CONF-810662--1

A-UR -81-1662

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MASTER

SUBMITTED TO:

To be presented at the Conference on Characterization and Quality Control of Nuclear Fuels, Karlsruhe, FRG on 2-5 June, 1981.

University of California

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Form No. 836 R3
St. No. 2629

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RECONSTRUCTION OF RADIAL FISSION-PRODUCT DISTRIBUTIONS IN REACTOR FUELS FROM A SMALL NUMBER OF PROJECTIONS

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Four mathematical techniques for reconstruction of the radial two-dimensional distribution of fission products using projections obtained by nondestructive gamma scanning were evaluated. Reconstruction of a picture from a finite set of projections is mathematically indeterminate; therefore, reconstruction techniques are heuristic, particularly when only a small number of projections are available. Of the techniques evaluated, the filtered backprojection algorithm provided the best reconstruction for simulated gamma-ray sources, as well as for actual irradiated fuel material.

1. INTRODUCTION

Nondestructive measurement techniques have been applied to a wide variety of measurement problems associated with the characterization of irradiated fuel materials. One particularly useful nondestructive technique is gamma scanning. Precision gamma scanning has been used by many investigators to measure the axial isotopic distribution of fission products. The radial isotopic distributions of fission products have usually been measured by sectioning the fuel rod and performing subsequent analyses by gamma scanning or electron microprobe. A technique was developed at the Los Alamos National Laboratory to obtain nondestructively the two-dimensional distribution of fission products at specific axial positions by measuring diametral scans, or projections at two or more angular orientations and analytically unfolding these projections.[1,2]

We have evaluated the applicability of four different reconstruction techniques to determine which one of the techniques provides the best reconstructed distribution. These four techniques include the TWODIM (1,2) algorithm developed at Los Alamos, a filtered backprojection Fourier transform technique,[3,4] a multiplicative algebraic iterative reconstruction technique (MART),[3,5] and finally, a recent adaptation of the Algebraic Reconstruction Technique algorithm (ART) to a maximum entropy criterion (MENT).[6] Each of these reconstruction techniques was applied to simulated gamma-ray sources that represented typical distributions of fission products within irradiated fuel pins. Also, the techniques were used to reconstruct the radial distributions of fission products from the experimentally measured projections of Light Water Reactor (LWR) and Fast Breeder Reactor (FBR) irradiated fuel rods.

2. THEORETICAL BACKGROUND

Reconstruction of images from projections has many scientific and medical applications where the invasion or destruction of the object is unacceptable. This noninvasive technique or procedure has been labeled many things: "computed tomography," "computerized axial tomography," "transaxial tomography," and "reconstruction from projections." Regardless of the varying names of the techniques, the procedure is that of mathematically combining projections from radiation emissions or transmissions at various angles to obtain an accurate representation of the original object.

The first work in the reconstruction of an object possessing circular symmetry from projections was published in 1826 by Abel.[7] Probably the first groundwork for the present-day procedures was laid in 1917 by the Austrian mathematician Radon,[8] who solved the basic mathematics of the problem. Radon's significant contribution was a set of integral equations that related two-dimensional, non-symmetric objects to their projections. He mathematically proved that a two- or three-dimensional object can be uniquely reconstructed from an infinite set of its projections. These formulas have served as the basis of the theoretical and practical development of the reconstruction technique. The recent application of reconstruction techniques has been tied to the development of computer technology because of the tremendous number of computations involved.

The input data for all of these techniques are projections obtained at two or more angular orientations. In Fig. 1 the function $g(x,y)$ represents the distribution of the physical property of interest -- in our particular case, the radial distribution of a

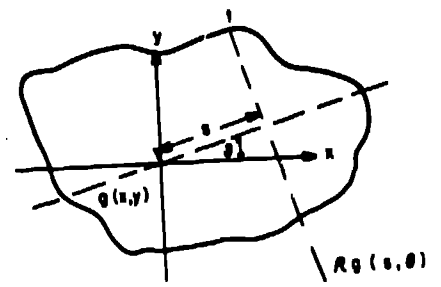

Figure 1: Each projection is an estimate of the line integral of the function $g(x,y)$, where the line of integration is specified by the parameters s and θ. Geometrical relation between a two-dimensional source $g(x,y)$ and the measured projection $\mathcal{R}g(s,\theta)$.

fission product. The projection data are estimates of the line integral of the function $g(x,y)$, where the line of integration is specified by the parameters s and θ. The s,t axes are rotated by an angle θ from the x,y axes. Then, the line integral of g along the line specified by (s,θ) can be denoted by $[\mathcal{R}g](s,\theta)$, where $\mathcal{R}$ is the Radon operator (named in honor of Radon). The function $\mathcal{R}g$ is obtained by a line integral along t as

$$[\mathcal{R}g](s,\theta) = \int_{-\infty}^{\infty} g(s\cdot\cos\theta - t\cdot\sin\theta,\; s\cdot\sin\theta + t\cdot\cos\theta)\,dt \quad . \quad (1)$$

Radon was the first to study this transformation that maps a function g into a function $\mathcal{R}g$, and subsequently, the inverse transformations to compute g from $\mathcal{R}g$.(9,10)

The reconstruction of images from projections is based on the development of techniques for solving variations of the above integral equation. No single technique has been found capable of satisfactorily processing the wide variety of projection-measurement geometries and the quantity and precision of data that occur in practical applications.

In the measurements of isotopic distributions of fission products in irradiated fuel pins, the data may be considered to be projection measurements or sets of line integrals of gamma-ray emission at definite steps across the diameter of the pin.

All of the reconstruction techniques are limited because they can only provide an estimate of the internal density distribution. Mathematical reconstruction from a finite set of projection data is known to be an indeterminate problem.(11,12) A very large number of algorithms exists for reconstructing objects from their projections. It would be useful to have some guidelines to determine which algorithms are most appropriate under specific circumstances. Unfortunately, little has been published in this direction.

Several criteria must be kept in mind when selecting a reconstruction algorithm. First and most important is the requirement that the reconstruction be consistent with the data available to the algorithm (artifacts are not introduced). This means that the projections (ray-sum) of the reconstructed picture should be the same as the ray-sum of the original distribution. With a finite number of projections, an approximation is the best result that can be attained.(13)

Secondly, a simple way must be available of deciding whether the reconstruction algorithm has successfully approximated the original distribution. A third factor involves the cost (manpower and financial) of applying a specific reconstruction algorithm. In actual applications, a large number of reconstructions and interpolations are involved and these may be cost prohibitive.

There are two ways of comparing reconstruction algorithms: theoretical and experimental. A theoretical comparison can be either a mathematical or a purely descriptive discussion of the nature of the methods under consideration (for example, method A is better than method B because it has a certain desirable property). The experimental method of comparison consists either of constructing test objects (phantoms) and physically taking their projections, or of designing test patterns and working out their projections mathematically; in either case, the various algorithms are used on the projection data, and the reconstructions are then compared to the original distribution.(11)

1. Backprojection

The most common reconstruction algorithm includes an operation referred to as backprojection, based on the previously described work of Radon.(8) Fundamental to understanding reconstruction from projection is the central-slice, or projection-slice, theorem. This theorem states "that a one-dimensional Fourier transform of a one-dimensional projection of a two-dimensional object is mathematically identical to one line (a slice) through the two-dimensional Fourier transform of the object itself. Thus, knowledge of all one-dimensional projections is sufficient to synthesize the two-dimensional transform of the object from which the object is readily obtained by an inverse two-dimensional transform."(14)

The quality of the projection images can be improved by using appropriate correcting functions. Correcting in the Fourier spaces is done by taking the Fourier transform of the degraded image, multiplying the transform by a correcting filter function, and performing the inverse Fourier transform. The filter we used performed a ramp function that tended to filter out high-frequency noise.

The Fourier method depends on transforming the projections into Fourier space, where they define part of the Fourier transform of the whole object. Each projection may be shown to yield values on a central section of Fourier space, which is a line or plane (corresponding to the two- or three-dimensional problem) through the origin at an angle corresponding to the direction of the projection in real space. An attempt is then made to interpolate the unknown values of the full Fourier transform from the values on the central sections. After interpolation a reverse Fourier transform provides an estimate of the object's structure.

2. Iterative Techniques

Iterative techniques are the basis of another set of algorithms used in the reconstruction of distributions from projections. One iterative technique is the algebraic reconstruction technique (ART). It can be called a "direct technique" because the reconstruction is done entirely in real space without using Fourier transforms.

As the number of projections increase, ART results gradually improve. In Herman,(15) eight projections seems to be a "critical" number with five iterations producing "superior" results. It should be emphasized that one can only hope to obtain a reasonable representation, since it has been proved that a nontrivial picture cannot be uniquely determined from a finite number of projections.(11)

Gordon explains the basic idea behind ART: "Starting from a blank picture, the ray-sums of all the projections are satisfied one after the other by distributing the difference between the desired ray-sum and the actual ray-sum equally among all the points in the ray. While satisfying the ray-sums of a particular projection, the process usually disturbs the ray-sums of previously satisfied projections. However, as we repeatedly go through satisfying all the projections, the disturbances get smaller and smaller, and eventually the method converges to a picture which satisfies all projections."(15)

Because changes are made fairly uniformly, the final product is the smoothest reconstruction satisfying the given projections. In general, the reconstructed image shows only those features that are forced on it by the projections and not those features introduced by the reconstruction process.

One iteration is considered to be the process of satisfying all projections one after the other only once. Predictably, accuracy of ART increases with the number of iterations. At first, the successive pictures become progressively better, but once an "optimum stopping point" is passed, the reconstructions become progressively worse. This phenomenon has been further examined by Herman et al.(16) One must remember that the "ideal outcome cannot be attained due to limitations on the amount and quality of data as well as the reconstruction algorithms themselves."(17)

Iterative methods may differ in the way the corrections are calculated and reapplied during each iteration. In Gordon's tutorial on ART (17) he states that the choice between additive ART and multiplicative ART (MART) depends on the physics of the radiation used. "For transmitted radiation, the form of the reconstructed object should be independent of an additive constant. Such a constant may result from variable exposure in an x-ray, variable development of the film, or an intervening filter...."

In studies by Minerbo and Sanderson,(5) and Gordon, Bender, and Herman,(18) MART reconstruction was claimed to be able to produce a solution with the largest maximum entropy. As early as 1971, Gordon, Bender, and Herman were trying to develop reconstruction algorithms that would give solutions that were minimally biased because of maximized entropy. It has been noted (18) but not proved for the general case that since the entropy of the source function increases with iteration when the multiplicative ART algorithm is used, there is a relation between MART and the maximum entropy solution.

One iterative algorithm, maximum entropy (MENT), has been implemented to produce a maximum entropy solution to the problem of reconstructing a source from a discrete set of projections.(6) In general, maximum entropy techniques are powerful but expensive with respect to computing time. From the standpoint of information theory, the maximum entropy technique is conceptually attractive. It yields the image with the lowest information content consistent with the available data. Thus with this approach one avoids introducing extraneous information or artificial structures. The problem of reconstructing a source distribution from a limited number of projections is known to be indeterminate. "A maximum entropy method thus seems attractive for this problem, especially when the available projection data are incomplete or degraded by noise errors."(6) By being an indeterminate problem, it is implied that an infinite number of nonunique solutions can satisfy the limiting criteria imposed on the solution by the actual data. When the data source has a simple structure or is close to circular symmetry in shape, one can use this a priori knowledge to eliminate various "unacceptable" results.

MENT may be considered an iterative technique for solving a constrained optimization problem. A constraint approach enhances a desired feature of the data at the expense of blurring other parts. The maximum entropy approach will not introduce new information but may not reproduce all "real" information either.(6)

Minerbo has shown (6) that with a small number of views (<10) iterative methods generally perform better than either Fourier space inversions or convolutional backprojection methods. MENT was originally developed to provide a maximum entropy solution that would not exhibit some of the "streaking" artifacts of ART and MART, which occur with a small number of projections.

3. Geometrical Unfolding

The original unfolding technique developed at Los Alamos, Two-dimensional (TWODIM),(1,2) involved the computation of the source intensity of successive rings that were defined by the stepsize of the diametral scans or projections. This successive solution of each ring has the tendency to be highly dependent on the noise of the projection data. A subsequent modification of the technique eliminated this instability problem but decreased the resolution of the technique.

4. Summary of Algorithms

We have evaluated the applicability of four techniques: backprojection, MART, MENT, and TWODIM, for reconstructing projections of a fission product to obtain the best estimate of its two-dimensional distribution. The filtered backprojection technique appeared to be superior for all of our test cases, as well as for the fuel specimens examined.

3. DISCUSSION

A simulated gamma-ray source was used to evaluate the applicability of each of the four algorithms for reconstructing the original distribution. Figure 2 shows one such simulated gamma-ray source: a narrow ring source superimposed on a solid source with the entire source skewed over the simulated scan region to reflect a 30 percent change across the diameter of the fuel rod. This type of source distribution is similar to the distribution of a volatile fission product that migrates radially to the pellet-cladding interface (i.e., cesium and iodine isotopes). The 30 percent change across the diameter was included to ensure asymmetrical projections. From this test case, six projections were calculated at 30° intervals from 0° to 150° and used as input for the reconstruction algorithms. The filtered backprojection Fourier transform technique resulted in the best reconstruction. The results are shown in Fig. 3.

The backprojection algorithm is best suited for a large number of projections; therefore, we used a linear interpolation technique to generate projections that would lie between the six projections. This interpolation process does not increase the stated information content of the reconstruction, but does produce a reconstruction more consistent with the known circular shape of the fuel pin. The concept is similar to that of smoothing data to improve its appearance even when some information is actually lost in the process.

The filtered backprojection reconstruction technique was used to analyze the projection data obtained from an FBR fuel pin. The fuel specimen was 0.538 cm in diameter and had a burnup of 19.2 at. percent. Figure 4 shows the results for the ^{137}Cs isotopic distribution with the ^{137}Cs being deposited in the outer regions of the fuel, just on the inside surface of the stainless steel cladding. The relative radial position of the cladding is given by the ^{60}Co distribution in Fig. 5.

4. CONCLUSIONS

Of the four reconstruction techniques evaluated, the filtered backprojection technique provided the most consistent results for both the simulated gamma-ray sources and experimentally-measured fuel pin. This is the only available technique for measuring nondestructively the two-dimensional radial distributions of fission products within irradiated fuel materials. Both FBR and LWR fuel rods have been characterized using this reconstruction technique, which is now an integral part of the fuel characterization program at the Los Alamos National Laboratory.

5. ACKNOWLEDGEMENTS

This work was funded by the Department of Energy and partly by the Electric Power Research Institute. The authors wish to express their appreciation to J. N. Quintana, A. S. Murray, D. K. Hamlin, E. G. Medina-Ortega for their assistance in the collection and analysis of the data.

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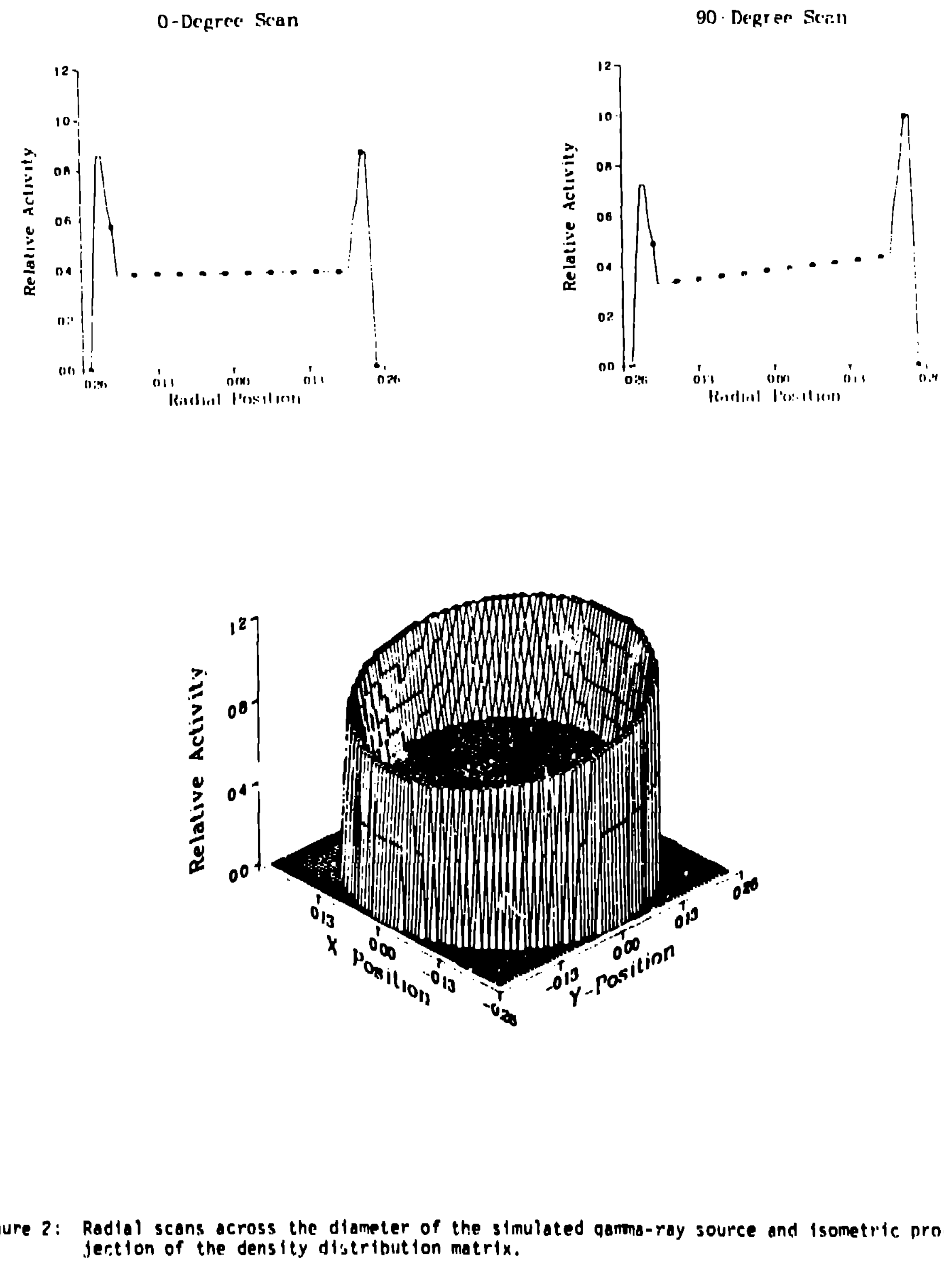

Figure 2: Radial scans across the diameter of the simulated gamma-ray source and isometric projection of the density distribution matrix.

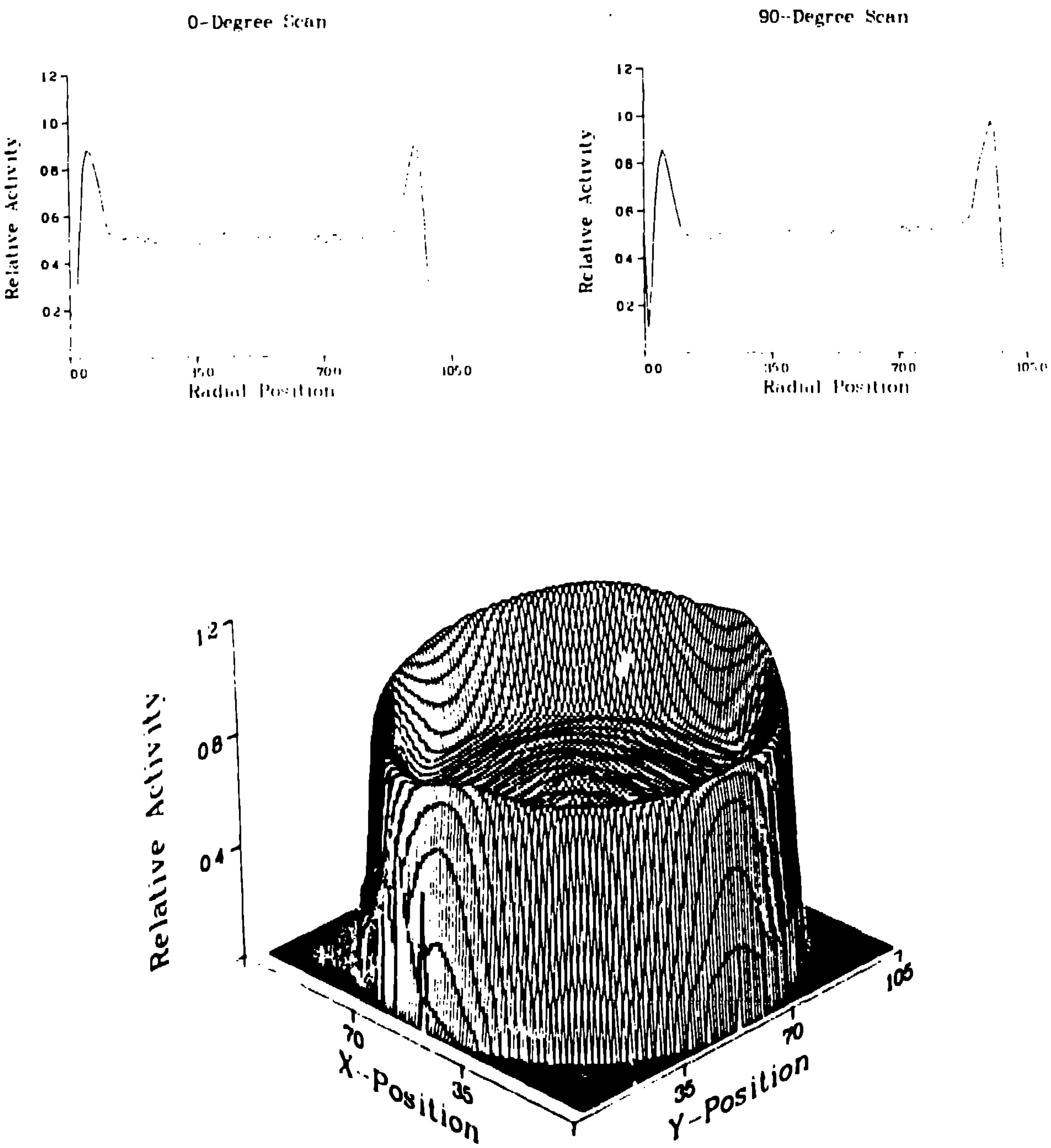

Figure 3: Reconstruction of the test case using the filtered backprojection technique.

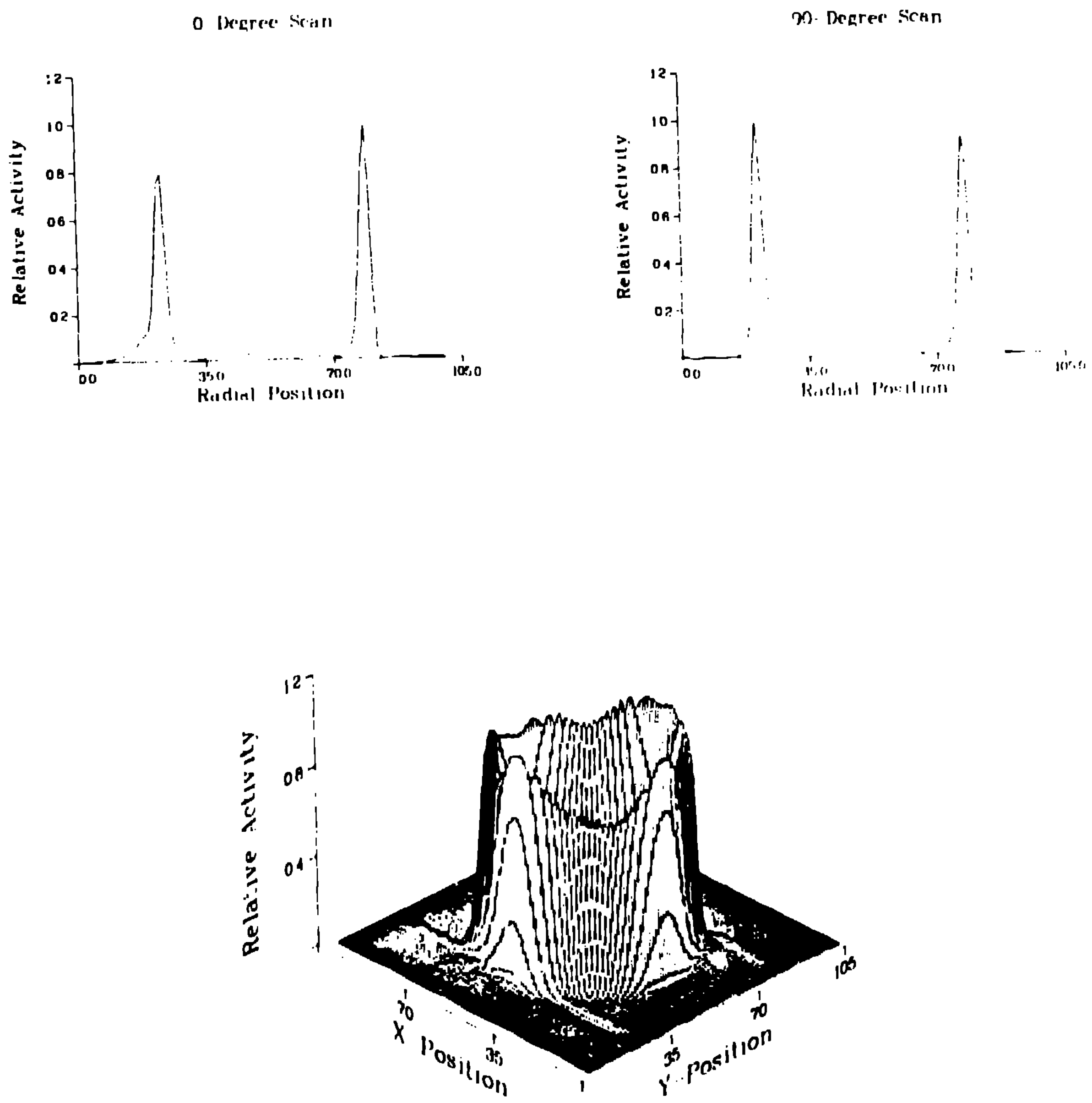

Figure 4: The results for ^{137}Cs from the filtered backprojection algorithm including attenuation correction.

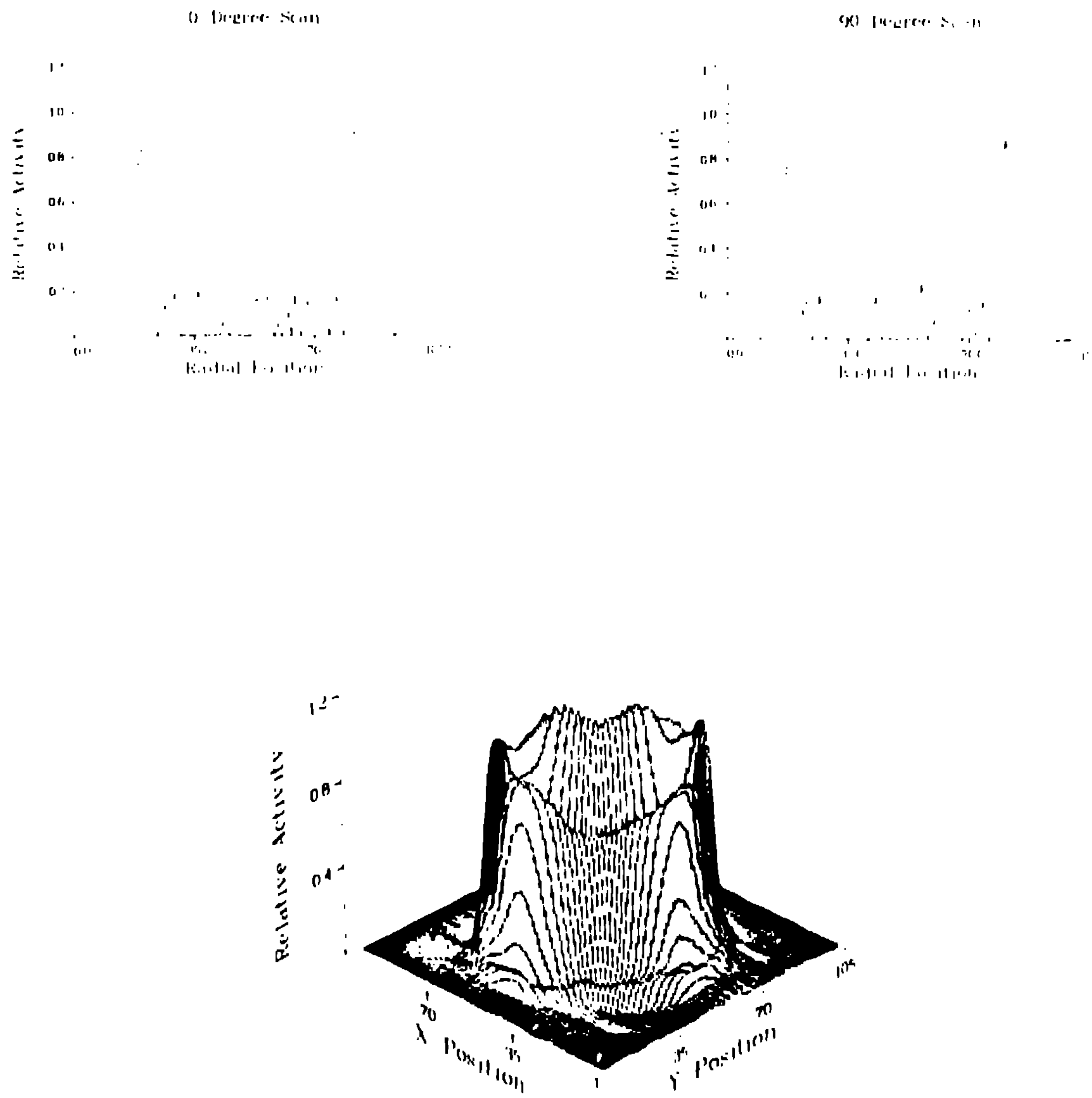

Figure 5: Input data and results for ^{60}Co from the filtered backprojection algorithm including attenuation correction.