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AN INSTABILITY LOCALIZED AT THE INNER SURFACE OF AN IMPLODING SPHERICAL SHELL S. J. Han

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ABSTRACT

It is shown that in an imploding spherical shell the surface instabilities are of two different types. The first, which occurs at the outer surface, is the Rayleigh-Taylor instability. The second instability occurs at the inner surface. This latter instability is not as disruptive as R-T modes, but it has three basic properties which differ considerably from those of the R-T instability: (1) it is oscillatory at early times; (2) it grows faster in the long wavelength modes; (3) it depends on the equation of state. It is further shown that this new instability is driven by amplified sound waves in the shell.

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We consider an imploding spherical shell that obeys the following ideal fluid equations:

$$\rho\left(\frac{d\underline{v}}{dt}\right) = -\underline{\nabla}p , \qquad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0 , \qquad (2)$$

$$\frac{d}{dt}(p/\rho^{\gamma}) = 0 \tag{3}$$

A self-consistent description of the shell motion can be obtained by introducing Sedov's hypothesis⁸ of self-similar motion, which in the Lagrangian representation is simply given as

$$\mathbf{R}(\mathbf{r}_{0},\mathbf{t}) = \mathbf{r}_{0} \mathbf{f}(\mathbf{t}) . \tag{4}$$

$$p(r_0,t) = p_0(r_0) f^{-3}, \quad p(r_0,t) = p_0(r_0) f^{-3\gamma}.$$
 (5)

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THE UNPERTURBED MOTION

Using the time-dependent pressure in Eq. (1), we obtain the unperturbed motion

$$\ddot{f}(t)f^{3\gamma-2}(t) = -\frac{1}{\rho_0 r} \frac{d}{dr} (\rho_0^{\gamma}(r)) = -\frac{1}{t_c^2} = -1 , \qquad (6)$$

The time-dependent part of Eq. (6) yields on integration

$$f = \left(\frac{2}{\alpha} \left(f^{-\alpha} - 1\right)\right)^{1/2} , \qquad (7)$$

where the initial values f(0) = 1 and f(0) = 0 have been used. Here $\alpha = 3(\gamma - 1)$. When $\gamma = 5/3$, Eq. (7) can be integrated once again to give

$$f^2 = 1 - t^2$$
, (8)

where $0 \le t \le 1$.

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$$\rho_{\rm o}(\mathbf{r}) = \hat{\rho}_{\rm o} \left((\mathbf{r}^2 - \mathbf{r}_{-}^2) / (\mathbf{r}_{+}^2 - \mathbf{r}_{-}^2) \right)^{1/(\gamma - 1)}, \qquad (9)$$

where r_+ and r_- are the outer and inner radii of the shell;

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$$p_{o}(r) = \hat{p}_{o}((r^{2} - r_{-}^{2})/(r_{+}^{2} - r_{-}^{2}))^{\gamma/(\gamma - 1)}, \qquad (10)$$

where \hat{p}_0 is a constant pressure at $r = r_+$.

STABILITY ANALYSIS

When a perturbation ξ is introduced, the position vector of a fluid element in Lagrangian variables becomes $\mathbf{R} + \boldsymbol{\xi}$. A straightforward, though somewhat tedious, calculation^{7,11} shows that $\boldsymbol{\xi}$ obeys the equation

$$f^{(3\gamma-1)} \ddot{\xi} = \frac{(\gamma-1)}{2} (r^2 - r_-^2) \nabla \sigma \div (\gamma-1) \sigma \underline{r} + \underline{r} \times \underline{\omega} + (\underline{r} \cdot \nabla) \underline{\xi} , \qquad (11)$$

where $\sigma \approx \nabla \cdot \xi$ and $\omega = \nabla \times \xi$.

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We now limit the discussion to the case of incompressible, irrotational perturbations for which the surface instabilities occur. That is, we choose the perturbations such that $\nabla \cdot \xi = 0$ and $\nabla \times \xi = 0$. This implies that $\xi = \nabla \chi$ with $\nabla^2 \chi = 0$. In spherical coordinates

$$\chi(\mathbf{r},\theta,\phi) = \sum_{\ell m} \left[Q_{+}^{\ell}(t) r^{\ell} + Q_{-}^{\ell}(t) r^{-(\ell+1)} \right] Y_{\ell m}(\theta,\phi) .$$
 (12)

To obtain $Q_{\pm}^{\ell}(t)$, we expand the perturbations in spherical harmonics¹¹

$$\hat{a}_1 = \hat{e}_r Y_{\ell m}$$
, $\hat{a}_2 = r \nabla Y_{\ell m}$, $\hat{a}_3 = r \times \nabla Y_{\ell m}$. (14)

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TIME-DEPENDENT EQUATION

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The Classical Rayleigh-Taylor Instability:

$$g(t) = \ddot{R}(t) = r \ \dot{f}(t) = -r \ f^{-3\gamma+2}$$
, (16)

t. in Eq. (15) can be written as

$$\ddot{Q}_{\pm}^{\ell}(t) + \left[\frac{3}{2} \mp (\ell + \frac{1}{2})\right] \frac{|g(t)|}{R(t)} Q_{\pm}^{\ell}(t) = 0 , \qquad (17)$$

Notice that $Q^{\ell}(t)$ modes localized at the inner surface are oscillatory in the static limit, whereas $Q^{\ell}_{+}(t)$ modes are unstable.

The growth rate of the unstable mode is given by

$$\gamma_{t}^{2} = |g_{0}| \left(\frac{m}{R_{0}}\right) = |g_{0}|k_{\theta},$$
 (18)

This is the growth rate of the classical Rayleigh-Taylor instability for static media.

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SOLUTIONS FOR AN ARBITRARY TIME

$$Q_{\pm}^{\ell}(t) = c_1 J_{\pm}(\ell, \gamma, t) + c_2 \mathcal{G}_{\pm}(\ell, \gamma, t)$$
 (19)

Here

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$$\mathcal{J}_{\pm}(\ell,\gamma,t) = {}_{2}F_{1}\left[\frac{1}{4} + \frac{2+id_{\pm}}{4\alpha}, \frac{1}{4} + \frac{2-id_{\pm}}{4\alpha}, \frac{1}{2}; 1 - f^{-\alpha}\right]$$
(20)

and

$$\mathcal{G}_{\pm}(\ell,\gamma,t) = (1 - f^{-\alpha})^{1/2} \, _{2}F_{1}\left[\frac{3}{4} + \frac{2 + id_{\pm}}{4\alpha}, \frac{3}{4} + \frac{2 - id_{\pm}}{4\alpha}, \frac{3}{2}; 1 - f^{-\alpha}\right], \quad (21)$$

where $d_{\pm} = \left[8\alpha \left\{\frac{3}{2} \mp (\ell + \frac{1}{2})\right\} - (\alpha + 2)^2\right]^{1/2}$.

As in Ref. 10, we obtain the stability criteria by evaluating the asymptotic limits:

$$\lim_{t \to 1} \frac{\mathcal{I}_{\pm}(\ell, \gamma, t)}{f(t)} \simeq a_{\pm}^{\ell} f^{(\alpha - 2 + id_{\pm})/2} + c.c. \qquad (22)$$

and

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$$\lim_{t \to 1} \frac{\mathcal{F}_{\pm}(\ell, \gamma, t)}{f(t)} \simeq b_{\pm}^{\ell} f^{(\alpha - 2 + 1d_{\pm})/2} + c. c., \qquad (23)$$

where

$$a_{\pm}^{\ell} = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{-id_{\pm}}{2\alpha})}{\Gamma(\frac{1}{4} + \frac{2-id_{\pm}}{7\alpha})\Gamma(\frac{1}{4} - \frac{2+id_{\pm}}{4\alpha})}, \quad b_{\pm}^{\ell} = i \frac{\Gamma(\frac{3}{2})\Gamma(\frac{-id_{\pm}}{2\alpha})}{\Gamma(\frac{3}{4} + \frac{2-id_{\pm}}{4\alpha})\Gamma(\frac{3}{4} - \frac{2+id_{\pm}}{4\alpha})}.$$
 (24)

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SUMMARY OF THE ANALYSIS

1) z = 0 mode. For $\gamma < 5/3$, the limits of \mathcal{F}_+/f and \mathcal{P}_+/f are finite.

If $\gamma < 5/3$, \mathcal{F}_{-}/f and \mathcal{R}_{-}/f diverge asymptotically, which signals instability.

2) The Q_+^{ℓ} modes with $\ell > 1$.

In the asymptotic limit, both \mathcal{F}_+ /f and \mathcal{F}_+ /f diverge as f^{κ} as $f \neq 0$, where κ is real and negative.

3) The asymptotic limits of both \mathcal{F}_{f} and \mathcal{P}_{f} of Q_{-}^{ℓ} diverge for $\ell \ge 1$ and $\gamma < 5/3$.

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Fig. 1. The absolute values of $a_{+}^{\ell}(\gamma)$ given in Eq. (20) are plotted against the mode number $\ell = 1$ to 50.

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Fig. 2. The exponent of the divergence factor κ is plotted against the mode number.

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Fig. 3. The absolute values of $a_{-}^{\ell}(\gamma)$ for $\gamma = 4/3$, 5/3, and 2 are plotted against the mode number $\ell = 1$ to 50.



Fig. 4. The absolute values of $a(\gamma, \mu_{g})$ for $\gamma = 4/3$, 5/3, and 2 are plotted against the mode number 2. Here μ_{g} is the smallest eigenvalue for a given 4 with $r_{+} = 7.0$ cm and $r_{-} = 5.0$ cm.

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