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AUTHOR(S): Masatoshi Imada

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PATTERN SELECTION AND INSTABILITY IN NONLINEAR WAVE EQUATION
-- AN ASPECT OF SOLITON AND CHAOS

Masatoshi Imada†
Institute for Theoretical Physics
and
Department of Physics
University of California
Santa Barbara, CA 93106

ABSTRACT

Pattern selection problems are found in a variety of phenomena. Fluid dynamical systems and nonlinear diffusion phenomena give typical examples of pattern formation problems in dissipative systems. In some cases the dissipation reduces the effective dimension of the system, and this fact leads to several strikingly universal behaviors which were initially found in simple model systems with a few degrees of freedom.

Nonlinear wave equations themselves, however, describes systems without dissipation in which the situation is more complicated. In spite of this complexity, many completely integrable systems are known in nonlinear wave equations, where neither ergodicity nor chaos is expected. With addition of small perturbation to completely integrable systems, one can see the growth of instability and the role of coherent structures in the pattern selection problem. Two aspects are briefly discussed in the following sections.

†Permanent address: Institute for Solid State Physics, University of Tokyo, Tokyo 106, JAPAN

I. NON-DISSIPATIVE SYSTEMS

Completely integrable systems are characterized by the fact that all the scatterings are elastic, which is equivalent to the infinite lifetime of each normal mode. In non-integrable systems, annihilation or creation of coherent structures like solitons is caused by inelastic scattering, which can cause the genesis of chaos. A computer simulation result[1] shows that orbital instability in the phase space is originated by the inelastic scattering of a soliton and an antisoliton in a perturbed sine-Gordon system. In this example, the phase space distance of two orbits whose initial conditions are very slightly different shows a sudden increase after the inelastic collisions. If one assumes continuing collisions of solitons at finite density in the infinite size system, this elementary process leads to positive Lyapunov exponents. Actually, the initial condition sensitivity is most prominent[2] at the threshold initial velocity of colliding solitons, below which a soliton and an antisoliton decay into oscillatory modes after the collision. The final spatial pattern is quite different when one changes the relative initial velocity of solitons around the threshold value. This kind of instability, that is, chaos, appears strongest for inelastic scattering of nonlinear, localized modes.

II. DISSIPATIVE SYSTEM

If a dissipative perturbation is added to an integrable system, the result may be quite different from the case of non-dissipative perturbation, because the final state of the system often falls onto a low-dimensional attractor. Because of the dissipation, only a small number of modes or spatial patterns survive after the transient time.

In this case the final state can be described by a system with a small number of degrees of freedom, and universal properties of chaos of simple systems may appear.

A characteristic problem in systems with many degrees of freedom is that of understanding which modes survive selectively after the transient time. Indeed it is expected that many attractors are possible candidates for the final state when the dissipation is small or the system is large. Although the final state is described by a small number of modes, the selected attractor may sensitively depend on the initial condition and the system parameter. Similar sensitivity is found also in systems with few degrees of freedom such as a damped driven pendulum.[3] The sensitivity becomes, however, incomparably strong in the partial differential equations. Actually, some universal routes to chaos such as the period doubling sequence seen in the damped driven pendulum are not found and interrupted by a change of the basin. An example of such a sensitivity is found in the system given by the following damped driven sine-Gordon equation:

$$\phi_{tt} - C_0^2\phi_{xx} + \sin\phi = -\gamma\phi_t + A \sin \omega t$$

$$- \mu \sin \omega t[\delta(x)-\delta(x-L)] \ .$$

Here γ is the strength of dissipation, and A is the amplitude of uniform driving force with frequency ω. The system is also driven by an ac field at the boundary of the system with length L and μ is the amplitude of this field, which corresponds to the magnetic field in the model of Josephson transmission line. If we fix other parameters and change the value A very slightly, the final spatial pattern often experiences a kaleidoscopic change. Although the final states are often periodic in time, the realized spatial patterns are quite different even when one changes the parameters very slightly. In some other cases, such a sensitive dependence leads to an intermittent behavior between two spatial patterns. The problem in partial differential equations with dissipation is to find a rule to decide which spatial pattern appears and how the pattern selection comes out. These are interesting open questions. If we compare the results shown in non-dissipative systems with the dissipative motion, chaotic behavior found in non-dissipative systems can be interpreted as the instability associated with changes of the spatial pattern. The change of spatial patterns is inhibited with increasing dissipation. In this sense the most characteristic feature of chaos in systems with many degrees of freedom is found in small dissipative cases.

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