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MAXIMUM ENTROPY RESTORATION OF LASER-FUSION TARGET X-RAY PHOTOGRAPHS*

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Burg[1,2,3] has suggested the application of the maximum entropy principle of analysis to the computation of power spectra. A number of publications followed (e.g., 4, 5, 6) which expanded the application to a variety of problems. Ball (ref. 7) has emphasized that the case for this method of analysis may have been overstated at times: caveat emptor!

Frieden[8,9,10] has applied the method to the analysis of image data. We briefly review his approach as we have applied it to the images of an x-ray pinhole camera. If the image detector (film in our case) were noiseless, the relation between object and image would be given by (ref. 11)

$$I(x_k', y_\ell') = \sum_{i=1}^{n_o} \sum_{j=1}^{n_o} S\,(x_k', y_\ell', x_i, y_j)\; O\,(x_i, y_j). \tag{1}$$

I is the irradiance of a square image cell centered on x_k', y_ℓ' and O is the radiance of a square object cell centered on x_i, y_j. The 4-dimensional matrix S is the pointspread or Green's function of the problem. Its normalization is such that for any object point its integral over the image plane is unity. The effects of various instrumental influences and physical phenomena are consolidated in S. The application of the present work to other problems (e.g., to astronomical mapping of the x-ray sky) requires, in principle, only a restructuring of the S calculation and manipulation. From a probability analysis Frieden then obtains

$$I(x_k', y_\ell') = \sum_i \sum_j \hat{O}(x_i, y_j)\; S\,(x_k', y_\ell', x_i, y_j) + \hat{N}(x_k', y_\ell') - B, \tag{2}$$

$$P_o = \sum_i \sum_j \hat{O}(x_i, y_j), \tag{3}$$

$$\hat{O}(x_i, y_j) = \exp[-1 - \mu - \sum_k \sum_\ell \lambda_{k,\ell}\; S(x_k', y_\ell', x_i, y_j)], \tag{4}$$

$$\hat{N}(x_k', y_\ell') = \exp[1 - \lambda_{k,\ell}/\rho]. \tag{5}$$

B is an adjustable parameter set to guarantee $N = n + B \geq 0$ where n is the actual noise in the system, ρ is an adjustable parameter, $\lambda_{k,\ell}$ and μ are variables to be solved for, $\hat{O}$ is the restored object matrix, $\hat{N}$ is the noise matrix and P_o is the total radiance of the object as established from I.

We now specialize to the particular problem where the object is a glowing laser-fusion target microsphere 0.95 cm from a pinhole of radius 2×10^{-4} cm, the image is 7.2 cm from the pinhole and the photon wavelength is likely to be 6.2×10^{-8} cm. The image matrix was constructed from a microdensitometer

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analysis of the image on Kodak RAR2490 film. R. F. Benjamin of this Laboratory kindly provided the film.

The circle of confusion of a pinhole camera can be estimated by simply adding the effects of geometric optics and diffraction. If the pinhole, of radius a, is illuminated by a spherically divergent wavefront, radius of curvature R, at image distance Z geometrical optics gives a circle of confusion radius $a(R + Z)/R$. The light is also diffracted through an angle δ/ka ($k = 2\pi/\delta$), $\delta \approx 1$, its exact value depending on the definition of the edge of the diffraction pattern. The radius of the circle then becomes $R_{eff} = a(R + Z)/R + Z\delta/ka$ and its minimum occurs at $(a_{opt})^2 = Z\delta(R/(R + Z))/k$; implying an optimum Fresnel number of $F_{opt} \approx 1/3$. In this case diffraction dominates the pointspread function. Light scattering in the emulsion (ref. 12) can also contribute. However, x-ray scattering is small and we neglect it for the nonce. The advantage of optimizing the camera are: best possible reso ution; design is as insensitive to pinhole contour as possible since $\partial R_{eff}/\partial a = 0$. To calculate the pointspread function we consider a circular aperture of radius a, in a screen located at $Z = 0$, uniformly illuminated by a spherically divergent wavefront of radius of curvature R. At the screen the field is $E(r,0) = \exp\{-ikr^2/2R\}$. Upon insertion of this into the Fresnel integral for axisymmetric diffraction we find at a distance Z from the screen

$$E(r,Z) = \frac{ik}{Z}\exp\{-ik(Z + r^2/2Z)\}\int_0^a e^{-ik\alpha\bar{r}^2/2Z}\, J_o\left(\frac{k\,r\,\bar{r}}{Z}\right)\bar{r}\,d\bar{r},$$

where $\alpha = (R + Z)/R$. For small Fresnel numbers the integral exponential is slowly varying and we can partially integrate setting $U = \exp(-ik\alpha\bar{r}^2/2Z)$, $dv = J_o(kr\bar{r}/Z)\bar{r}d\bar{r}$. Repeating the process indefinitely we find

$$E(r,Z) = \frac{1}{\alpha}\exp\{-ik[Z + (r^2 + \alpha^2 a^2)/2Z]\}\sum_{n=1}^{\alpha}(ik\alpha a^2/Z)^n\,\frac{1}{\rho^n}\,J_n(\rho), \qquad (6)$$

which converges for all values of the Fresnel number; $F = k\alpha a^2/2\pi Z$ and $\rho = kar/Z$. To a sufficient approximation for our purposes

$$|E(r,Z)|^2 \sim \left[\sum_1^5(-1)^n\left(\frac{k\alpha a^2}{Z\rho}\right)^{2n} J_{2n}(\rho)\right]^2 + \left[\sum_1^5(-1)^n\left(\frac{k\alpha a^2}{Z\rho}\right)^{2n-1} J_{2n-1}(\rho)\right]^2 \qquad (7)$$

where we now take $\rho = ka|\vec{x} - \vec{x}'|/Z$: $\vec{x}$ being a point in image space and $\vec{x}'$ being an object space point projected on the image plane. This photon distribution is then used in its nonfactorable form to calculate the pointspread function.

We now consider the computational aspects of the problem. Let the image space be overlayed by a square grid n by n, and the object space by a square grid m by m. Since we want to use an n of 100 or more we cannot handle the complete discretized pointspread matrix, which with diffraction, would have $(mn)^2$ elements. Instead we truncate the original pointspread function at a spot size which includes about 95% of the energy. In the present case this reduces the number of elements by a factor of 250 and permits storage of the matrix on disk (after deletion of zeroes by compaction).

Finding the m by m object matrix which maximizes the (constrained) entropy requires the solution of $n^2 + 1$ simultaneous nonlinear equations. This is done by a Newton Raphson method which requires at each iteration, calculation of the $(n^2 + 1)$ by $(n^2 + 1)$ symmetric Jacobian and solution of the $n^2 + 1$ linear system. The truncation of the discretized pointspread function also serves, fortunately, to make the Jacobian sparse and of a particular pattern. The n^2 by n^2 part, if considered to be an n by n block matrix is a band matrix, with, say, d sub and super diagonals, where $(d + 1)/n$ is approximately the ratio of the spot diameter to the side of the image space. In the limit of a very fine grid (large n) this ratio is constant: the spot approaches the point source image of the pinhole. Furthermore, the n by n matrices of ordinary numbers which constitute the nonzero blocks of the Jacobian have the same structure. The one extra row and column of the Jacobian, which corresponds to the overall power constraint, are full.

We can just afford to store on disk the approximately $2\ d^2n^2$ distinct nonzero elements of the symmetric Jacobian but two tricks are required for solving the linear system $Jx = y$ inside the Newton Raphson iteration loop. The first takes care of the single full row and column. The second avoids fill-in of the sparse Jacobian during the solution process for the linear system. Let

$$J = \left(\begin{array}{c|c} K & R \\ \hline R^T & J_o \end{array}\right),\ x = (u,\ x_o),\ y = (v,\ y_o),$$

where R, u and v are n^2 by one and J_o, x_o, y_o are scalars. Then to solve $Jx = y$ we first solve $Ku_1 = v$, $Ku_2 = R$. Then we write $u = u_1 + x_ou_2$ and solve $R^T(u_1 + x_ou_2) = J_o$ for x_o. To solve $Kw = Z$ without using any extra storage we write the decomposition $K = D + G$, where D is the block diagonal and G is the rest. Thus D is a symmetric band matrix of order n^2 with d sub and super diagonals and the system $Da = b$ can be solved efficiently with no extra space required. We then iterate the system $Dw^{(q)} = Z - Gw^{(q-1)}$ starting with $w^{(o)} = 0$. This can probably be proved convergent when we are near the solution, and it is of course desirable in any event to start the Newton Raphson procedure fairly near the solution; for example by solving the problem first with a fairly coarse mesh. We note that the work required for each q is of order d^2n^2; for $q = 1$ this work is required for initial trianglularization of D, and for $q \neq 1$ it is required for the multiplication of G (which has about $d(2d + 1)$ nonzero elements per row).

With these techniques in hand we have analysed the microdensitometer traces of a laser-fusion target photograph. We are presently generating restored object matrices of 80 x 80 pixels from image matrices of 64 x 64 pixels. We have the prospect of increasing both of these numbers significantly. During the oral presentation of this paper we will examine an original x-ray photograph, its microdensitometer profile and restorations made with various values of the quasi-free parameters in the calculation. We will also consider computer time requirements.

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