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PROMPT FISSION NEUTRON SPECTRA AND $\bar{\nu}_p$

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ABSTRACT

Methods used to obtain the evaluated prompt fission neutron spectrum N(E) and the average prompt neutron multiplicity $\bar{\nu}_p$ are reviewed. The relative influence of experimental data; interpolated, extrapolated, and fitted experimental data; systematics; and nuclear theory are considered for the cases where (a) abundant experimental data exist, (b) some experimental data exist, and (c) no experimental data exist. The Maxwellian and Watt distributions, and the determination of the parameters of these distributions by data fitting, are described and compared to recent new theoretical work on the calculation of N(E). Similarly, various expressions for $\bar{\nu}_p$ that have been obtained by data fitting and systematics are described and compared to recent new theoretical work. Complications in the evaluation of N(E) and $\bar{\nu}_p$ due to the onset of multiple-chance fission and the interrelationships between N(E), $\bar{\nu}_p$ and the multiple-chance fission cross section are discussed using the example of the fission of ^{235}U. Some statistics and comments are given on the evaluations of N(E) and $\bar{\nu}_p$ contained in ENDF/B-V, and a number of concluding recommendations are made for future evaluation work.

I. INTRODUCTION

The prompt fission neutron spectrum N(E) and the average prompt neutron multiplicity $\bar{\nu}_p$ are quantities of crucial importance in a number of practical considerations. Accordingly, there is strong interest in the accuracy and detail with which they are described in evaluated nuclear data files. In this work the methods of evaluating these quantities in preparation of such files is reviewed.

In Sec. II a number of methods are reviewed by which prompt fission neutron spectra are evaluated. These methods are then classified according to the amount of experimental data available for the evaluation. In Sec. III models and two new theories of the prompt fission neutron spectrum are discussed and reviewed. Specific recommendations for the evaluation of N(E) in the cases of least-squares fitting of experimental data and direct calculation are made in Sec. IV. Sections V and VI, for the methods of evaluating the average prompt neutron multiplicity $\bar{\nu}_p$, are completely analogous to Secs. II and IV, respectively. A summary together with some concluding remarks is given in Sec. VII and a number of statistics are presented on the evaluation methods used to determine N(E) and $\bar{\nu}$ in ENDF/B-V.

The most recent reviews on prompt fission neutron spectra were held at conferences in 1971 and 1975 [1-2]. A comprehensive review of $\bar{\nu}$ by Manero and Konshin [3] appeared in 1972.

II. METHODS TO EVALUATE PROMPT FISSION NEUTRON SPECTRA

A number of methods exist by which a representation of the prompt fission neutron spectrum N(E) can be obtained from experimental data, theoretical calculation, or a combination of the two. These methods include the following:

A. Direct use of experimental data. N(E), where E is the energy of the secondary or emitted neutron, is usually given by the experimentalist in histogram form, that is, N_i is given for the i'th emitted neutron energy bin $(E_i - E_{i-1})$ or for some representative energy $\bar{E}_i$ of that bin, where $\bar{E}_i$ has either been measured or defined. The spectrum N(E) is usually measured for a fixed incident energy E_n of the neutron inducing fission or for spontaneous fission. Typically, the experimental data are not presented in absolute units, that is, N(E) is usually unnormalized.

The ENDF MF=5 LF=1 law (Arbitrary Tabulated Function) can be used to tabulate a histogram of N(E). The emitted neutron energy bin limits or a representative bin energy can be used to construct the tabulation. A complete experimental data set for such a tabulation would perhaps consist of 10 to 15 fission spectra which span emitted neutron energy ranges of ~10 keV to ~ 20 MeV and which span an incident neutron energy range of thermal energy to ~20 MeV. However, no such data set yet exists. Points to consider in this approach for the data sets that do exist include the following:

1. The experimental data cover a limited range of the emitted neutron energy E. The problem therefore exists as to what to use in the external regions below the lowest emitted neutron energy cutoff (~200 keV to ~1 MeV) and above the highest emitted neutron energy cutoff (~5 MeV to ~15 MeV).

2. The problem (1) is further compounded by the fact that the integral of N(E) over all emitted neutron energies E must be unity, that is, N(E) is a normalized spectrum.
3. If one is specifying the energy bin of the histogram by a defined representative bin energy $\bar{E}_i$ some amount of care must be exercised in the definition especially in the tail region (E > ~2-3 MeV) of the spectrum where the behavior is approximately exponential. For example, the choice of the mean energy of the bin in this region would distort the measured shape of the spectrum.

B. Use of experimental data to determine parameters of models which approximate the prompt fission neutron spectrum. Least-squares or "eyeball" fitting procedures of experimental fission neutron spectra can be applied to determine the parameters of a model of the fission neutron spectrum. The parameters extracted from the fitting analysis can then be used in the model expression to represent the fission neutron spectrum at, or near, the incident neutron energy corresponding to the experiment. Of course, if experimental spectra exist for a number of incident neutron energies for the isotope of interest, then the model parameters can be extracted and used in the model expression to represent the fission neutron spectrum on a set of incident neutron energies. In this case, if the experimental spectra are sufficiently extensive and accurate, the dependence of the model parameters upon the incident neutron energy can be determined provided that the model is a realistic physical approximation.

Some of the more widely used model expression include the Maxwellian distribution, the simple form of the evaporation spectrum [4], and the Watt distribution [5]. These expressions will be discussed in more detail in Secs. III. A. and III.B. They are options for representing the fission neutron spectrum in ENDF MF=5 under LF=7, 9, and 11 laws, respectively.

Points to consider in the approach of using experimental fission neutron spectrum data to determine model parameters include the following:

1. The most obvious question is which of the commonly used model expressions best approximates the fission neutron spectrum? This topic will be addressed in Secs. III and IV.
2. A good example of the use of least-squares fitting procedures in the determination of Maxwellian and Watt distribution parameters, on prompt fission neutron spectra measured at a single incident neutron energy, is given by Johansson and Holmqvist [6].
3. If the least-squares minimization procedure involves an experimental data set comprised of measurements from several sources, the possibility exists that quite different experimental error analyses were performed. This condition can lead to over-biased and possibly erroneous results unless the degree of thoroughness, or lack thereof, of each error

analysis is understood by the evaluator. Error adjustment, of course, should only be done in collaboration with the experimentalist.

4. At sufficiently high incident neutron energy (E_n > ~6-7 MeV) multiple-chance fission processes (n,xnf) begin to occur in which x neutrons are sequentially evaporated prior to the fission of the last compound nucleus. In an experiment 0, 1, 2, ... up to x evaporated neutrons are measured together with pure fission spectrum neutrons in the same fission event (coincidence gate) to comprise the total fission neutron spectrum. The model expression used in the least-squares fitting procedure is therefore made up of a certain number of evaporation spectrum terms and a certain number of pure fission spectrum terms, depending upon the magnitude of E_n. The number of terms and how they are combined is explained in Sec. III.C.

C. Direct use of models which approximate the prompt fission neutron spectrum. This approach is identical to the one just discussed (Sec. II.B) except that the parameters of the model expressions are obtained by methods other than least-squares or "eyeball" fitting procedures using experimental fission neutron spectra. Therefore, the comments of Sec. II.B.1 and Sec. II.B.4 apply to the present discussion. The key to this approach is the determination of an expression for the model parameter, such as the Maxwellian temperature T_M, either empirically or by calculation in terms of other known quantities of the fissioning system.

One well known example due to Terrell [7,8] is the relationship between the average energy of the prompt fission neutron spectrum $\langle E \rangle$ and the average prompt neutron multiplicity $\bar{\nu}_p$, given by

$$\langle E(E_n) \rangle = \alpha + \beta\sqrt{\bar{\nu}_p(E_n) + 1} \tag{1}$$

where $\alpha \simeq 0.75$, $\beta \simeq 0.65$, and where α and β vary slowly with E_n and fissioning nucleus. Since the average energy of the Maxwellian distribution is (3/2) T_M, Eq. (1) determines T_M in terms of $\bar{\nu}_p$ which can be accurately measured. In Sec. III.B it will be shown that Eq. (1) is an approximation of a more general expression also calculated on the basis of Terrell's experiments [7].

D. Use of systematics to approximate the prompt fission neutron spectrum. The use of systematics in this case means simply that the fission neutron spectrum desired is identified with an existing fission neutron spectrum, or average of existing fission neutron spectra, for fissioning nuclei that share some property, or properties, in common with the one of interest. An example might be that thermal fission is a common property and that the mass and charge numbers of the compound fissioning nuclei are very nearly the same. The cautions that need be exercised in the application of this method are obvious.

E. Theoretical calculation of the prompt fission neutron spectrum. Two new theoretical calculations of the prompt fission neutron spectrum exist. The first of these is a statistical model approach by Browne and Dietrich [9,10] which employs a full Hauser-Feshbach calculation for forty fission fragments representative of the fragment mass distribution. The second calculation is an approach based upon nuclear-evaporation theory by Madland and Nix [11-16] which accounts for a number of physical effects which have usually been ignored. Both of these calculations show significant advances in reproducing experimental fission neutron spectra. This gives credence to their use in evaluation work. They will be discussed in more detail in Sec. III.

Evaluation of the prompt fission neutron spectrum for a given isotope will generally involve the use of one or more of the five methods, A through E, just discussed. It is useful to consider the suitability of these methods as a function of the amount of quality experimental data available to the evaluator. We consider three cases: abundant experimental data, some experimental data, and no experimental data. In the case of abundant experimental data the methods A, B, and E may be suitable. If accuracy is the main consideration in the construction of the evaluated file then methods A and E may dominate, whereas if minimal computing time in the use of the file is the main concern then methods B and, to some extent, E may dominate. In the case of some experimental data the methods B, C, and E may be suitable. If methods B and C are both employed, but at different ranges of incident neutron energy, then overlap calculations should be performed to insure continuity. In the case of no experimental data the methods C, D, and E may be suitable. Comparison of the results using methods C and E would perhaps be useful in assessing a confidence level.

III. MODELS AND THEORIES OF THE PROMPT FISSION NEUTRON SPECTRUM

In the following E and N(E) are laboratory expressions, and ϵ and $\Phi(\epsilon)$ are center-of-mass expressions, where E and ϵ are secondary or emitted neutron energies. Unless otherwise noted N(E) and $\Phi(\epsilon)$ are normalized to unity when integrated from zero to infinity. All energies and temperatures are in units of MeV.

A. Previous work. The Maxwellian distribution is given by

$$N_M(E) = (2/\sqrt{\pi T_M^3})\ E^{1/2}\ \exp(-E/T_M) \qquad (2)$$

where T_M is the Maxwellian temperature. The average energy of the Maxwellian distribution is given by

$$\langle E\rangle_M = (3/2)T_M \; . \tag{3}$$

The Maxwellian distribution is properly a center-of-mass expression and its use to describe fission neutron spectra measured in the laboratory system means that the temperature T_M is accounting not only for the emission of neutrons from the fission fragments, but also the center-of-mass motion of the fragments as well. We use the notation for the laboratory system here because the distribution is used in this way, historically and at present, to describe measured fission neutron spectra.

The center-of-mass neutron evaporation spectrum [4] is given by

$$\phi(\varepsilon) = k(T_e)\sigma_c(\varepsilon)\varepsilon\,\exp(-\varepsilon/T_e) \tag{4}$$

where $k(T_e)$ is the temperature-dependent normalization, $\sigma_c(\varepsilon)$ is the cross section for the inverse process of compound nucleus formation, and T_e is the temperature of the residual nucleus following neutron emission. The center-of-mass simple neutron evaporation spectrum is obtained by assuming $\sigma_c(\varepsilon)$ constant. In this case $k(T_e) = 1/\sigma_c\,T_e^2$ and

$$\phi(\varepsilon) = (\varepsilon/T_e^2)\,\exp(-\varepsilon/T_e) \tag{5}$$

for which the average energy is given by

$$\langle\varepsilon\rangle = 2T_e \; . \tag{6}$$

One use of Eqs. (4) and (5) is in the description of the evaporation-neutron component of the total fission neutron spectrum in the case of multiple-chance fission, as discussed in Sec. II.B.4.[†]

The Watt [5] distribution is given by

$$N_W(E) = [\exp(-E_f/T_W)/\sqrt{\pi E_f T_W}]\times [\exp(-E/T_W)\,\sinh(2\sqrt{EE_f}/T_W)] \tag{7}$$

[†] In this circumstance the neutrons are evaporating from a compound fissioning nucleus just prior to fission and the distinction between laboratory and center-of-mass systems is negligible.

where T_W is the Watt temperature and E_f is the average fission-fragment kinetic energy per nucleon. The average energy of the Watt distribution is given by

$$\langle E\rangle_W = E_f + (3/2)T_W \ . \tag{8}$$

The Watt distribution is obtained by assuming that the center-of-mass fission neutron spectrum is Maxwellian, that both fission fragments are moving with the same average kinetic energy per nucleon E_f, and transforming to the laboratory system. In the limit that E_f approaches zero, $N_W(E)$ approaches $N_M(E)$. The Watt distribution is clearly more physical than the Maxwellian distribution because the contributions due to center-of-mass fragment motion and neutron emission from the fragments are separately taken into account.

B. New work. Two new theoretical calculations of the prompt fission neutron spectrum exist. They have been briefly described in Sec. II.E.

The first new calculation is the statistical-model Hauser-Feshbach calculation of Browne and Dietrich [9,10]. This approach may ultimately provide the most exact agreement with experiment. At the present time, however, this is not the case because of insufficient knowledge of three main input quantities to the calculation. These are the initial fragment spin and excitation energy distributions, the fragment nuclear level densities, and the neutron plus fragment transmission coefficients. The status of this work is that fairly good agreement with experiment has been achieved in calculating the ^{252}Cf spontaneous-fission neutron spectrum. The next step would be to improve the physics content of the three main input quantities. This would perhaps be crucial in using this approach in evaluation work. The Hauser-Feshbach formalism is sufficiently complex that the final expression for the fission neutron spectrum cannot be written without considerable preliminary definition. Accordingly, a reading of Ref. [9] is recommended.

The second new calculation is the approach based upon standard nuclear-evaporation theory by Madland and Nix [11-16] which accounts for the physical effects of (1) the center-of-mass motion of each fission fragment, (2) the distribution of fission-fragment residual nuclear temperature, (3) the energy-dependence of the cross section $\sigma_c(\varepsilon)$ for the inverse process of compound nucleus formation, and (4) the occurrence of multiple-chance fission processes at high excitation energy. This approach is somewhat more easily applied for various fissioning nuclei and incident neutron energies. Moreover, the formalism simultaneously yields expressions for N(E) and $\bar{\nu}_p$, for high- as well as low-excitation fission. In addition, this work gives formulas for the parameters of the Maxwellian and Watt distributions as a function of fissioning nucleus and incident neutron energy. A cautionary note similar to that for the Hauser-Feshbach approach is that certain input quantities to the calculation, such as nuclear level-density parameter and average fission energy release, could have improved physical content.

The fission neutron spectrum has been calculated for two different assumptions concerning the cross section $\sigma_c(\varepsilon)$. Use of a constant cross section yields a closed expression for the spectrum while use of an energy-dependent cross section, calculated with the optical model, yields a numerical integral expression. The expressions for the prompt fission neutron spectrum $N(E)$, the average energy of the spectrum $\langle E \rangle$, and the average prompt neutron multiplicity $\bar{\nu}_p$, under the two assumptions are as follows:

$\underline{\sigma_c(\varepsilon) = \text{constant}}$

$$N(E) = \frac{1}{2}[N(E,E_f^L) + N(E,E_f^H)] \tag{9}$$

where E_f^L and E_f^H are the average fission-fragment kinetic energy per nucleon of the light and heavy fragment, respectively, and

$$N(E,E_f) = (1/3\sqrt{E_f T_m})\,[u_2^{3/2}\,E_1(u_2) - u_1^{3/2}\,E_1(u_1) + \gamma(3/2,u_2) - \gamma(3/2,u_1)] \tag{10}$$

where

$$u_1 = (\sqrt{E} - \sqrt{E_f})^2 ,$$

$$u_2 = (\sqrt{E} + \sqrt{E_f})^2 ,$$

E_1 is the exponential integral [17], γ is the incomplete gamma function [17], and T_m is the maximum temperature of the fission-fragment residual nuclear-temperature distribution. The average energy of the spectrum is given by

$$\langle E \rangle = \frac{1}{2}[E_f^L + E_f^H] + (4/3)T_m . \tag{11}$$

The average prompt neutron multiplicity $\bar{\nu}_p$ is given by

$$\bar{\nu}_p = \frac{\langle E^* \rangle - \langle E_\gamma^{tot} \rangle}{\langle S_n \rangle + (4/3)T_m} , \qquad (12)$$

where $\langle E^* \rangle$ is the initial total average fragment excitation energy, $\langle E_\gamma^{tot} \rangle$ is the total average prompt gamma-ray energy, and $\langle S_n \rangle$ is the average fission-fragment neutron separation energy.

$\sigma_c(\varepsilon)$ calculated using the optical model

N(E) is given by Eq. (9) where

$$N(E,E_f) = (1/2\,\sqrt{E_f}\,T_m^2) \times \int_{u_1}^{u_2} \sigma_c(\varepsilon)\,\sqrt{\varepsilon}\,d\varepsilon \int_0^{T_m} k(T)\,T\,\exp(-\varepsilon/T)dT . \qquad (13)$$

The average energy of this spectrum and the average prompt neutron multiplicity are given exactly by Eqs. (11) and (12) with the quantity $(4/3)T_m$ replaced by $\langle\varepsilon\rangle$ which is the average energy of the center-of-mass neutron spectrum obtained by numerical integration (see Ref. [15]).

Equation (10) can be evaluated easily on any modern computer with a scientific program library. Similarly, Eq. (13) can be numerically integrated by a number of techniques; for example, Gaussian quadrature is used in Refs. [12-16].

The initial average fragment excitation energy $\langle E^* \rangle$ and the maximum temperature T_m are related by the Fermi-gas law

$$T_m = (\langle E^* \rangle / a)^{1/2} \qquad (14)$$

where a is the nuclear level-density parameter. The average excitation energy is obtained from

$$\langle E^* \rangle = \langle E_r \rangle + B_n + E_n - \langle E_f^{tot} \rangle , \qquad (15)$$

where $\langle E_r \rangle$ is the average energy release given by the difference between the ground-state mass of the fissioning compound nucleus and the ground-state masses of the two fission fragments, B_n and E_n are the separation energy and kinetic energy of the neutron inducing fission, and $\langle E_f^{tot} \rangle$ is the total average fission-fragment kinetic energy. T_m is obtained by substitution of Eq. (15) into Eq. (14).

The quantities $<E_r>$ of Eq. (15) and $<S_n>$ of Eq. (12) must be carefully calculated by averaging over the peaks of the fragment mass distribution [15]. The remaining quantities in the two equations together with E_f^L and E_f^H can be obtained directly from experiment or empirically based formulas [13, 15, 18, 19].

discussion

Expressions for the Maxwellian and Watt temperatures T_M and T_W in terms of the above physical quantities are obtained by equating the average energies of the two distributions to that of the exact calculation. In the σ_c = constant case, using Eqs. (3) and (11), one obtains

$$T_M = \frac{1}{3}[E_f^L+E_f^H] + (8/9)T_m , \qquad (16)$$

while use of Eqs. (8) and (11) yields

$$T_W = (8/9)T_m , \qquad (17)$$

where T_m is given by Eqs. (14) and (15). Going one step further, a quadratic relation between T_m and $\bar{\nu}_p$ is obtained by substitution of Eq. (14) into Eq. (12) yielding

$$T_m = (2\bar{\nu}_p/3a) + \sqrt{(2\bar{\nu}_p/3a)^2 + (\bar{\nu}_p <S_n> + <E_\gamma^{tot}>)/a} , \qquad (18)$$

which can be used in Eqs. (16) and (17), respectively, to obtain T_M and T_W in terms of $\bar{\nu}_p$. The well-known expression due to Terrell [7] given by Eq. (1) is obtained in the limit that the first term in the square-root expression is negligible compared to the second and that $<S_n> \simeq <E_\gamma^{tot}>$. Substitution of the approximate T_m into Eq. (16) and multiplying by (3/2) gives Eq. (1).

The prompt fission neutron spectrum for the fission of ^{235}U induced by 0.53-MeV neutrons is shown in Fig. 1 for the present calculation, where σ_c is assumed constant, given by Eqs. (9) and (10), for the Maxwellian distribution given by Eq. (2), and for the Watt distribution given by Eq. (7). The average energies of the Maxwellian and Watt distributions are identical to that of the exact calculation by construction using Eqs. (16) and (17). In Fig. 2 where the ratios of these two approximations to the exact calculation are plotted, the Watt spectrum is accurate to within ~5% for laboratory neutron energies below ~7 MeV, but for higher energies is less than the exact calculation because T_W is less than the maximum temperature T_m. In fitting experimental fission neutron spectra to the Watt distribution, T_W is usually increased and E_f decreased to somewhat unphysical values, in order to simultaneously optimize the fit at intermediate and higher energies where most of the data exist [6]. The Maxwellian spectrum is a less accurate approximation, especially at high energy because T_M is substantially greater than T_m. In fitting experimental spectra to the Maxwellian distribution, T_M is usually decreased in order to preserve the fit at high energy [6,20,21]. This simultaneously increases the spectrum somewhat at lower energies because of the normalization.

The present calculation predicts a definite dependence of the prompt fission neutron spectrum (and $\bar{\nu}_p$) upon both the fissioning nucleus and the incident energy of the neutron inducing fission. Figure 3 shows the changes in the spectrum, at both low and high energy, as the charge and mass of the fissioning nucleus increases, for thermal-neutron induced fission. Figure 4 shows the dependence of the spectrum upon the incident neutron energy, for the first-chance fission of ^{235}U.

Figures 5 and 6 show comparisons of the present calculations, for the two cases of σ_c = constant and $\sigma_c(\varepsilon)$ calculated using the optical model, with the experimental data of Johansson and Holmqvist [6]. The optical-model parameters of Becchetti and Greenlees [2?] are used for reasons given in Ref. [15]. In Fig. 6 it is apparent that the energy-dependent cross section calculation has introduced some structure into the spectrum and has softened the high-energy portion. It is clear that the calculation performed with the energy-dependent $\sigma_c(\varepsilon)$ is more exact than the calculation performed with σ_c = constant. The same effect and the same conclusion is obtained in a comparison to the experimental data of Boldeman [20] for the spontaneous fission of ^{252}Cf, shown in Fig. 7. The calculations shown in Figs. 5-7, together with those in Figs. 1-4 discussed above, have all been performed assuming a nuclear level-density parameter a in Eq. (14) given by

$$a = A/(11\ \text{MeV})\ , \tag{19}$$

where A is the mass number of the fissioning compound nucleus (see Ref. [15]).

A final note of this discussion is that the σ_c = constant calculation given by Eqs. (9) and (10) is much simpler, with respect to both computing time and coding time, than is the optical-model $\sigma_c(\varepsilon)$ calculation given by Eqs. (9) and (13). The question arises as to whether the energy dependence of the compound-nucleus formation cross section $\sigma_c(\varepsilon)$ can be simulated. An approximate solution is found by a slight readjustment of the level-density parameter from the value given by Eq. (19) to the value

$$a_{eff} = A/(10\ \text{MeV}) \ . \tag{20}$$

A comparison of Figs. 5 and 8, which differ only in the choice of the level-density parameter for the σ_c = constant calculations given by the dashed curves, indicates that the approximation is reasonably good.

C. Multiple-chance fission. At high incident neutron energy E_n multiple-chance fission processes (n,xnf) occur in which x neutrons are sequentially evaporated prior to fission where x = 0,1,2, ... In this circumstance the total prompt fission neutron spectrum is comprised of evaporation-spectrum terms and pure fission-neutron spectrum terms depending upon the magnitude of E_n. If E_n ~ 7 MeV first- and second-chance fission occur whereas if E_n ~ 14 MeV up to third-chance fission is possible. The total prompt fission neutron spectrum where first-, second-, and third-chance fission are energetically possible is given by

$$N(E) = \{P_1\bar{\nu}_1N_1(E) + P_2[\phi_1(E)+\bar{\nu}_2N_2(E)] + P_3[\phi_1(E)+\phi_2(E)+\bar{\nu}_3N_3(E)]\}/\{P_1\bar{\nu}_1 + P_2(1+\bar{\nu}_2) + P_3(2+\bar{\nu}_3)\} \ . \tag{21}$$

The average prompt neutron multiplicity, as a function of the incident neutron energy E_n, where first-, second-, and third-chance fission become energetically possible as E_n increases, is given by

$$\bar{\nu}(E_n) = \{P_1(E_n)\bar{\nu}_1(E_n) + P_2(E_n)[1 + \bar{\nu}_2(E_n)] + P_3(E_n)[2 + \bar{\nu}_3(E_n)]\}/\{P_1(E_n) + P_2(E_n) + P_3(E_n)\} \ . \tag{22}$$

In these two expressions P_m, N_m, and $\bar{\nu}_m$ are the fission probability, pure fission neutron spectrum, and average prompt neutron multiplicity, respectively, for m'th chance fission whereas ϕ_m is the evaporation spectrum for the m'th evaporation neutron.

A method to calculate the fission probabilities is developed and used in Ref. [15].

Figures 9 and 10 illustrate the influence of multiple-chance fission processes on the prompt fission neutron spectrum. The curve labeled "σ_c = Constant" is calculated using Eqs. (9) and (10) under the assumption of first-chance fission only. The same is true for the curve labeled "$\sigma_c(\varepsilon)$ First-chance fission" except that Eqs. (9) and (13) are used. The curve labeled "$\sigma_c(\varepsilon)$ Multiple-chance fission" is calculated using Eqs. (4), (9), (13), and (21). Figure 9 shows how the high energy tail of the spectrum softens and the evaporation-neutron peak appears as multiple-chance fission processes are switched on. In Fig. 10 the evaporation-neutron peak is seen more clearly as a ~1-2 MeV wide peak centered at ~0.5 MeV.

IV. PROMPT FISSION NEUTRON SPECTRUM RECOMMENDATIONS

General recommendations for evaluating the prompt fission neutron spectrum depending upon the amount of quality experimental data available have been given at the end of Sec. II. More specific recommendations as a consequence of the developments outlined in Sec. III are given here.

A. Least-squares fitting of experimental prompt fission neutron spectra. The recommended expressions to use in fitting procedures are given in decreasing order of physical content:

1. The prompt fission neutron spectrum N(E) for $\sigma_c(\varepsilon)$ = constant given by Eqs. (9) and (10). The three fitting parameters are E_f^L, E_f^H, and T_m.
2. The Watt distribution $N_W(E)$ given by Eq. (7). The two fitting parameters are E_f and T_W.
3. The Maxwellian distribution $N_M(E)$ given by Eq. (2). The single fitting parameter is T_M.

If multiple-chance fission spectrum data are fit, a combination of N(E) terms and neutron evaporation terms $\phi(E)$ must in principle be included in the fitting expression according to Eq. (21).

If extracted T_m, T_W, or T_M values are themselves parameterized in terms of the incident neutron energy E_n or $\bar{\nu}_p$, it is recommended that the dependencies follow those given in Eqs. (14) through (18).

B. Direct calculation of prompt fission neutron spectra. The recommended expressions to use in calculating prompt fission neutron spectra are given in decreasing order of physical content:

1. The prompt fission neutron spectrum N(E) for $\sigma_c(\varepsilon)$ calculated with the optical model and given by Eqs. (9), (13), (14), and (19).
2. The prompt fission neutron spectrum N(E) for $\sigma_c(\varepsilon)$ = constant given by Eqs. (9), (10), (14), and (20).

3. The Watt distribution $N_W(E)$ given by Eqs. (7), (14), (17), and (20). Note that Eq. (18) can be used in place of Eq. (14).
4. The Maxwellian distribution $N_M(E)$ given by Eqs. (2), (14), (16), and (20). Note that Eq. (18) can be used in place of Eq. (14).

Equation (21) should be used when calculating multiple-chance fission neutron spectra. Equation (4) should be used for energy-dependent $\sigma_c(\epsilon)$ calculations and Eq. (5) should be used for σ_c = constant calculations.

V. METHODS TO EVALUATE THE AVERAGE PROMPT FISSION NEUTRON MULTIPLICITY $\bar{\nu}_p$

A number of methods exist by which a representation of the average prompt fission neutron multiplicity $\bar{\nu}_p$ can be obtained from experimental data, theoretical calculation, or a combination of the two. These methods include the following:

A. Direct use of experimental data. The experimentalist usually gives $\bar{\nu}_p$ as a function of the incident energy E_n of the neutron inducing fission. A relative measurement is usually performed by which $\bar{\nu}_p$ for the isotope of interest is measured relative to that for some standard reaction such as the spontaneous fission of ^{252}Cf. Accordingly, care must be exercised in the use of experimental $\bar{\nu}_p$-ratio sets from diverse sources because different standard reactions may have been used or the accepted value of $\bar{\nu}_p$ for a common standard may have changed with time.

The ENDF MF=1 LNP=2 law (Tabulated pairs) can be used to tabulate experimental $\bar{\nu}_p$ values directly, as a function of E_n.

B. Use of experimental data and systematics to determine parameters of models which approximate $\bar{\nu}_p$. It is well known experimentally [3] that $\bar{\nu}_p$ varies approximately linearly with the incident neutron energy E_n. The simplest approach of all, therefore, is to fit the experimental data for a given isotope to the linear expression

$$\bar{\nu}_p(E_n) = b + cE_n \tag{23}$$

by the method of least-squares, where b and c are the parameters to be determined. This approach has been wisely used, a good example being that of Soleilhac [23]. The ENDF MF=1 LNP=1 law (Polynomial representation) can be used to represent $\bar{\nu}_p$ as a polynomial in E_n up to third degree. Note that use of quadratic and cubic terms in this approach can yield unphysical results when extrapolating the fit to high E_n values where, in many instances, little or no experimental data exist. However, a careful and extensive study of the polynomial representation of $\bar{\nu}_p(E_n)$ by least-squares analysis has been performed by Manero and Konshin [3] in which, for some cases, fifth-degree terms were statistically significant. The theoretical energy dependence of $\bar{\nu}_p$ is obtained from Eqs. (12), (14), and (15) and is of the form

$$\bar{\nu}_p(E_n) = (d+E_n)/(f+g\sqrt{h+E_n}) \tag{24}$$

where d, f, g, and h are approximately constant with E_n [15]. The energy-dependence of Eq. (24) is dominated by the linear term of the numerator, but is modified to turn slightly downward with increasing E_n due to the energy-dependent term of the denominator.

Much attention has been given to the determination of the parameters b and c of Eq. (23) by a study of the systematic behavior of $\bar{\nu}_p$ in actinide and trans-actinide nuclei [3]. In particular, Gordeeva and Smirenkin [24] determined a linear expression for b in terms of A and Z of the fissioning nucleus. Their expression, valid for thermal-neutron-induced fission, ultimately proved to be accurate to within ~8-9% [3]. Similarly, Ping-Shin Tu and Prince [25] found that b could be described in terms of $Z^2\sqrt{A}$ with an accuracy of ~10%.

Howerton [26], in a study of the systematics of neutron-induced fission for incident neutron energies ranging from thermal to ~6 MeV determined expressions for both b and c of Eq. (23). The parameter b is described in terms of A and Z of the target nucleus and a quantity E_{Th}. The parameter c is expressed in terms of A. When E_{Th} is optimally adjusted to fit the $\bar{\nu}_p(E_n)$ data of a given isotope the overall agreement is better than ~5% for the data of that isotope. In this approach E_{Th} simply shifts the zero-energy intercept of Eq. (23) to the optimal value. Obtaining, in this way, a set of E_{Th} values for the $\bar{\nu}_p(E_n)$ data of a set of actinide nuclei, overall agreement is obtained to within ~5% for the entire set of experimental data. The results obtained in Ref. [26] are therefore useful in representing the experimental $\bar{\nu}_p(E_n)$ data that were used to obtain those results.

C. Theoretical Calculation of $\bar{\nu}_p$. A new theoretical calculation of $\bar{\nu}_p$ exists based upon standard nuclear-evaporation theory. A discussion of the calculations [11-16] and presentation of the results has been given in Sec. III.B and III.C. In summary: $\bar{\nu}_p(E_n)$ is obtained using experimentally known or calculated quantities as input. Calculations of $\bar{\nu}_p(E_n)$ are performed for two different assumptions concerning the cross section $\sigma_c(\varepsilon)$ for the inverse process of compound-nucleus formation. In the case σ_c = constant, $\bar{\nu}_p(E_n)$ is given by Eqs. (12), (14), (15), and (20). In the case $\sigma_c(\varepsilon)$ is calculated using the optical model, $\bar{\nu}_p(E_n)$ is given by Eq. (12) with $(4/3)T_m$ replaced by $\langle\varepsilon\rangle$, and Eqs. (14), (15), and (19). As shown in Ref. [15], either of these assumptions can be u for the calculation of $\bar{\nu}_p(E_n)$. The energy dependence of $\bar{\nu}_p(E_n)$ is given, approximately, by Eq. (24). When multiple-chance fission occurs Eq. (22) is used to calculate $\bar{\nu}_p(E_n)$. A calculation of $\bar{\nu}_p(E_n)$ is compared to experimental data in Fig. 11. The calculations were performed using the energy-dependent cross section assumption. In one case it is assumed that only first-chance fission occurs whereas in the other case first-, second-, and third-chance fission contributions are combined according to Eq. (22).

As in Sec. II, it is useful to consider the suitability of the methods A, B, and C as a function of the amount of quality experimental data available to the evaluator. In the case of abundant experimental data the methods A and the least-squares fitting procedures of method B may be most suitable. Method C may serve as a guide. In the case of some experimental data the methods B and C may be most suitable. If both are used, overlap calculations are required. In the case of no experimental data method C is preferred.

VI. RECOMMENDATIONS FOR $\bar{\nu}_p$

General recommendations for evaluating $\bar{\nu}_p(E_n)$ as a function of the amount of quality experimental data available have just been given at the end of Sec. V. More specific recommendations are given here.

A. Least-squares fitting of experimental $\bar{\nu}_p(E_n)$ data. The recommended expressions to use in fitting procedures are given in decreasing order of physical content:

1. The expression for $\bar{\nu}_p(E_n)$ given by Eq. (12). See Eq. (24) for a parametric form.
2. The expression for $\bar{\nu}_p(E_n)$ given by Eq. (23).

B. Direct calculation of $\bar{\nu}_p(E_n)$. The recommended expressions to use in calculating $\bar{\nu}_p(E_n)$ are given in decreasing order of physical content:

1. The expression for $\bar{\nu}_p(E_n)$ given by Eq. (12) with $(4/3)T_m$ replaced by $\langle\varepsilon\rangle$, and Eqs. (14), (15), and (19).
2. The expression for $\bar{\nu}_p(E_n)$ given by Eqs. (12), (14), (15), and (20).
3. The expression for $\bar{\nu}_p(E_n)$ given by Howerton [26].

Equation (22) is used to calculate $\bar{\nu}_p(E_n)$ in the multiple-chance fission region.

VII. SUMMARY

General recommendations on the evaluation of the prompt fission neutron spectrum N(E) and the average prompt neutron multiplicity $\bar{\nu}_p$ as a function of the amount of quality data available have been given at the ends of Secs. II and V, respectively. Specific recommendations with respect to least-squares fitting procedures and direct calculation of N(E) and $\bar{\nu}_p$ have been given in Secs. IV and VI, respectively, where the available options are listed in decreasing order of physical content. Recommendations have been given for the evaluation of N(E) and $\bar{\nu}_p$ at high incident neutron energy where multiple-chance fission processes occur. All of the recommendations given in this work have been made from the perspective of the most accurate physical representation of N(E) and $\bar{\nu}_p$. In actual practice, however, the evaluator must weigh these recommendations against the other constraints of his task.

A new calculation of N(E) and $\bar{\nu}_p$ based upon standard nuclear-evaporation theory has been outlined and shown to yield good results in the prediction of N(E) and $\bar{\nu}_p$ for high- as well as low-excitation fission. The theory has demonstrated the dependence of N(E) and $\bar{\nu}_p$ upon fissioning nucleus and incident neutron energy. It has been shown by derived relationships that N(E) and $\bar{\nu}_p$ should be calculated and evaluated simultaneously.

In conclusion, a survey was made of the forty ENDF/B-V actinide and trans-actinide evaluations having MF=1 and MF=5 $\bar{\nu}$ and N(E) files. The intent was to gather statistics on the methods used to evaluate N(E) and $\bar{\nu}_p$. The results of the survey are given in Tables I and II, which are self-explanatory. Some noteworthy comments are the following. In Table I (C) for the statistics of the total fission neutron spectrum (MT=18) it is seen that in 35 of 40 cases the Maxwellian distribution is used. In 16 of the 35 cases a single Maxwellian temperature represents the complete energy dependence. Moreover, in 15 of these 16 cases, the single Maxwellian temperature has the same value, namely, T_M = 1.33 MeV. This constitutes an unphysical description of N(E) for the 15 cases in that the dependence of N(E) upon fissioning nucleus and incident neutron energy is ignored. In Table II(B) it is seen that only in 8 cases out of 40 is a distinction made between the total and prompt $\bar{\nu}$. Other over-simplifications in the description of N(E) for multiple-chance fission processes can be seen in Table I. The two tables illustrate that much work can be done to improve the physical descriptions of N(E) and $\bar{\nu}_p$ in future work.

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TABLE I

Statistics on the Fission Neutron Spectrum Files of Forty ENDF/B-V Actinide and Trans-Actinide Evaluations

(A) Nuclide Mass Number and Charge Number Ranges

$230 \leq A \leq 253$ $90 \leq Z \leq 98$

(B) Distribution of Fission Neutron Spectrum Files According to Total Fission and Multiple-chance Fission Components (MT=18 is total fission, MT=19 is first-chance fission, MT=20 is second-chance fission, MT=21 is third-chance fission, and MT=38 is fourth-chance fission)

File Types Included	Number of Cases
MF=5, MT=18	22
MF=5, MT=18,19	0
MF=5, MT=18,19,20	12
MF=5, MT=18,19,20,21	4
MF=5, MT=18,19,20,21,38	2
total number of cases	40

(C) Distribution of Fission Neutron Spectrum Representations Used for MT=18 Files (Total Fission)

Representation	Number of Cases
Maxwellian spectrum (single temperature)	16
Maxwellian spectrum (array of temperatures)	19
energy-dependent Watt spectrum	5
total number of cases	40

(D) Distribution of Fission Neutron Spectrum Representations Used for MT=19 Files (First-chance Fission)

Representation	Number of Cases
Maxwellian spectrum (single temperature)	12
Maxwellian spectrum (array of temperatures)	3
energy-dependent Watt spectrum	3
total number of cases	18

(E) Distribution of Fission Neutron Spectrum Representations Used for MT=20 Files (Second-chance Fission)

Representation	Number of Cases
Maxwellian spectrum (single temperature)	12
Maxwellian spectrum (array of temperatures)	2
Maxwellian spectrum (array of temperatures) and evaporation spectrum (array of temperatures)	1
energy-dependent Watt spectrum and evaporation spectrum (array of temperatures)	3
total number of cases	18

(F) Distribution of Fission Neutron Spectrum Representations Used for MT=21 Files (Third-chance Fission)

Representation	Number of Cases
Maxwellian spectrum (array of temperatures)	2
Maxwellian spectrum (array of temperatures) and evaporation spectrum (array of temperatures)	1
energy-dependent Watt spectrum and evaporation spectrum (array of temperatures)	2
energy-dependent Watt spectrum and two evaporation spectra (arrays of temperatures)	1
total number of cases	6

(G) Distribution of Fission Neutron Spectrum Representations Used for MT=38 Files (Fourth-chance Fission)

Representation	Number of Cases
energy-dependent Watt spectrum and evaporation spectrum (array of temperatures)	1
energy-dependent Watt spectrum and three evaporation spectra (arrays of temperatures)	1
total number of cases	2

TABLE II

Statistics on the $\bar{\nu}$ Files of Forty ENDF/B-V Actinide and Trans-Actinide Evaluations

(A) Nuclide Mass Number and Charge Number Ranges

$230 \leq A \leq 253$ $90 \leq Z \leq 98$

(B) Distribution of $\bar{\nu}$ Files According to Total ($\bar{\nu}_t$), Prompt ($\bar{\nu}_p$), and Delayed ($\bar{\nu}_d$) Components (MT=452 is $\bar{\nu}_T$, MT=456 is $\bar{\nu}_p$, and MT=455 is $\bar{\nu}_d$)

File Types Included	Number of Cases
MF=1, MT=452	32
MF=1, MT=452,456	1
MF=1, MT=452,456,455	7
total number of cases	40

(C) Distribution of $\bar{\nu}$ Representations Used for MT=452 Files ($\bar{\nu}_t$)

Representation	Number of Cases
linear	25
tabulation	15
total number of cases	40

(D) Distribution of $\bar{\nu}$ Representations Used for MT=456 Files ($\bar{\nu}_p$)

Representation	Number of Cases
linear	2
tabulation	6
total number of cases	8

(E) Distribution of $\bar{\nu}$ Representations Used for MT=455 Files ($\bar{\nu}_d$)

Representation	Number of Cases
tabulation	7
total number of cases	7

FIGURE CAPTIONS

Fig. 1. Prompt fission neutron spectrum in the laboratory system for the fission of ^{235}U induced by 0.53-MeV neutrons. The solid curve give the present spectrum calculated from Eqs. (9) and (10), the dashed curve gives the Watt spectrum calculated from Eq. (7), and the dot-dashed curve gives the Maxwellian spectrum calculated from Eq. (2). The values of the three constants appearing in the present spectrum are E_f^L = 1.062 MeV, E_f^H = 0.499 MeV, and T_m = 1.019 MeV, whereas those in the Watt spectrum are E_f = 0.780 MeV and T_W = 0.905 MeV. The value of the single constant appearing in the Maxwellian spectrum is T_M = 1.426 MeV. The mean laboratory neutron energies of the three spectra are identical.

Fig. 2. Ratio of the Watt spectrum and the Maxwellian spectrum to the present spectrum, corresponding to the curves shown in Fig. 1.

Fig. 3. Dependence of the prompt fission neutron spectrum upon the fissioning nucleus, for thermal-neutron-induced fission. The values of the constants are E_f^L = 1.106 MeV, E_f^H = 0.457 MeV, and T_m = 0.989 MeV for ^{229}Th+n, are E_f^L = 1.033 MeV, E_f^H = 0.527 MeV, and T_m = 1.124 MeV for ^{239}Pu+n, and are E_f^L = 0.995 MeV, E_f^H = 0.575 MeV, and T_m = 1.304 MeV for ^{249}Cf+n.

Fig. 4. Dependence of the prompt fission neutron spectrum upon the kinetic energy of the incident neutron, for the fission of ^{235}U. The maximum temperature T_m is 1.006 MeV when the incident neutron energy is 0 MeV, is 1.157 MeV when the incident neutron energy is 7 MeV, and is 1.290 MeV when the incident neutron energy is 14 MeV. The values of the average kinetic energy per nucleon are for each case held fixed at E_f^L = 1.062 MeV and E_f^H = 0.499 MeV. For the latter two cases, the spectra are calculated for first-chance fission only.

Fig. 5. Prompt fission neutron spectrum in the laboratory system for the fission of ^{235}U induced by 0.53-MeV neutrons. The dashed curve gives the present spectrum calculated from Eqs. (9) and (10) assuming a constant cross section whereas the solid curve gives the present spectrum calculated from Eqs. (9) and (13) using the optical-model parameters of Becchetti and Greenless [22]. The values of the three constants appearing in the calculated spectra are in both cases E_f^L = 1.062 MeV, E_f^H = 0.499 MeV, and

T_m = 1.019 MeV. The experimental data are those of Johansson and Holmqvist [6].

Fig. 6. Ratio of the present spectrum calculated using the optical-model parameters of Becchetti and Greenless [22] and the experimental data of Johansson and Holmqvist [6] to the present spectrum calculated assuming a constant cross section, corresponding to the curves shown in Fig. 5.

Fig. 7. Prompt fission neutron spectrum for the spontaneous fission of ^{252}Cf. The dashed curve gives the present calculation calculated assuming a constant cross section whereas the solid curve gives the present calculation using the optical-model parameters of Becchetti and Greenless [22]. The values of the three constants appearing in the calculated spectra are in both cases E_f^L = 0.984 MeV, E_f^H = 0.553 MeV and T_m = 1.209 MeV. The experimental data are those of Bolderman et al [20].

Fig. 8. Prompt fission neutron spectrum for the fission of ^{235}U induced by 0.53-MeV neutrons illustrating the simulated energy dependence of $\sigma_c(\varepsilon)$. The two calculated spectra are in every respect identical to those of Fig. 5 except that the level-density parameter used in the constant cross section calculation, shown by the dashed curve, is given by Eq. (20) instead of Eq. (19).

Fig. 9. Prompt fission neutron spectrum for the fission of ^{235}U induced by 14.0-MeV neutrons. The dashed and dot-dashed curves give the present spectrum calculated for first-chance fission assuming, respectively, a constant cross section and an energy-dependent cross section calculated using the optical-model parameters of Becchetti and Greenless [22]. The solid curve gives the present spectrum calculated for first-, second-, and third-chance fission contributions using Eq. (21) and assuming an energy-dependent cross section calculated with the same optical potential. The values of the three constants appearing in the spectra for the first compound nucleus, ^{236}U, are E_f^L = 1.062 MeV, E_f^H = 0.499 MeV, and T_m = 1.290 MeV.

Fig. 10. Ratio of the present spectra calculated using energy-dependent cross sections and assuming either first-chance or multiple-chance fission to the present spectrum calculated using a constant cross section and assuming first-chance fission, corresponding to the curves shown in Fig. 9.

Fig. 11. Average prompt neutron multiplicity as a function of incident neutron energy for the neutron-induced fission of ^{235}U. The dashed curve, for first-chance fission, is calculated using Eq. (12), but replacing the term $(4/3)T_m$ with the corresponding quantity obtained by numerical integration in the energy-dependent cross section calculation. The solid curve, for first-, second-, and third-chance fission contributions, is also calculated using energy-dependent cross sections, but Eq. (22) is used. The optical-model parameter of Becchetti and Greenless [22] were used. The Ref. experimental data references are compiled in [15].