The pairing-vibration coupling term is not considered here, since it vanishes for large pairing strength. When the Inglis cranking inertia approaches zero for large pairing, the inertia containing the  $\Delta_1$  contribution remains finite and close to



1. Dependence of the  $\beta$ -vibrational inertia upon the pairing gap  $\Delta$  in a harmonic-oscillator potential at the equilibrium deformation. The solid curve gives the present result calculated in the general cranking model with 15 oscillator shells, the long-dashed curve corresponding result calculated in the Inglis cranking model and -dashud curve gives the irrotational result.

the limiting irrotational value. The deviation arises from the slow convergence of the cranking inertia with increasing basis size [5].

For a harmonic-oscillator potential with an effective mass, relation (4) holds, and the reaction of the pairing field to the collective motion is given by

$$\Delta_1 \propto Y_{20} .$$

For a more realistic modified-harmonic-oscillator potential we expect similar results. In particular, we expect that the proper inclusion of the effective-mass term  $h_1$  for  $\beta$ -vibrational inertias may account for the renormalization factor of 0.8 that was originally needed to reproduce experimental spontaneous-fission lifetimes [4].

#### REFERENCES

- 1) D. R. Inglis, Phys. Rev. 96 (1954) 1059 and 97 (1955) 701.
- 2) A. Sobiczewski, Z. Szymański, S. Wycech, S. G. Nilsson, J. R. Nix, C. F. Tsang, C. Gustafson, P. Möller and B. Nilsson, Nucl. Phys. A131 (1969) 67.
- 3) M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky and C. Y. Wong, Rev. Mod. Phys. 44 (1972) 320.
- 4) J. Rindrup, S. E. Larsson, P. Möller, S. G. Nilsson, K. Pomorski and A. Sobiczewski, Phys. Rev. C13 (1976) 229.
- 5) J. Kunz and J. R. Nix, Los Alamos Preprint LA-UR-83-3624 (1983).
- 6) A. B. Migdal, Nucl. Phys. 13 (1959) 655.
- 7) S. T. Belyaev, Nucl. Phys. 64 (1965) 17.
- 8) D. J. Thouless and J. G. Valatin, Nucl. Phys. 31 (1962) 211.
- 9) J. Kunz and J. R. Nix, to be published.
- 10) M. J. Giannoni, F. Moreau, P. Quentin, D. Vautherin, M. Veneroni and D. H. Brink, Phys. Lett. 65B (1976) 305.