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THE EFFECT OF ROTATION ON THE HYDRODYNAMICS OF STELLAR COLLAPSE

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I. INTRODUCTION

To date, most of the hydrodynamic calculations that have followed the details of the physics during an iron core collapse (see Bowers, these proceedings, for a review) have been restricted to spherical symmetry and therefore have neglected the role that rotation may play in the hydrodynamics of the collapse (see, however, LeBlanc and Wilson, 1970). If rotation is important, the core will flatten to an oblate spheroidal shape allowing some loss of energy through gravitational radiation; the core could conceivably, dynamically evolve to a toroidal configuration, as has been observed in some models of rotating protostellar clouds (Tohline, 1980b; Boss, 1980b and references cited therein); and it may, through a rotational instability, eventually evolve into a non-axisymmetric structure. It is important to know just how much rotational energy must be present in the pre-collapse core in order for these, or any other significant deviations from spherical symmetry, to become important considerations during a core collapse.

Several authors, most notably Saenz and Shapiro (1978, 1979; but see also: Thuan and Ostriker, 1974; Novikov, 1975; Shapiro, 1977, 1979; Chia, Chau and Henriksen, 1977), have integrated the collapse of rotating, uniform-density spheroids in an attempt to measure the importance of gravitational radiation during core collapse. The restriction to uniform density allows an analytic description of the two-dimensional equation of motion for the core collapse and hence offers a relatively cheap way to explore parameter space. These investigators are, however, unable to comment on the role that density gradients, non-spherical shocks, and velocity flow patterns arising from a non-homologous contraction may play during the core's collapse and its subsequent bounce(s). In order to study the non-homologous aspects of an evolution, we have used the two-dimensional, axisymmetric versions of two different multidimensional hydrodynamic computer codes (Tohline, 1980a; Boss, 1980a) to numerically follow the Newtonian collapse of a rotating, 1.4 $M_\odot$ core. Müller, Rożyczka, and Hillebrandt (1980) have performed an experiment similar to the one described here. We have not considered the detailed physical reactions and transport properties in the core, but have adopted, instead, a simple adiabatic prescription of the collapse as has been developed and used by Van Riper (1978, 1979) in his studies of the hydrodynamics of spherical core collapses. The core's initial structure is chosen to be that of an equilibrium, n = 3 polytrope with the central density $\rho_c = 4 \times 10^9$ g cm^{-3}. In order to initiate collapse, the pressure P throughout the core is dropped uniformly from its equilibrium value P_0 according to the prescription: $P = d \cdot P_0$ ($d \leq 1.0$). During the ensuing collapse, variations in the pressure at any point in the core are governed by the relation $P \propto \rho^\Gamma$, where the power Γ is itself a well-defined function of the local density. The function $\Gamma(\rho)$ suggested by Van Riper and implemented here is shown in Figure 1. Initially, and throughout most of the collapse, Γ is held constant at a value γ_{min} ($\leq 4/3$). Then at nuclear densities ($\rho_{nuc} = 2 \times 10^{14}$ g cm^{-3}, here), the stiffening of the equation of state is mimicked by making Γ increase (linearly in log ρ) up to a maximum value γ_{max}. For the models discussed here, we have selected values of d, γ_{min}, and γ_{max} that Van Riper has found are most promising for ejecting the envelope of the star via the hydrodynamic bounce of the core: $\gamma_{min} = 1.33$, $\gamma_{max} = 1.40$ or 2.75, and d = 0.95 or 0.88.

2. ROTATION IN THE PRE-COLLAPSE CORE

The following terms are useful in discussing the degree of importance of rotation in the core:

ω = angular velocity,

$$\chi \equiv \frac{\text{centrifugal force}}{\text{gravity}}$$

$$\beta \equiv \frac{\text{total rotational kinetic energy}}{|\text{gravitational potential energy}|} .$$

The subscript $\underline{i}$ will be used to designate values of these parameters in the initial core model. For experimental purposes we have chosen to model:

A. Cores that are initially in solid body rotation—ω_i defines these models;

B. Differentially rotating cores in which χ_i = constant in the equatorial plane and rotation is uniform on cylinders. In these cases, $\omega_{center} \simeq 5 \times \omega_{edge}$ initially.

Very little is known about the rate of rotation of the cores of highly evolved stars, therefore our choice of ω_i or χ_i is somewhat arbitrary. We can, however, gain some guidance from the following consideration. Under conservation of angular momentum and a perfectly spherical contraction, χ increases as $\chi \propto \rho^{1/3}$. This relation allows us to pinpoint values of χ_i and ω_i that may have some astrophysical relevance. (i) In order for $\chi \rightarrow 1$ (centrifugal balance) as $\rho \rightarrow \rho_{nuc}$, χ_i must be $\simeq 0.02$ (i.e., $\omega_i \simeq 5\ s^{-1}$ in the center of the core). (ii) It is believed that at birth, the Crab pulsar was rotating no more than twice its presently observed rate (Ruderman, 1972). Using, then, $\omega \sim 60\ s^{-1}$ at nuclear densities, the core that gave birth to the Crab pulsar would have had $\chi_i \simeq 10^{-5}$ (i.e., $\omega_i \simeq 0.01\ s^{-1}$ in the center of the core). (iii) Massive main sequence stars having central densities $\rho \sim 1\ g\ cm^{-3}$ are observed to have surface rotational velocities $\sim 100\ km\ s^{-1}$. If these stars are in solid body rotation and the core can conserve angular momentum during the star's evolution up the giant branch, then ω at the onset of core collapse could be as large as $30\ s^{-1}$. This implies $\chi_i > 1$! From these three points, we can conclude that $\chi_i \lesssim 10^{-5}$ is certainly a realistic choice, but it is not likely to produce significant deviations from a perfectly spherical collapse. On the other hand, it is possible to construct an evolutionary scenario in which $\chi \sim 1$ in a star's core at the onset of core collapse. With these thoughts in mind, columns 2-4 of Table I show the values of χ, β, and ω that we have selected for our 6 initial models from Case B. Similar choices were used in our models from Case A.

3. THE EVOLUTIONS

All of the models that we have tested, whether from Case A or Case B, evolved in qualitatively the same manner. As rotational forces slowed the collapse perpendicular to the rotation axis somewhat, the core began to flatten. As the core flattened to only an axis ratio of $\sim 1.3:1$, it stopped its contraction and a spheroidal shaped shock front began to propagate from near the core center toward its surface. Density contours on a meridional slice through one core are shown in Figure 2. This figure illustrates the shape of the core at its typically flattest configuration.

It is easy to understand why the core stopped collapsing at only a slightly flattened structure in all of our evolutions. Consider a perfectly spherical collapse: If Γ is even slightly less than 4/3 (the classically derived "critical" value of Γ), then for a given increase in density, gravitational forces always increase more rapidly than do pressure forces and contraction will continue. However, when rotational forces (which do increase more rapidly than gravity) are included, the core must flatten somewhat. During the contraction of an oblate spheroid, the gravitational force at the pole increases less rapidly for a given increase in ρ than it does during a spherical contraction. Therefore, for values of Γ less than 4/3, pressure forces are able to grow more rapidly than gravity along the rotation axis once the core has flattened to a sufficient degree. The degree required is not large if Γ is only slightly less than 4/3, as it is in these models.

In our models, the density to which the core collapsed before the induced flattening stopped the contraction depended strongly on the initial rotational energy in the core. For large initial β, centrifugal forces were able to exert an influence early in the evolution and to stop the contraction at a density much lower than nuclear densities. For smaller β's, the core approached nuclear densities before undergoing centrifugal flattening. Column 5 of Table I lists the maximum density to which the core evolved before its infall was stopped in each initially uniform-χ model. Columns 6-8 of the table give a few other properties of the core near its point of maximum contraction: ω_c is the central angular velocity, W is the core's gravitational potential energy (to be compared with the initial value $|W| = 4 \times 10^{51}$ ergs), and E_{kin} is the translational kinetic energy of the infalling material. In all models, the <u>final</u> β for the core was only a few x 10^{-2}.

The strength of the shock front correlated, understandably, with the ratio $E_{kin}/|W|$ at the time the core bounced. The shock front propagated outward noticeably faster in the equatorial plane than it did in other directions, but in no case was the deviation from spherical symmetry extreme.

4. SUMMARY

Using values of Γ, γ_{min}, and γ_{max} that Van Riper (1978) has found most promising for a hydrodynamic envelope ejection, we have shown that even a small amount of rotation in the initial core can stop its collapse before nuclear densities are reached. We expected $\chi_i > 0.02$ to produce significant deviations from a spherically symmetric collapse, but have found that χ_i as much as ten times smaller than this will not allow the core to reach densities as high as in the spherical collapse. In no case, however, does the core flatten very much, nor does the value of β become very large. Low final β's preclude the formation of an axisymmetric torus. They also indicate that deformation of an iron core into a triaxial configuration or fragmentation of the core during its collapse is an extremely unlikely event. (Note: Classically, β must exceed 0.27 before a dynamic instability to non-axisymmetric perturbations is encountered.)

The small degree of flattening of the core also suggests that the reduced moment of inertia I of the core will always be relatively small in magnitude and hence that the third time derivative of I, which is proportional to the energy emitted in gravity wave radiation, will not be very significant. Numerically calculated estimates of I during some of these model evolutions supports this suspicion. If the γ_{min} and d used here are found to be realistic values after the detailed physics of the core collapse is well understood, it is clear that gravitational radiation from a core collapse will be difficult to measure.

Finally, we should point out that it is the relatively large values of γ_{min} (near 4/3) combined with values of d near unity that (a) prevented the core from flattening significantly in these models and (b) prevented the core from reaching high β configurations. If "realistic" values of either one (or both) of these parameters are found to be much smaller in more complete models of the core collapse, then the core will have to become flatter (and denser) before pressure gradients will support it along the rotation axis. All of the conclusions drawn here would be modified accordingly under those circumstances. It should also be noted that in general relativistic models, the critical Γ for spherical collapse is somewhat larger than 4/3 (Van Riper, 1979). Therefore, we predict that when fully general relativistic core collapses are performed including rotation, a given choice of γ_{min} and β_i will produce a slightly flatter and slightly denser core than the corresponding model that has been presented here.

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