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TITLE: A NUCLEAR STRUCTURE STUDY OF THE PROPOSED GAMMA-RAY LASER CANDIDATE NUCLEUS ^{186}RE

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ABSTRACT

We present results of theoretical nuclear structure model calculations for the gamma-ray laser candidate nucleus ^{186}Re proposed by Collins. Our calculations of this odd-odd transitional nucleus are based on an axially-asymmetric (particle plus triaxial rotor) model for constructing the orbitals of the odd nucleons that couple under the influence of the residual neutron-proton interaction. We include pairing correlations in the determination of these orbitals by using the BCS approximation with newly determined pairing strengths. The matrix elements of the residual neutron-proton interaction are obtained using phenomenological spin-dependent δ function potentials of both surface and volume forms. We examine the sensitivity of the calculated low-excitation level structure of ^{186}Re to the strength of these potentials. Calculated energy levels of ^{186}Re will be presented and compared with experiment. The impact of our results on the proposed use of ^{186}Re as a gamma-ray laser will be discussed. In addition, based upon these and other model calculations to be described, we assess the level of effort necessary in a full-scale theoretical search for a viable candidate nucleus for a gamma-ray laser.

1. INTRODUCTION

The odd-odd transitional nucleus ^{186}Re has an experimentally known isomeric state of spin and parity $J^{\pi} = 8^+$ at an excitation energy of approximately 150 keV. The state has a halflife $T_{1/2}$ of 2 x 10^5 years. Collins[1] has proposed ^{186}Re as a candidate nucleus for a gamma-ray laser on the basis of an as yet to be identified 7^- state lying close by the 8^+ isomeric state. The 8^+ storage level could be depopulated by E1 radiation to the 7^- transfer level, which would then cascade to the upper lasing level--the 3^- state at an excitation energy of 99 keV. The viability of the proposal of Collins rests upon the existence of the 7^- state, its excitation energy relative to the 8^+ storage state, and the strength of the E1 transition rate between these two states.

We report in this paper on our theoretical investigation of the low-lying level structure of ^{186}Re with particular emphasis on the excitation energies and nuclear structure properties of any 7^- states near the known 8^+ isomeric state at approximately 150 keV. Our investigation consists of nuclear structure

model calculations that are carried out in the following manner. We view the odd-odd transitional nucleus $^{186}_{75}Re_{111}$ as the even-even core nucleus $^{184}_{74}W_{110}$ plus an odd unpaired proton and an odd unpaired neutron that couple together under the influence of the attractive residual neutron-proton interaction. We obtain the orbitals (wave functions) of the unpaired proton by reproducing the low-lying level structure of $^{185}_{75}Re_{110}$, viewing this nucleus as the $^{184}_{74}W_{110}$ core plus a proton. Because $^{185}_{75}Re_{110}$ is a transitional nucleus, we use an axially-asymmetric model to calculate its low-lying level structure. Specifically, we use the particle plus triaxial-rotor model as formulated, for example, by Larsson, Leander, and Ragnarsson.[2] Similarly, to obtain the orbitals (wave functions) of the unpaired neutron, we use the same model to reproduce the low-lying level structure of $^{185}_{74}W_{111}$, viewing this nucleus as the $^{184}_{74}W_{110}$ core plus a neutron. The calculations of these orbitals are described in detail in Sec. 2.

In Sec. 3, we calculate the matrix elements of the attractive residual neutron-proton interaction using the wave functions determined in Sec. 2. We consider two phenomenological spin-dependent δ function potentials to represent this interaction. These are surface and volume δ function potentials whose strengths are determined by reproducing optimally the *known* low-lying levels in ^{186}Re. With this information, the *remaining*, unmeasured, low-lying levels of ^{186}Re can be reliably predicted. We calculate the ^{186}Re low-lying excitation spectrum in this way for both choices of the residual interaction, and compare our results with the available experimental level information. Our conclusions from this work, expectations for future work, and assessment of the scope of a full-scale theoretical search for a viable candidate nucleus for a gamma-ray laser will be given in Sec. 4.

2. SINGLE-PARTICLE WAVE FUNCTIONS

Our starting point for the calculation of the two sets of single-particle wave functions that we require is the particle plus triaxial-rotor model as formulated and coded by Larsson, Leander, and Ragnarsson.[2,3] The particle-rotor Hamiltonian is derived from the many-body Bohr-Mottelson Hamiltonian of an odd-mass system[4] and is given by

$$H = \sum_{k=1}^{3} \frac{\hbar^2}{2I_k}(J_k - j_k)^2 + H_{part}(\alpha) , \qquad (1)$$

where the I_k are the moments of inertia, and J_k and j_k are components of the total and single-particle angular momentum, respectively. The second term is the single-particle Hamiltonian in an adiabatic core field described by collective coordinates α. Equation (1) is actually obtained from the Bohr-Mottelson Hamiltonian by choosing the strong-coupling basis, which has the advantage that the particle-rotation interaction then appears in the equation *only* in a Coriolis term,

$$\text{Coriolis term} = -\sum_{k=1}^{3} \frac{\hbar^2}{2I_k}(J_k - j_k)^2 + H_{part}(\alpha) , \quad (1)$$

and a recoil term,

$$\text{recoil term} = \sum_{k=1}^{3} (\hbar^2 j_k^2 / 2I_k) , \qquad (2)$$

The triaxial core assumption, appropriate to the transitional nuclei, is introduced to Eqs. (1)-(3) through a hydrodynamical model for the three moments of inertia, namely,

$$I_k = (4/3)I_0 \sin^2(\gamma + 2\pi k/3) , \qquad (4)$$

where γ measures the degree of axial asymmetry or triaxiality and I_0 is the moment of inertia associated with axial symmetry.

The single-particle Hamiltonian of Eq. (1), $H_{part}(\alpha)$, required to calculate the single-particle wave functions Z_ν, so that the diagonalization of Eq. (1) in the strong-coupling basis can occur, is constructed using the modified oscillator potential[5]

$$V_{osc} = (1/2)\hbar\omega_0(\varepsilon_2, \varepsilon_4)\rho^2 [1 - (2/3)\varepsilon_2 P_2(\cos\theta_t) + 2\varepsilon_4 P_4(\cos\theta_t)] . \qquad (5)$$

This potential is expressed here in the stretched coordinate system[3] for the case of quadrupole (ε_2) and hexadecapole (ε_4) deformations. In our actual calculations we include even higher deformation order, *viz.*, ε_6. Two other ingredients of H_{part} necessary to fit accurately the level spectra of deformed nuclei are a spin-orbit term,

$$\text{spin-orbit term} = -2\kappa\hbar\mathring{\omega}_0 \vec{l} \cdot \vec{s} . \qquad (6)$$

with coupling strength κ, and a quadratic orbital angular momentum term (to truncate the oscillator potential with increasing mass number A),

$$\text{quadratic } \vec{\ell} \text{ term} = -\kappa\mu\hbar\omega_0 \vec{\ell}\cdot\vec{\ell}, \tag{7}$$

with coupling strength $\kappa\mu$. With the total single-particle potential given by Eqs. (5)-(7), one obtains solutions of $H_{part}(\alpha)$ of the form

$$\chi_\nu = \sum_{\alpha\Omega} C^{(\nu)}_{\alpha\Omega} \psi^{\alpha}_{\Omega} . \tag{8}$$

with $\alpha = N,\ell$, where N is the principal oscillator quantum number, ℓ and j are the orbital and total angular momentum of the particle, and Ω is the projection of j along any one of the three principal axes. The basis wave functions ψ^{α}_{Ω} are coupled harmonic oscillator wave functions referred to the stretched coordinate system.[5] However, we wish to note that the entire formalism presented here can also be referred to the unstretched coordinate system in which no approximations are made in the coupling calculation. We have used the unstretched coordinate system in all of the calculations presented in this paper.

Finally, we account for the effects of pairing correlations by performing a standard BCS calculation on the adiabatic single-particle levels χ_ν. Therefore, the Fermi level and the pairing occupation factors are uniquely determined without additional adjustable parameters. In the BCS calculation, the pairing strengths $G_0 \pm G_1[(N-Z)/A]$ are determined by reproducing the experimental pairing gaps Δ_n and Δ_p obtained from fourth-order mass difference equations.[6]

We now present our results in calculating the single-particle wave functions χ_ν for the odd-proton nucleus $^{185}_{75}Re_{111}$ and the odd-neutron nucleus $^{185}_{74}W_{111}$. The final values of the input parameters for the two nuclei are presented in Table 1. In this table, the values of κ and μ are taken from the extensive studies of Nilsson et al.[5] and these were held fixed throughout. The values of the quadrupole (ε_2) and hexadecapole (ε_4) deformations were constrained to lie close to the transforms of the corresponding β_2 and β_4 values determined in the analysis of inelastic scattering experiments of A ~ 185 nuclei. The value of ε_6 for ^{185}Re was taken from the systematic theoretical analysis of ground-state masses by Möller and Nix.[7] The value of ε_6 for

^{185}W was taken to be zero for the present calculations. For ^{185}Re a value of $\gamma = 15.04^{\circ}$ was determined by adjustment to the measured excitation energy 646.0 keV of the bandhead of the $K = 1/2$ γ-vibrational band. A value of $\gamma = 0.0^{\circ}$ is used for ^{185}W, as the only experimental evidence for a γ-vibrational band in this nucleus is tentative, for a $K = 3/2$ γ-vibrational band above 1 MeV in excitation. Since the presence of the odd nucleon in the ^{184}W core polarizes this core somewhat, the excitation energy of the core 2^+ state, $E(2^+)$, is adjusted to reproduce the experimental excitation energy of the first-excited state in the odd-nucleon nucleus. The values of $E(2^+)$ in Table 1 should be compared to the true value of 0.1112 MeV. As already mentioned, the values of the pairing strengths G_0 and G_1 are adjusted to reproduce the experimental pairing gaps Δ_n and Δ_p. Finally, the parameters ξ_{pair} and $\xi_{Coriolis}$ are attenuation factors on the pairing factor (exponential) and the Coriolis term (multiplicative), which are normally of unit value. However, it is well known that attenuation of these two terms can sometimes improve agreement with experiment in axially-symmetric cases. We have found that a value of $\xi_{Coriolis} = 0.9$ for ^{185}W (axially symmetric in our present treatment) gives better agreement with the experimental level spectrum than does the unit value.

The levels (and therefore the corresponding wave functions) calculated using the parameters contained in Table 1 are compared with the experimental levels in Tables 2 and 3. In Table 2 we compare calculated and experimental levels for the first three even-parity bands of ^{185}Re. The quite good agreement indicates that the corresponding wave functions can be used with confidence in calculating the residual n-p interaction matrix elements for ^{186}Re, to be discussed in Sec. 3. We reach a similar conclusion with Table 3, in which we compare calculated and experimental levels for the first three odd-parity bands of ^{185}W. Here, the agreement between calculation and experiment is not as good overall as for ^{185}Re. However, the errors in the ^{185}W wave functions that are required in predicting the locations of any 7^- states in ^{186}Re are quite small. We now turn to a discussion of the calculations of the residual n-p interaction matrix elements using the single-particle wave functions that have been described here.

Table 1. Parameters for Constructing the Wave Functions of the Odd-Proton Nucleus $^{185}_{75}Re_{110}$ and the Odd-Neutron Nucleus $^{185}_{74}W_{111}$.

Parameter	Odd-Proton Nucleus	Odd-Neutron Nucleus
κ	0.062	0.0636
μ	0.614	0.393
ϵ_2	0.230	0.240
ϵ_4	0.130	0.068
ϵ_6	-0.013	0.0
γ (deg)	15.04	0.0
$E(2^+)$ (MeV)	0.1187	0.1200
G_0 (MeV)	20.435	20.150
G_1 (MeV)	7.4	7.4
Δ_p (MeV)	0.826	—
Δ_n (MeV)	—	0.783
ξ_{pair}	1.0	1.0
$\xi_{Coriolis}$	1.0	0.9

Table 2. Comparison of Calculated and Experimental Low-Lying Even-Parity Bands of the Odd-Proton Nucleus $^{185}_{75}Re_{110}$

Band	J^π	Calcuated (keV)	Experimental (keV)
$\frac{5}{2}[402]$	5/2+	0.0	0.0
	7/2+	125.1	125.348
	9/2+	284.5	284.1
	11/2+	476.8	475.6
	13/2+	699.0	697.0
	15/2+	950.5	949.5
	17/2+	1227.2	1230.0
$K=\frac{1}{2}\gamma$-vib.	1/2+	646.0	646.11
($\frac{5}{2}[402]$)	3/2+	708.5	717.42
	5/2+	786.9	767.3
$\frac{1}{2}[411]$	3/2+	869.5	874.81
	1/2+	888.9	880.27

Table 3 Comparison of Calcuated and Experimental Low-Lying Odd-Parity Bands of the Odd-Neutron Nucleus $^{185}_{74}W_{111}$

Band	I^π	Calcuated (keV)	Experimental (keV)
$\frac{3}{2}$[512]	3/2⁻	0.0	0.0
	5/2⁻	64.4	65.86
	7/2⁻	204.1	173.68
	9/2⁻	313.9	302.0
	11/2⁻	564.9	480.0
$\frac{1}{2}$[510]	1/2⁺	8.9	23.54
	3/2⁺	87.6	93.30
	5/2⁺	195.1	187.88
	7/2⁻	343.5	338.0
	9/2⁻	544.3	490.0
	11/2⁻	745.5	706.0
$\frac{7}{2}$[503]	7/2⁻	246.8	243.5
	9/2⁻	419.5	390.8
	11/2⁻	630.4	570.0

3. RESIDUAL NEUTRON-PROTON INTERACTION AND THE ^{186}RE SPECTRUM

The simplest generalization of Eq. (1) for two nucleons is

$$H = \sum_{k=1}^{2} \frac{\hbar^2}{2I_k}(I - j_n - j_p)_k^2 + H_{sp} + V_{np} , \qquad (9)$$

where the rotational and single particle terms of H have the same significance as in Eq. (1), and V_{np} in the residual neutron-proton interaction. Historically, only the axially-symmetric case was considered[8,9,10]. This was in part due to a lack of detailed experimental information in odd-odd nuclei, a lack of the requisite computational abilities, and to a realization that the differences due to assumed axial asymmetries were offset by the uncertainties in V_{np}.

However, each of these difficulties has been, or is currently being overcome. With the advent of new heavy ion accelerators and 4π detectors, prodigious amounts of data are becoming available. Larger computers allow more basis states--an essential point since relaxing the requirement of axial symmetry results in Hamiltonian matrices an order of magnitude larger than for the case of axially symmetric systems. Finally, the increased computing power also allows the calculation of the neutron-proton interaction from potentials based on nucleon-nucleon scattering. Further, considerable work on the magnitude of V_{np} has been done by R. Hoff and his collaborators[11].

In this paper we shall calculate V_{np} from simple phenomenological interactions such as a delta function potential. Our choice for the delta function includes both a spin-independent as well as a spin-dependent part. Our selection follows closely the work of Sood *et al.*[12,13], although in addition to a purely central delta interaction, we also allow a tensor interaction with no radial dependence to contribute.

It is relevant to briefly discuss the case of axial symmetry, since the systematics of the matrix elements of the neutron-proton interaction is similar for the two instances. If Ω_p and Ω_n are the projections of the proton and neutron single-particle angular momenta along the symmetry axis, then $K = \Omega_p + \Omega_n$ where K is the projection of the angular momentum of the core along the axis. It was demonstrated by Gallagher and Moszkowski[8] that if the projection of each of the single nucleon's spin is aligned or anti-aligned, the matrix element of V_{np} is attractive. If one is aligned with the direction of the projection of the orbital angular momentum and the other is anti-aligned, then the matrix element is repulsive.

A simple representation of V_{np} is

$$\langle I K | V_{np} | I K \rangle = A + (-)^I \delta_{K,0} B \qquad (10)$$

The first term is usually referred to as a Gallagher-Moszkowski matrix element. The second term is the so-called Newby term[9]. The magnitudes of A and B are typically a few hundreds and tens of keV, respectively. In our work, the matrix element is explicitly calculated.

From the single particle wave functions and energies calculated, as described in Sec. 2, and with the matrix elements of V_{np} calculated as described above, one may calculate the structure of ^{186}Re. Our results for the negative parity states are shown in Fig. 1. We omit here a discussion of the positive parity states, since we are primarily interested in the possibility of a low-lying 7^- level. In fact, there are several 7^- levels; one is a dominant member of the ground-state band; others may be formed from other bands. The physical levels are, of course, mixtures of the simple states thus formed. The lowest 7^- lies above 300 keV and therefore ^{186}Re can unfortunately be eliminated from consideration as a candidate for a gamma ray laser.

This points out two important facts: nuclear structure calculations can be useful as a guide to whether a particular nucleus may be a possible candidate. It also reaffirms that it is easier to eliminate a nucleus than to demonstrate that a given nucleus meets the necessary criteria.

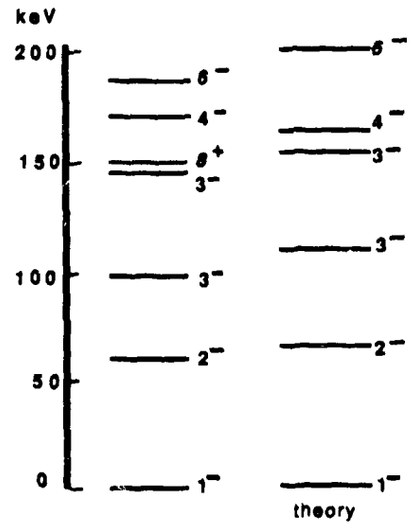

Fig. 1. The low lying negative parity states of ^{186}Re calculated as described in the text. The interaction used in the calculation has not been optimized.

4. DISCUSSION AND CONCLUSION

We have demonstrated in this paper that it is possible to eliminate ^{186}Re from consideration as a candidate for a gamma-ray laser. Similar techniques may be applied to other nuclei to test their applicability. However, this only addresses one aspect of the problem--that of eliminating candidates. The other aspect, namely, that of actually proposing promising possibilities, is more difficult. In order to do this, several refinements to our model currently under way will have to be completed.

The neutron-proton interaction V_{np} was calculated above from a simple central delta interaction. However, it is known that the true interaction contains appreciable non-central components. These are currently being calculated using G-matrix techniques pioneered by Kuo and Brown.

A further refinement applies both to the odd-A and odd-odd cases. In the above discussion the pairing interaction was included in an *ad hoc* manner, which leads to uncertainty. For example, should ^{185}Re be considered as a particle outside a ^{184}Re core or a hole in ^{186}Re? If the mass 184 and 186 nuclei do not differ appreciably, it is

probably irrelevant from a practical point of view, although the ambiguity is still unseemly. Both of these objections may be overcome by using the results of Dönau and Frauendorf[14], which are based on earlier work by Kerman and Klein[15]. In this case the Hamiltonian takes the form of an array:

$$H = \begin{pmatrix} E_R + \Gamma + \lambda & \Delta \\ \Delta & E_R - \Gamma - \lambda \end{pmatrix} \qquad (11)$$

The basis states are the core states of A+1 and A-1. Since the energies of these core states enters explicitly, they may have very different properties. Further, these even-even core states may be calculated explicitly. Ideally they would be calculated by self-consistent, multi-configurational Hartree-Fock. Such an approach would be more basic, but prohibitively expensive if one wishes to explore many nuclei. We choose to obtain the core states by numerically solving the Bohr Hamiltonian using a variant of the techniques pioneered by Hess *et al*[16]. This is somewhat similar to recent work by Bennour *et al.*[17]. The analog of eq. (11) has been derived for the case of two nucleons outside a core. Work in these directions is continuing.

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