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## Decision Analysis for Dynamic Accounting of Nuclear Material

by

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### ABSTRACT

Effective materials accounting for special nuclear material in modern fuel cycle facilities will depend heavily on sophisticated data-analysis techniques. Decision analysis, which combines elements of estimation theory, decision theory, and systems analysis, is a framework well suited to the development and application of these techniques. Augmented by pattern-recognition tools such as the alarm-sequence chart, decision analysis can be used to reduce errors caused by subjective data evaluation and to condense large collections of data to a smaller set of more descriptive statistics. Application to data from a model plutonium nitrate-to-oxide conversion process illustrates the concepts.

KEYWORDS: Nuclear safeguards, materials accounting, decision analysis, CUSUM, Kalman filter, alarm-sequence chart, plutonium nitrate-to-oxide conversion

### INTRODUCTION

Materials accounting for safeguarding special nuclear material (SNM) has two important aspects: (1) the collection of materials accounting data, and (2) the analysis of materials accounting data. The collection function is a broad, highly developed subject (e.g., see Refs. 1-5 and the references therein) that we will not pursue here; in this paper we are primarily concerned with the analysis of materials accounting data.

The data collection function is usually structured to facilitate performance of the analysis function, commonly by providing sufficient measurements of SNM within a facility to allow the drawing of material balances around selected portions of the facility on a reasonable time scale. The data, which are always corrupted by measurement errors, often appear as time sequences of material balances, one sequence from each part of the facility for which material balances are drawn.

Therefore, the data-analysis function must operate on imperfect data that become available sequentially in time. Its primary goals are (1) detection of the event(s) that SNM has been diverted, (2) estimation of the amounts diverted, and (3) determination of the significance of the estimates. Furthermore, data analysis must search for evidence of diversion that may have occurred in any of several patterns.

These goals are ideal for statistical treatment using sequential, probabilistic techniques that have been developed for applications to communications and control systems.[6-9] Decision analysis[10,11] is a framework of such tools, and combines techniques from estimation theory and hypothesis testing, or decision theory, with systems analysis for treating complex, dynamic problems. The decision-analysis framework is general enough to allow a wide range in the level of sophistication in examining SNM accounting data, while providing guidelines for the development and application of a variety of powerful methods.

## THE DECISION PROBLEM

Let us suppose that over some time period we have acquired materials accounting data consisting of a set of in-process inventory measurements at discrete times and a set of material transfer measurements between those times. Let $I(k)$ be the $k^{th}$ inventory measurement, and let $T(k)$ be the measurement of the net transfers that occurred between the $I(k)$ and $I(k+1)$ inventory measurements. If the measurements were exact and there had been no diversion of SNM, then the continuity equation would be satisfied:

$$I(k+1) = I(k) + T(k) \quad . \tag{1}$$

However, we never have perfect measurements of bulk material, and SNM may or may not have been diverted, so that (1) should be rewritten as

$$I(k+1) = I(k) + T(k) - M(k+1) \quad , \tag{2}$$

where $M(k+1)$ is the imbalance in the continuity equation (1) at time $k+1$ caused by measurement errors and diversion. We call $M(k)$ the material balance value at time $k$, or the $k^{th}$ material balance for short. Clearly, $M(k)$ is a random variable, and the sequence $\{M(k), k = 2,3,\ldots\}$ is a stochastic process, about which we can make probabilistic statements if something is known of the nature of the measurement errors.

For convenience, denote the set of inventory measurements $\{I(k), k = 1,2,\ldots,N\}$, the set of net transfer measurements $\{T(k), k = 1,2,\ldots,N-1\}$, and the statistical information on the measurement errors by the quantity $Z(N)$. Then $Z(N)$ contains all information available for the N materials accounting intervals. Thus, the decision problem is to determine, based on observation of $Z(N)$, whether the set $\{M(k+1), k = 1,2,\ldots,N-1\}$ ($M^N$ for simplicity) contains diversion.

### The Likelihood Ratio

For any particular $Z(N)$ that is observed, $M^N$ may or may not have contained diversion. Define the hypotheses

H: $M^N$ does not contain diversion,

K: $M^N$ does contain diversion.

Then the actual probability density function that governs $Z(N)$ is determined by which of H, K is true; i.e.,

$Z(N) \sim p[Z(N)|H]$ for H true,

$Z(N) \sim p[Z(N)|K]$ for K true,

where $\sim$ means "has the density function." These two conditional density functions are also called <u>likelihood functions</u> for the hypotheses H and K. The values of the likelihood functions for a particular $Z(N)$ are relative measures of the likelihood that $Z(N)$ is governed by one or the other density function, or in other words, that H is true or K is true.

The usual statistical test consists of forming the <u>likelihood ratio</u>,[6-9] L, and comparing it to a threshold:

$$\text{If } L[Z(N)] \triangleq \frac{p[Z(N)|K]}{p[Z(N)|H]} \begin{cases} < T, \text{ accept H,} \\ > T, \text{ accept K,} \end{cases} \tag{3}$$

where T is the threshold to be determined below. Roughly speaking, if $Z(N)$ is "enough" more likely to have occurred as a result of H being true than of K being true, then decide that H is true; otherwise, decide that K is true.

## Conversion to Sufficient Statistics

The likelihood functions are difficult to work with because they are joint density functions of many variables, in general. Under certain circumstances, which usually hold for SNM accounting, it is possible to condense the quantity Z(N) to a single number S(N) without loss of information. The number S(N) is called a sufficient statistic[7] and is equivalent to knowledge of Z(N). If such a S(N) can be found, and if its density function can be calculated, then the likelihood ratio test (3) can be replaced by

$$\text{If } L[S(N)] = \frac{p[S(N)|K]}{p[S(N)|H]} \begin{cases} < T', \text{ accept } H, \\ > T', \text{ accept } K. \end{cases} \tag{4}$$

Now, the density (i.e., likelihood) functions are univariate and, therefore, much more tractable mathematically. The hidden problem is to find a sufficient statistic that is significantly indicative of whether H or K is true.

## Sequential Formulation

As we have seen, the likelihood ratio test, (3) or (4), for a fixed number N of points consists of comparing the likelihood ratio to a single threshold. However, in practical situations we seldom will know when the test should start or end. Therefore, we will want to begin the test at all possible starting points and let the test itself determine when it should be terminated. This procedure also has the provident property of requiring fewer samples, on the average, than a fixed-sample-size test having the same characteristics.[8]

For the sequential likelihood ratio test, also called the sequential probability ratio test or SPRT,[8] there are three possible results at each decision time, rather than two:

$$\text{If } L[S(k)] \begin{cases} \le T_0, \text{ accept } H, \\ \ge T_1, \text{ accept } K, \\ \text{otherwise, take another observation,} \end{cases} \tag{5}$$

and the SPRT is repeated for all possible starting points. The thresholds $T_0$ and $T_1$ can be found from the Neyman-Pearson criterion or by minimizing the Bayes risk,[7] but that may require some information that is unavailable. The following approximation, devised by Wald, gives useful thresholds that can be shown to be conservative.

Let $P_M$ and $P_F$ be the desired (given) miss and false-alarm probabilities, respectively, for the SPRT. Then the thresholds are[7,8]

$$T_0 = \frac{P_M}{1 - P_F} \ , \qquad T_1 = \frac{1 - P_M}{P_F} \ . \tag{6}$$

The probability of detecting diversion, related to $1-P_M$, is called the power or size of the test; $P_F$ is called the significance or level of the test.

## SOME SUFFICIENT STATISTICS

For any decision problem, there is a large number of sufficient statistics that may be calculated, but some are more useful than others because of computational reasons, closer relationship to physically meaningful quantities, or better discriminatory powers between the two hypotheses. Following are several statistics that have been effective in various applications.

## The CUSUM Statistic

The CUSUM[1,12-16] (cumulative summation) of material balances is just the sum of the material balances over the time period of interest.

$$\text{CUSUM}(k+1) = M(2) + M(3) + \ldots + M(k+1) \quad ,$$

where the M(i) are found from (2):

$$M(i+1) = - I(i+1) + I(i) + T(i) \quad , \qquad i = 1,2,\ldots,k \quad .$$

The CUSUM can also be written as

$$\text{CUSUM}(k+1) = - I(k+1) + I(1) + \sum_{i=1}^{k} T(i) \quad ,$$

which emphasizes that the material balances are negatively correlated through the common inventory measurement. For uncorrelated transfer measurements, the CUSUM variance is

$$VC(k+1) = VI(k+1) + VI(1) + \sum_{i=1}^{k} VT(i) \quad ,$$

where VI(·) and VT(·) are the inventory and transfer measurement error variances, respectively. In recursive form suitable for a SPRT, the cusum and its variance can be written as

$$\text{CUSUM}(k+1) = \text{CUSUM}(k) - I(k+1) + I(k) + T(k) \quad , \tag{7}$$

$$VC(k+1) = VC(k) + VI(k+1) - VI(k) + VT(k) \quad . \tag{8}$$

The corresponding SPRT can be shown to reduce to

$$\text{If } \frac{\text{CUSUM}(k+1)}{\sqrt{VC(k+1)}} \begin{cases} \leq -\sqrt{2|\ln T_0|} & \text{, accept H,} \\ \geq +\sqrt{2|\ln T_1|} & \text{, accept K,} \\ \text{otherwise, take another observation.} \end{cases} \tag{9}$$

The CUSUM statistic is interesting because it is an estimate of the total amount of missing material during the period. However, the CUSUM is not a minimum-variance statistic unless the variances of the material balance measurements are all equal, and unless the knowledge of how the material balances combine inventory and transfer measurements is unimportant. The last condition would hold if the inventory were small or well measured compared to the transfers.

## The Two-State Kalman Filter Statistic

The two-state Kalman filter statistic[2,17-20] estimates the average amount of missing material per balance. It uses all available information from the continuity equation (2) and from the statistics of the measurement errors. The two-state Kalman filter statistic can be shown to be optimal in the sense that it is the minimum-variance, unbiased, linear estimate whenever the measurement error probability densities are symmetric about their means.[21]

The two-state Kalman filter yields estimates of both the inventory and the material balance at each time. In recursive form, the equations are[21,22]

$$\hat{I}(k+1) = \hat{I}(k+1|k) + K_1 \, [I(k+1) - \hat{I}(k+1|k)] \quad ,$$
$$\hat{M}(k+1) = \hat{M}(k) + K_2 \, [I(k+1) - \hat{I}(k+1|k)] \quad , \tag{10}$$

where

$$\hat{I}(k+1|k) = \hat{I}(k) + T(k) - \hat{M}(k) \quad ,$$

and $\hat{I}(k+1)$ and $\hat{M}(k+1)$ are the inventory and material balance estimates, respectively, at time k+1 based on all information through time k+1. The filter gains, $K_1$ and $K_2$, are given by

$$K_1 = \frac{VIE(k+1)}{VI(k+1)} \quad ,$$
$$K_2 = \frac{VMIE(k+1)}{VI(k+1)} \quad , \tag{11}$$

where VIE(k+1) and VMIE(k+1) are respectively the inventory estimate error variance and the covariance between the inventory and material balance estimate errors. They are given recursively by

$$VIE(k+1) = \frac{VIE(k+1|k)\ VI(k+1)}{VIE(k+1|k) + VI(k+1)} \quad ,$$
$$VMIE(k+1) = \frac{VMIE(k+1|k)\ VI(k+1)}{VIE(k+1|k) + VI(k+1)} \quad , \tag{12}$$

with

$$VIE(k+1|k) = VIE(k) - 2VMIE(k) + VME(k) + VI(k) \quad ,$$
$$VMIE(k+1|k) = VMIE(k) - VME(k) \quad . \tag{13}$$

The variance of the error of the material balance estimate at time k+1, VME(k+1), is

$$VME(k+1) = VME(k) - \frac{VMIE^2(k+1|k)}{VIE(k+1|k) + VI(k+1)} \quad . \tag{14}$$

See References 2 and 17-24 for more detail.

The resulting SPRT is similar to that for the CUSUM, and reduces to

$$\text{If} \quad \frac{\hat{M}(k+1)}{\sqrt{VME(k+1)}} \begin{cases} \leq -\sqrt{2|\ln T_0|} & \text{, accept H,} \\ \geq +\sqrt{2|\ln T_1|} & \text{, accept K,} \\ \text{otherwise, take another observation.} \end{cases} \tag{15}$$

## Other Sufficient Statistics

All sufficient statistics such as those just discussed are called parametric because they depend upon knowledge of the statistics of the measurement errors. They also happen to work best when the measurement errors are Gaussian, a quite common but by no means all-inclusive situation. If the measurement error statistics are unknown or non-Gaussian, then nonparametric[25] sufficient statistics may give better test results. In addition, nonparametric tests can provide independent support for the results of parametric tests even though nonparametric tests are generally less powerful than parametric ones under conditions for which the latter are designed.

The two most common nonparametric tests are the sign test and the Wilcoxon rank sum test. The sufficient statistic for the sign test is the total number of positive material balances. That for the Wilcoxon test is the sum of the ranks of the material balances. The rank of a material balance is its relative position index under a reordering of the material balances according to magnitude. Other, possibly more effective, nonparametric tests are being investigated. Further discussion of nonparametric tests is beyond the scope of this paper.

## TEST APPLICATION

### Procedure

As discussed above, we seldom will know beforehand when diversion has started or how long it will last. Therefore, the decision tests must examine all possible, contiguous subsequences of the available materials accounting data. That is, if at some time we have N material balances, then there are N starting points for N possible sequences, all ending at the $N^{th}$, or current, material balance, and the sequence lengths range from N to 1. Because of the sequential application of the tests, sequences with ending points less than N have already been tested; those with ending points greater than N will be done if the tests do not terminate before then.

Another procedure that helps in interpreting the results of tests is to do the testing at several significance levels, or false-alarm probabilities. This is so because, in practice, the test thresholds are never exactly met; thus, the true significance of the data is obscured. Several thresholds corresponding to different false-alarm probabilities give at least a rough idea of the actual probability of a false alarm.

### Displaying the Results

Of course, one of the results of most interest is the sufficient statistic. Common practice is to plot the sufficient statistic, with symmetric error bars of length twice the square root of its variance, vs the material balance number. The initial material balance and the total number of material balances are chosen arbitrarily, perhaps to correspond to the shift or campaign structure of the process. For example, if balances are drawn hourly, and a day consists of three shifts, then the initial material balance might be chosen as the first of the day, and the total number of material balances might be 24, covering three shifts. Remember, however, that this choice is for display purposes only; the actual testing procedure selects all possible initial points and sequence lengths, and any sufficient statistic may be displayed as seems appropriate.

The other important results are the outcomes of the tests, performed at the several significance levels. A new tool, called the alarm-sequence chart,[1-3,12] has been developed to display these results in compact and readable form. To generate the alarm-sequence chart, each sequence causing an alarm is assigned (1) a descriptor that classifies the alarm according to its false-alarm probability, and (2) a pair of integers $(r_1,r_2)$ that are, respectively, the indexes of the initial and final material balance numbers in the sequence.* The alarm-sequence chart is a point plot of $r_1$ vs $r_2$ for each sequence that caused an alarm, with the significance range of each point indicated by the plotting symbol. One possible correspondence of plotting symbol to significance is given in Table I. The symbol T denotes

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* It is also possible to define $(r_1,r_2)$ as the sequence length and the final material balance number of the sequence. The two definitions are equivalent.

TABLE I

ALARM CLASSIFICATION FOR THE ALARM-SEQUENCE CHART

| Classification (Plotting Symbol) | False-Alarm Probability |
|---|---|
| A | $10^{-2}$ to $5 \times 10^{-3}$ |
| B | $5 \times 10^{-3}$ to $10^{-3}$ |
| C | $10^{-3}$ to $5 \times 10^{-4}$ |
| D | $5 \times 10^{-4}$ to $10^{-4}$ |
| E | $10^{-4}$ to $10^{-5}$ |
| F | $< 10^{-5}$ |
| T | $\sim 0.5$ |

sequences of such low significance that it would be fruitless to examine extensions of them; the letter T indicates their termination points. It is always true that $r_1 \leq r_2$ so that all symbols lie to the right of a 45° line through the origin. Examples of these charts are shown in the section on results.

## AN EXAMPLE

### The Process

To illustrate the application of decision analysis, we present results from a study of materials accounting in a plutonium nitrate-to-oxide conversion facility.[3] The reference process is based on plutonium (III)-oxalate precipitation; a simplified block diagram is shown in Fig. 1. Nominal capacity is 116 kg of plutonium per day, processed in 2-kg batches. Some of the most important design parameters for the main process stream are given in Table II.

TABLE II

CONVERSION PROCESS DESIGN PARAMETERS

| Function | Volume or Weight Per Batch | Concentration | Frequency |
|---|---|---|---|
| Receipt tank feed | 200.0 L | 30.0 g/L | 1/0.41 h |
| Valence adjust feed | 66.67 L | 30.0 g/L | 1/0.41 h |
| Precipitator fee | 75.44 L | 26.5 g/L | 1/0.41 h |
| Pu oxalate boat to furnace | 4.65 kg | 0.422 kg/kg | 1/0.41 h |
| Pu oxide to accountability | 2.21 kg | 0.882 kg/kg | 1/0.41 h |
| Pu product to storage | 2.18 kg | 0.882 kg/kg | 1/0.41 h |
| Filtrate | 154.9 L | 66.4 mg/L | 1/0.41 h |
| Precipitator flush | 109.2 L | 4.6 g/L | 3/day |
| Furnace sweeping | 0.85 kg | 0.882 kg/kg | 1/week |
| Boat flush | 34.4 L | 2.9 g/L | 10/day |
| Dump station sweep | 0.85 kg | 0.882 kg/kg | 2/day |

### The Materials Accounting System

Many different ways of drawing material balances for the conversion process can be defined. Based on the conversion study,[3] one strategy that works very well is to consider the main process stream from the receipt tank to the product dump and assay station as one unit process. Thus, the transfers consist of feed into the receipt tank, product out of the dump and assay station, and recycle solids and liquids. All these transfers must be measured, and we must obtain an estimate of the in-process inventory. Table III gives the required measurements and some possible measurement methods and associated uncertainties.

TABLE III

MATERIALS ACCOUNTING MEASUREMENTS FOR THE CONVERSION PROCESS

| Measurement Point | Measurement Type* | Instrument Precision (%) | Calibration Error (%) |
|---|---|---|---|
| Receipt tank | Volume | 0.2 | 0.1 |
| | Concentration (by L-edge densitometry) | 1 | 0.3 |
| Wet boat (precipitator output) | Mass (by neutron well counter) | 2 | 5 |
| Precipitator holdup | Mass (by He-3 neutron counter) | 2 | -- |
| Filtrate | Volume | 0.2 | 0.1 |
| | Concentration (by alpha monitor) | 10 | 2 |
| Precipitator flush | Volume | 0.2 | 0.1 |
| | Concentration (by L-edge densitometry) | 1 | 0.3 |
| Boat flush | Volume | 0.2 | 0.1 |
| | Concentration (by x-ray fluorescence) | 1 | 0.3 |
| Furnace sweep | Mass (by neutron well counter) | 2 | 0.5 |
| Dump station sweep | Mass (by neutron well counter) | 2 | 0.5 |
| Product cans | Mass (by neutron well counter, calorimeter, or gamma spectrometer | 1 | 0.5 |

* See References 1-3 for detailed discussions of measurement techniques.

## Results

The techniques of decision analysis described earlier have been used to evaluate the diversion sensitivity of this materials accounting strategy and others. Part of the evaluation consists of examining test results, in the form of estimate (sufficient statistic) and alarm-sequence charts, for various time intervals. Examples of typical one-day charts for the CUSUM and two-state Kalman filter are shown in Figs. 2 and 4; the corresponding alarm-sequence charts are given in Figs. 3 and 5.[3] In the course of evaluation, many such sets of charts are examined so that the random effects of measurement errors and normal process variability can be assessed; that is, we perform a Monte Carlo study to estimate the sensitivity to diversion. In applying decision analysis to data from a facility operating under actual conditions, only one set of data will be available for making decisions, rather than the multiple data streams generated from a simulation. The decision-maker will have to extrapolate from historical information and from careful process and measurement analysis to ascertain his true diversion sensitivity.

The results of the evaluation are given in Table IV. By comparison, current regulations require that the material balance uncertainty be less than 0.5% ($2\sigma$) of throughput for each two-month accounting period, which corresponds to 33 kg of plutonium for this process. Such large improvement in diversion sensitivity is possible through the combination of timely measurements with the powerful statistical methods of decision analysis.

TABLE IV

DIVERSION SENSITIVITY FOR THE CONVERSION PROCESS

| Detection Time | Average Diversion Per Batch (kg Pu) | Total at Time of Detection (kg Pu) |
|---|---|---|
| 1 batch (1.35 h) | 0.13 | 0.13 |
| 1 day | 0.03 | 0.63 |
| 1 week | 0.01 | 1.24 |
| 1 month | 0.005 | 2.65 |

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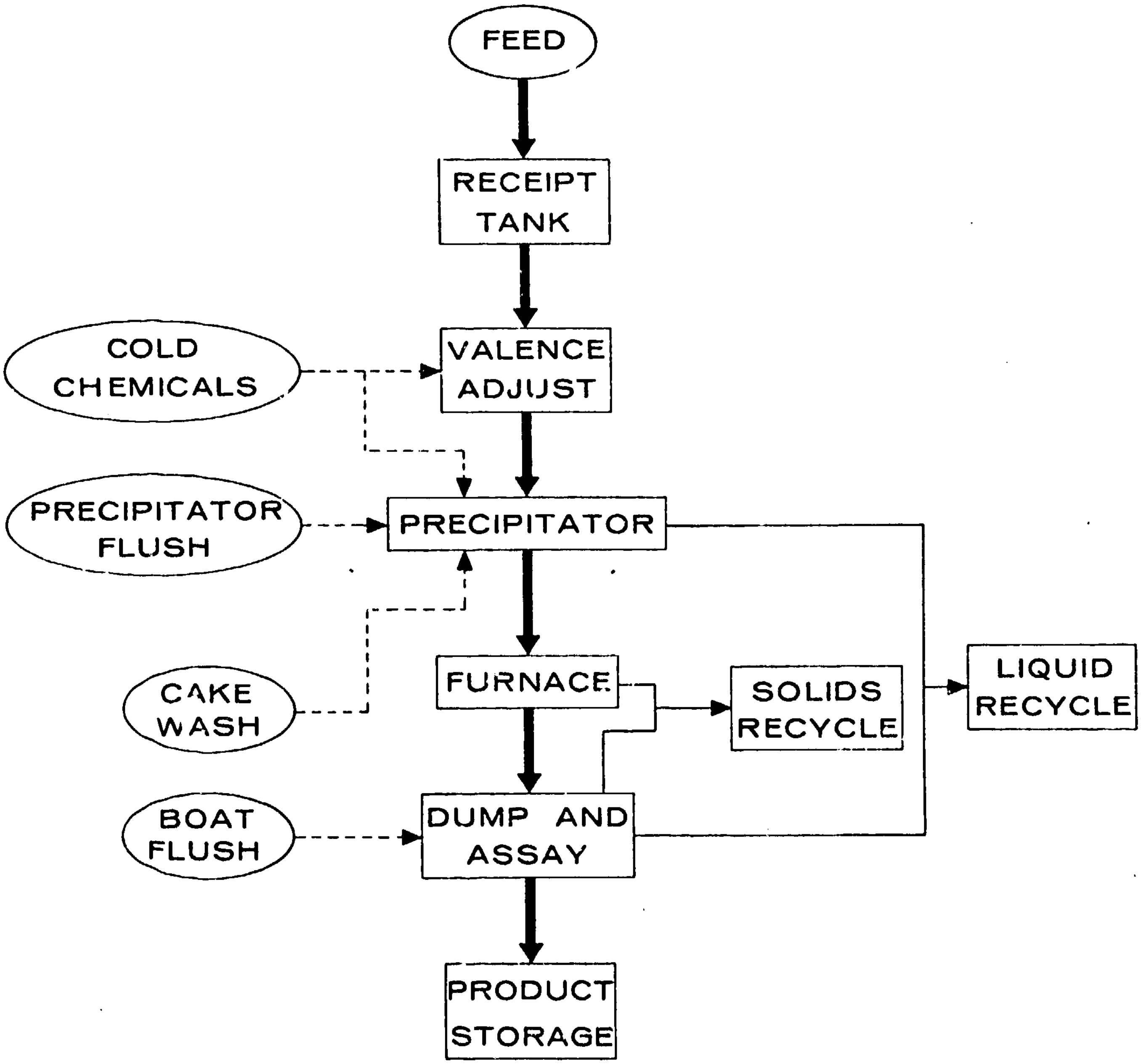


Fig. 1. Simplified block diagram of the plutonium nitrate-to-oxide conversion process.

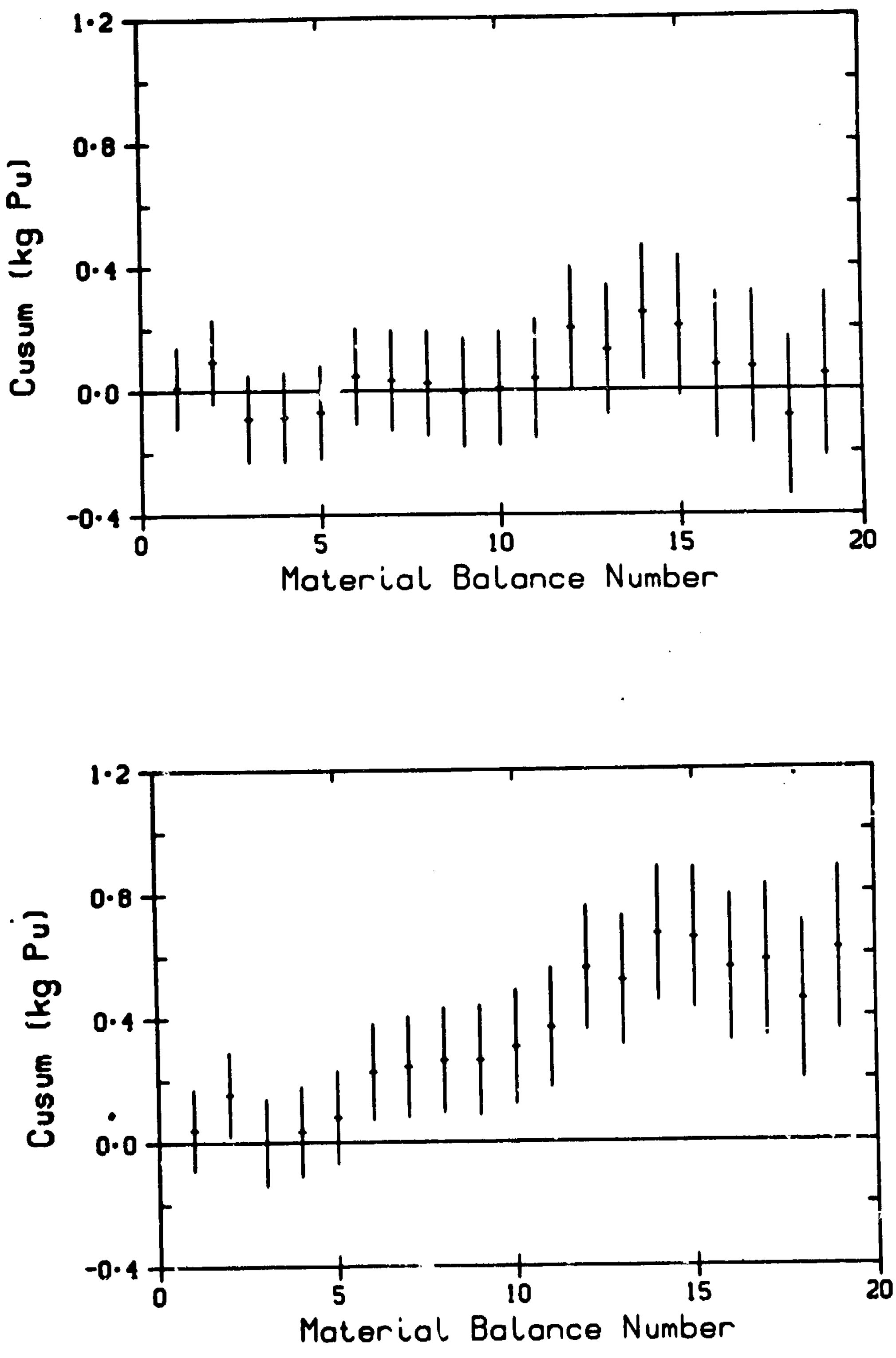


**Fig. 2.** CUSUM charts for one day with no diversion (upper), with diversion (lower).

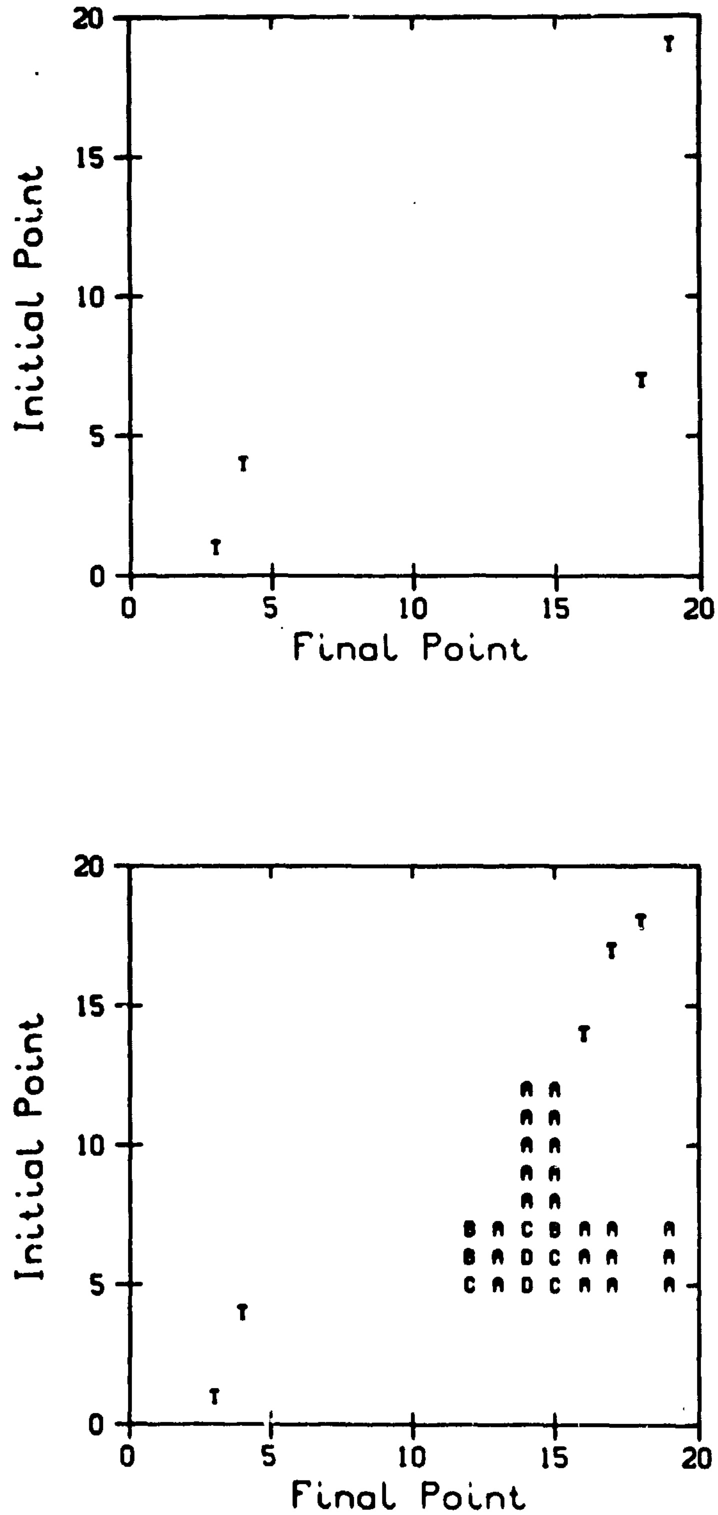


Fig. 3. Alarm-sequence charts for CUSUM: with no diversion (upper), with diversion (lower).

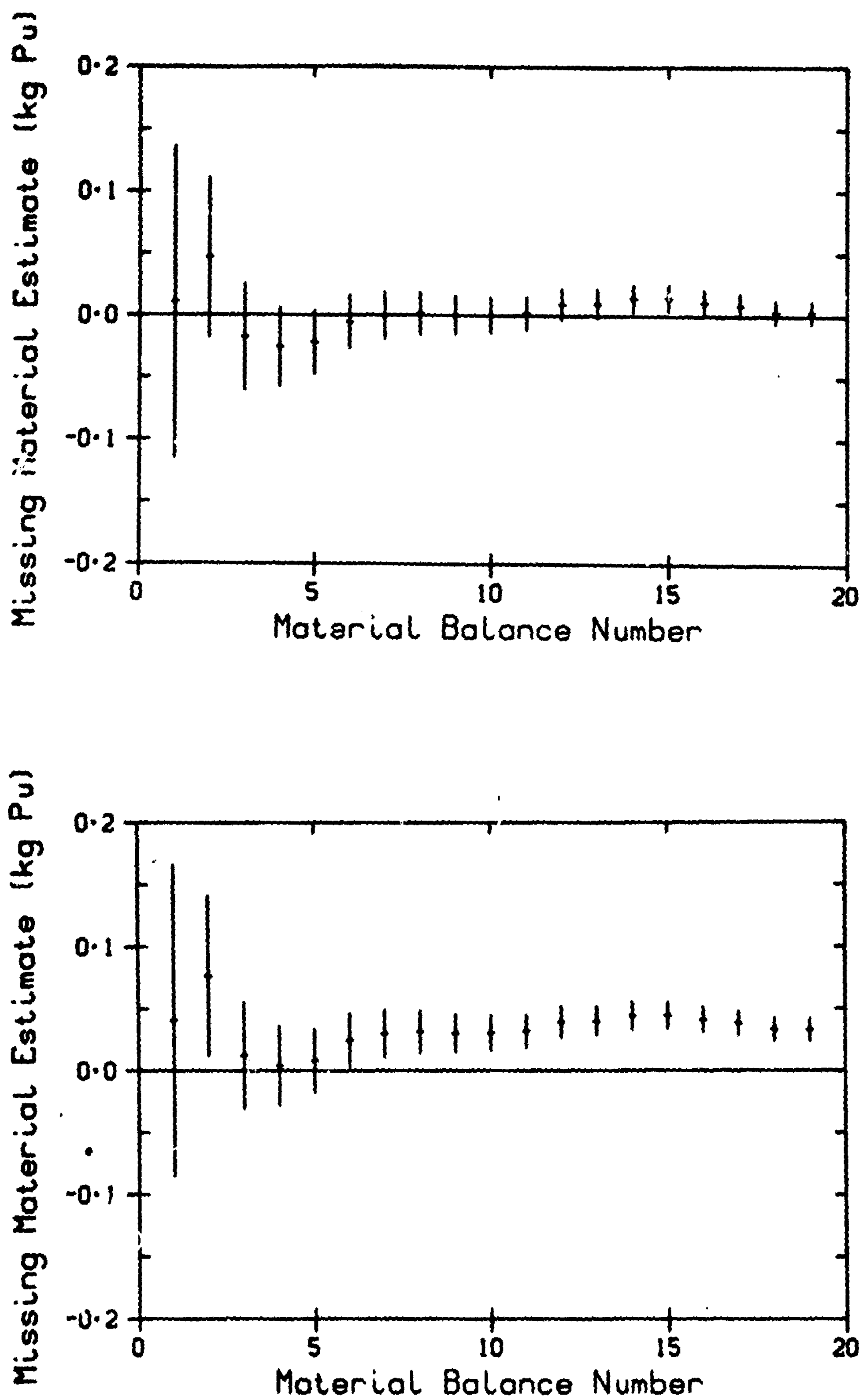


Fig. 4. Kalman filter estimates of average missing material for one day with no diversion (upper), with diversion (lower).

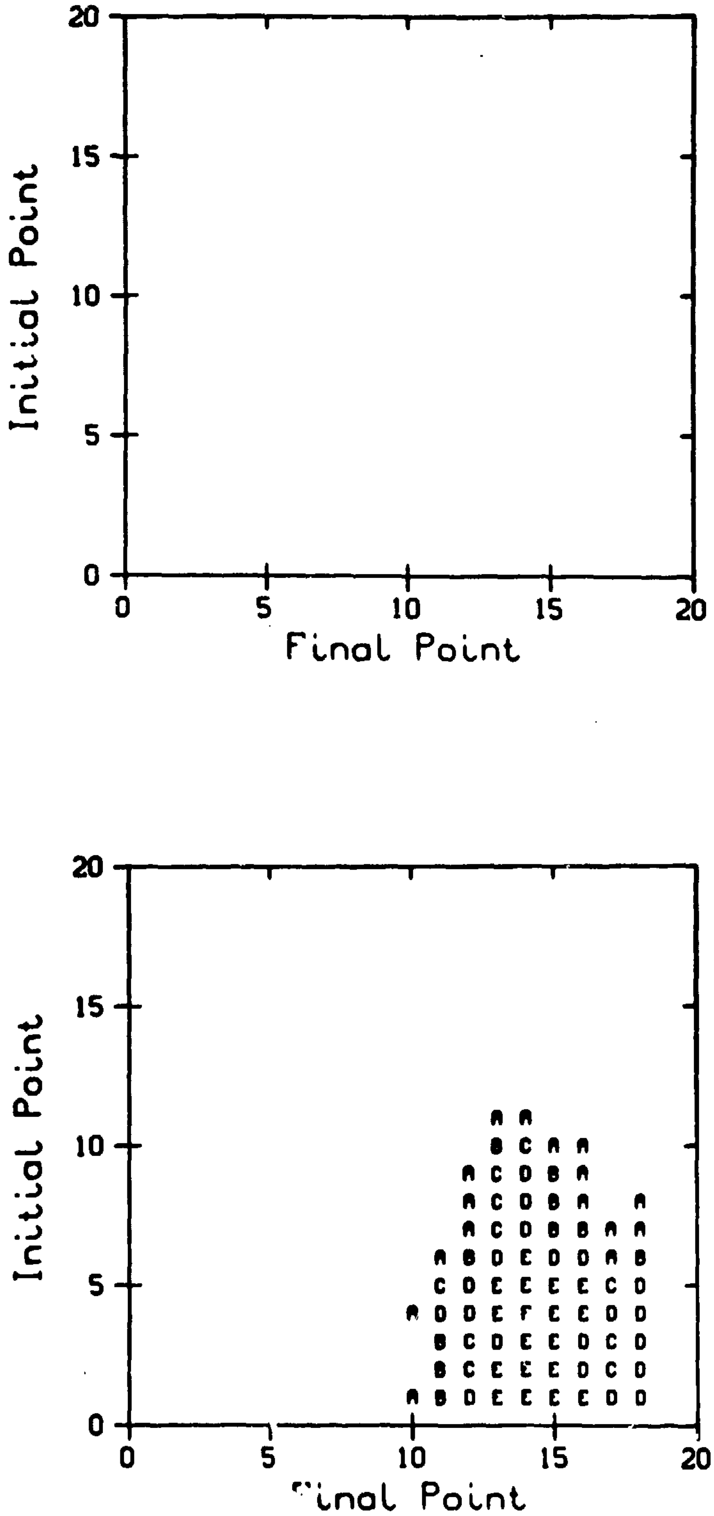


Fig. 5. Alarm-sequence charts for Kalman filter estimates: with no diversion (upper), with diversion (lower).