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TITLE INTRODUCTORY REMARKS ON DOUBLE BETA DECAY AND NUCLEAR PHYSICS

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INTRODUCTORY REMARKS ON DOUBLE BETA DECAY AND NUCLEAR PHYSICS

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I. Introduction

In these introductory remarks I want briefly to review the particle physics aspects of double beta decay and the theory of the phenomenon. Then I shall discuss some of the things we can learn from existing data and what we might anticipate from future experiments.

The study of double beta decay has always included a search for the sequence of processes in which two neutrons are transformed into two protons and two electrons:



Alternatively, in more modern parlance, one can think of two down-quarks transforming into two up-quarks plus two electrons



$$"v" + d_2 \rightarrow u_2 + e^- .$$

Racah,⁽¹⁾ in 1937, proposed the sequence of eq. (1.1) as a test for the "symmetrical" theory of leptons introduced by Majorana⁽²⁾ in the same year. He took the neutrino exchanged between the neutrons to be a real particle, and hence assumed that one would need a source of neutrinos produced by nuclear processes in order to carry out the test. Furry,⁽³⁾ in 1939, realized that the neutrino could also be a "virtual particle", in the sense of perturbation theory, and thus invented the process of no-neutrino double beta decay in nuclei:

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- . \quad (1.3)$$

R. Davis⁽⁴⁾ carried out a test for the Racah sequence of eq. (1.1) using real neutrinos from a reactor in 1955, but for the past thirty years the emphasis has been on the virtual neutrino process of eq. (1.3) because it is a much more sensitive probe for the existence of Majorana neutrinos.

2. Dirac and Majorana Neutrinos

To appreciate the distinction between Dirac and Majorana neutrinos, we need to compare the neutrino ν_ρ^- which accompanies a negatively charged lepton ℓ^- in some hadronic decay process.

$$h^0 \rightarrow h^+ + \ell^- + \nu_\rho^- \quad (2.1)$$

with the neutrino ν_ρ^+ which accompanies a positively charged lepton ℓ^+ in some other decay process:

$$H^+ \rightarrow H^0 + e^+ + \nu_\ell^- . \quad (2.2)$$

For Dirac neutrinos, ν_ℓ^- and ν_ℓ^+ are distinct particles: consequently the Racah sequence of eq. (1.1) is forbidden and we may have a conservation law for the number of ℓ -type leptons.

For Majorana neutrinos, the ν_ℓ^- and ν_ℓ^+ are identical particles. The Racah sequence is allowed and there is no conservation law for ℓ -type leptons. There is, however, an important caveat concerning the neutrino mass. In the limit of zero mass for the neutrino and pure (V-A) for charged weak currents (the so-called γ_5 invariance), the ν_ℓ^- always has perfect positive helicity and the ν_ℓ^+ perfect negative helicity. It follows that, in this case, the Racah sequence is forbidden no matter whether the neutrino be a Majorana particle or not.

Therefore for the Majorana versus Dirac distinction to be operationally meaningful, we must break the γ_5 -invariance in one of two ways:

(1) charged currents must have an admixture of (V+A) and $m_\nu = 0$;

or

(2) charged currents are pure (V-A) and $m_\nu \neq 0$.

It is important to recognize that we are talking about the dominant properties of the theory in lowest order. In higher orders, the presence of an admixture of (V+A) in the dominantly (V-A) charged weak current will generate a mass for the neutrino; and likewise, the existence of a neutrino mass will give rise, in higher order, to an admixture of (V+A) currents. However, we expect these higher order

effects to be small, and we are therefore concerned with the question of which is the dominant mechanism for γ_5 -breaking in lowest order, (1) or (2).

In gauge theories, the option (1) is not possible because the same neutrino cannot couple, in general, to both (V-A) and (V+A) currents. This follows from the need to conserve gauge theoretic quantum numbers. Therefore, in gauge theories, the distinction between Majorana and Dirac neutrinos is only meaningful when the neutrino mass is not zero. It follows that the existence of Majorana neutrinos requires the existence of neutrino mass.⁽⁵⁾

Now the typical mass term in a Lagrangian always couples a left-handed field ν_L to a right-handed one N_R :

$$-\mathcal{L} = \frac{1}{2} m \bar{\nu}_L N_R + \text{hermitian conjugate} \quad (2.3)$$

The question whether this is a Dirac or a Majorana mass term depends upon the signs of the charged leptons to which ν_L and N_R couple in the charged weak current. If ν_L and N_R couple to leptons with the same charge, for example e_L^- and e_R^- respectively, then the Lagrangian is invariant under a global phase transformation,

$$\begin{bmatrix} e_{L,R}^- \\ \nu_L \\ N_R \end{bmatrix} \rightarrow e^{i\phi} \begin{bmatrix} e_{L,R}^- \\ \nu_L \\ N_R \end{bmatrix} \quad (2.4)$$

and we have a lepton conservation law. This corresponds to Dirac neutrinos and a Dirac mass term.

The same would hold true if N_R were a singlet under the weak

gauge group; but if N_R coupled to the lepton of opposite charge it would not. Suppose for example that ν_L coupled to e_L^- and N_R to e_R^+ (as is the case for $(\nu_L)^c$): in this case we do not have a global phase invariance and therefore no conservation law. This is the case of a Majorana mass term. It is a unique possibility which exists for neutrinos, but not for charged leptons.

In general both Dirac and Majorana mass terms can be present in the Lagrangian:

$$- \mathcal{L} = \frac{1}{2} \{ [\bar{\nu}_L, \overline{(N_R)^c}] \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} (\nu_L)^c \\ N_R \end{bmatrix} + \text{hermitian conjugate} \quad . \quad (2.5)$$

Under the presumption that ν_L and N_R couple to e_L^- and e_R^+ respectively, the diagonal elements m_L and m_R yield Majorana mass terms and the off-diagonal ones yield Dirac mass terms. As long as $(m_L + m_R) \neq 0$, the eigenvectors of this mass matrix are always Majorana neutrinos. By taking m_L close to zero, m_D equal to a charged lepton or quark mass, and m_R very large, we obtain the see-saw mechanism with a very light left-handed neutrino and a very heavy right-handed one.

3. Double Beta Decay

Two modes of double beta decay have been thoroughly analyzed in the literature. One is the two-neutrino mode

$$(A, Z) \rightarrow (A, Z + 2) + 2 e^- + 2 \bar{\nu}_e \quad (3.1)$$

which is expected as a second-order effect of the same Hamiltonian as

is responsible for single beta decay, and which should occur independently of whether the neutrino is a Dirac or Majorana particle. The other is the no-neutrino mode of eq. (1.3), which is also regarded as a second-order weak process, although it could arise from some entirely new interaction. A third mode, in which the two electrons are accompanied by a Majoron has also been considered.

In schematic form, the lifetime for the two-neutrino mode can be expressed as:

$$(1/\tau_{1/2}^{2\nu}(A)) = \{\text{Phase Space} \otimes \text{Coulomb Factor}\} \\ \times \left| M_{GT}^{2\nu} \right| \langle \mu \rangle^2 \quad (3.2)$$

where the phase space factor is a polynomial of degree 10 or 11 in the energy release Q_0 corresponding to the four-lepton final state, and $\langle \mu \rangle$ is an average energy denominator. Closure has been used to obtain the nuclear matrix element

$$M_{GT}^{2\nu} = \langle f | \sum_{i \neq j} \tau_i^+ \tau_j^+ \vec{\sigma}_i \cdot \vec{\sigma}_j | i \rangle \quad (3.3)$$

Further details can be found in an excellent series of review articles recently written by our hosts and other authors. (6)

The major problem in two-neutrino double beta decay is that the theoretical estimates of the nuclear matrix elements are generally larger than the values derived from experiment by a factor of order 3. Why does this happen, especially when all the naive arguments suggest that the matrix elements should be small?

Could there be some symmetry, for example the Wigner supermultiplet scheme, linking the ground states of (A,Z) and (A,Z + 2) together? The supermultiplet scheme is based on an SU(4) which contains the direct product of isospin and intrinsic spin, and which includes the commutator

$$\sum_{\ell} [\tau^+ \sigma_{\ell}, \tau^- \sigma_{\ell}] = 3\tau_3 \quad (3.4)$$

as part of its algebra.

Could it be that closure is a bad approximation? Professor Klapdor and his colleagues⁽⁷⁾ have pointed out that one gets roughly the correct nuclear matrix element by taking the lowest 1^+ intermediate state as dominant contributor to the sum over intermediate states.

Could it be that double beta decay is trying to tell us something new about the nucleus, as Petr Vogel⁽⁸⁾ has suggested? It is important to understand this problem in two-neutrino decay, especially if we want to extract reliable limits on the neutrino mass from no-neutrino decay.

Although no-neutrino double beta decay could, at least in principle, arise from some hitherto unknown interaction which violates lepton number conservation by two units, we are going to take the point of view that it occurs as a second-order weak effect brought on by the exchange of one or more neutrinos between two neutrons inside the nucleus. This exchange can only take place if the neutrinos are Majorana particles and at least one of them has a non-zero mass.

It has been pointed out by various authors⁽⁹⁾ that the actual observation of no-neutrino double beta decay implies the existence of a Majorana mass term for neutrinos independently of the interaction responsible for the process. This Majorana mass term may be a higher order effect, and hence small, but it does mean that neutrino mass eigenstates must correspond to Majorana particles.

Another general feature of no-neutrino double beta decay is that it can be used to set an upper bound on the effective (Majorana) mass of light neutrinos, or a lower bound on the effective mass of heavy neutrinos.⁽¹⁰⁾ Roughly speaking, the neutrino propagator which enters the decay amplitude engendered by neutrino exchange is of the form

$$P \approx \frac{m_\nu \langle p \rangle}{(m_\nu^2 + \langle p \rangle^2)} \quad (3.5)$$

where the average neutrino momentum $\langle p \rangle$ corresponds to the average separation between nucleons in the nucleus and lies somewhere between 10 and 100 MeV. When $m_\nu \ll \langle p \rangle$, the propagator reduces to $(m_\nu / \langle p \rangle)$ and a lower limit on the no-neutrino lifetime yields an upper bound on m_ν . When $m_\nu \gg \langle p \rangle$, the propagator becomes $(\langle p \rangle / m_\nu)$ and the lifetime bound yields a lower limit on m_ν . In practice one must also include in the bound the mixing matrix element $|u_{ei}|^2$, and so we find that either $|u_{ei}|^2 m_\nu < \text{few ev}$ or $> \text{few GeV}$. As the latter bound approaches the order of 100 GeV for the mass itself, we approach an effective point interaction for no-neutrino decay and must take the structure of the nucleon into account, for example by a quark model.

An interesting variant of this argument occurs when we assume

that the decay is engendered by the exchange of one light neutrino of order 10-20 eV and one heavy neutrino of opposite CP, both coupled to the same helicity current. (This is necessary for coherent interference between the two neutrinos.) In this case the requirement that the effective mass, which is of the form

$$m_{\beta\beta} = (\cos^2\theta m_{\text{Light}} - \sin^2\theta m_{\text{Heavy}} F) \quad (3.6)$$

where F is the ratio of propagation factors for heavy and light neutrinos in the nuclear medium, be less than a few eV, leads to an upper bound on the product $(\sin^2\theta)m_{\text{Heavy}}$ of mixing angle times heavy neutrino mass.⁽¹¹⁾ This has been used recently by Langacker, Sathiapalan and Steigman⁽¹²⁾ to exclude large regions of the heavy mass domain.

In terms of neutrino mass as the lepton number violating parameter, the half-life for no-neutrino decay takes the form:

$$\left[1 / T_{1/2}^{\text{ov}}(A) \right] = \{ \text{Phase Space} \oplus \text{Coulomb Factor} \} \times \left| \frac{m_{\beta\beta}}{\text{eV}} \cdot M_{\text{GT}}^{\text{ov}}(1-X_F) \right|^2 \quad (3.7)$$

where the phase space factor is now a polynomial of only the fifth or sixth degree in Q_0 because there are only two fermions in the final state. The nuclear matrix elements are given by:

$$M_{\text{GT}}^{\text{ov}} = \langle f | \sum_{ij} \tau_i^+ \tau_j^+ \vec{\sigma}_i \cdot \vec{\sigma}_j h^+(r_{ij}) | i \rangle \quad (3.8)$$

$$X_F M_{GT}^{0\nu} = (g_V^2/g_A^2) \langle f | \sum_{i,j} \tau_i^+ \tau_j^+ h^+(r_{ij}) | i \rangle$$

where $h^+(r_{ij})$ is the configuration space form of the neutrino propagator. More general expressions involving both the effective mass and right-handed current parameters can be found in the reviews mentioned above. (6)

On grounds of helicity, we can show that for $0^+ \rightarrow 0^+$ nuclear double beta decay transitions, the angular distributions for the two electrons are predominantly back-to-back for the mass-induced no-neutrino mode and for two-neutrino decay because the electrons have the same helicity. For the right-handed current induced no-neutrino mode, the electrons have opposite helicities, and hence they are emitted in a predominantly parallel configuration.

4. Comparison of Theory and Experiment for Selected Isotopes.

I now want to consider what we can learn from the existing data on double beta decay. To some extent, I am de-emphasizing the question of bounds on neutrino mass, and concentrating instead on the properties of the phenomenon itself.

In Table 1, the data and theoretical analyses for four specific isotopes are given. The first two rows consist of the Q_0 values and experimental data on lifetimes; the next three give the phase space Φ Coulomb factor for the two-neutrino mode, the corresponding nuclear matrix element (as determined by Haxton, Stephenson, and Strottman), and the predicted half-life for two-neutrino decay. The last three rows give the same calculations for the no-neutrino mode.

Besides the data we have on the tellurium isotopes, we are now in possession of two new and important pieces of data. One is the lower bound on the no-neutrino lifetime for $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ of 4×10^{23} years,⁽¹³⁾ and the other is the apparent consistency between the geochemical and directly-observed lifetimes for $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$, each being about 1.3×10^{20} years.⁽¹⁴⁾ Here I am presuming that a significant fraction of the events reported at this meeting by Mike Moe will turn out to be double beta decay.

From the last rows of Table 1, we see that the lifetime for no-neutrino decay in ^{82}Se is roughly 1/3 of the corresponding lifetime for ^{76}Ge . Therefore from the measured limit on $^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2e^-$, we deduce that the no-neutrino lifetime for ^{82}Se is

$$T_{1/2}^{0\nu} (^{82}\text{Se}) \gtrsim 1.3 \times 10^{23} \text{ years} . \quad (4.1)$$

In other words less than 0.1% of all ^{82}Se decays are expected to occur via the no-neutrino mode.

Since the ^{82}Se lifetime comes essentially from the two-neutrino mode alone, we can turn the above analysis around and predict the two-neutrino lifetime for ^{76}Ge from that for ^{82}Se . The ^{76}Ge lifetime for the two-neutrino mode is expected to be about 16 times longer than that for ^{82}Se ; hence we expect that

$$T_{1/2}^{2\nu} (^{76}\text{Ge}) \approx 2 \times 10^{21} \text{ years} . \quad (4.2)$$

Since Elliott, Hahn, and Moe⁽¹⁴⁾ have been striving heroically to observe a lifetime of order 10^{20} years in ^{82}Se , the question arises as

to whether the ^{76}Ge lifetime, being an order of magnitude longer, is actually observable in the laboratory.

In these arguments, we have tacitly assumed that, although the absolute magnitudes of nuclear matrix elements are too large by a significant factor, the ratios of different matrix elements are reasonably correct within the $A = 76 - 82$ range of nuclei. What happens when we compare these nuclei with much heavier ones?

Both the Heidelberg and the Missouri groups have measured directly, by the geochemical method, the ratio of lifetimes for ^{130}Te and ^{82}Se double beta decay. The geochemical method, since it involves the detection of the daughter nucleus only, does not distinguish between different modes of decay, but we may use a comparison with the limits on ^{76}Ge decay to argue that the no-neutrino mode is no more than a few percent of all ^{130}Te decays. Thus the ratio of lifetimes is essentially the ratio of lifetimes for the two-neutrino mode.

Now, quite remarkably, the phase space plus Coulomb factor for the two-neutrino mode in ^{82}Se is equal to that for ^{130}Te , as can be seen from Table 1. This is purely coincidental: the phase space favors ^{82}Se , which has the larger Q -value, while the Coulomb favors ^{130}Te , which has the larger Z , and there is no obvious reason why these effects should cancel each other out. Nevertheless, the equality means that the ratio of two-neutrino lifetimes is a direct measure of the corresponding nuclear matrix elements.

Experimentally the Heidelberg group⁽¹⁵⁾ finds a ratio of 13 ± 2 , while the Missouri group⁽¹⁶⁾ gives a value of 7-8. Allowing for the

average energy denominators*, we then find that the ratio of matrix elements must be in the range of 3-4:

$$\left\{ \frac{|M_{GT}^{2\nu}|_{82}}{|M_{GT}^{2\nu}|_{130}} \right\} \approx 4.3 \text{ (Heidelberg)}$$
$$\approx 3.4 \text{ (Missouri)} \quad . \quad (4.3)$$

This constitutes the first direct evidence for a significant difference between two-neutrino nuclear matrix elements. It should provide a bench-mark test for all theoretical calculations of such matrix elements.

5. Possibilities for New Experiments

There is a considerable degree of activity these days regarding isotopes ^{100}Mo and ^{136}Xe . Here at Osaka, Professor Ejiri is engaged in an experiment with the former isotope, and he has already reported preliminary results at the Sendai Conference. At other institutions there are groups actively working on Xenon TPC's. The advantages of ^{100}Mo are its relatively large Q-value and a recent suggestion by Vogel that its matrix element will be large; ^{136}Xe is a relatively abundant isotope, and the TPC allows one to combine source and detector in a single unit.

Another isotope which I believe is worthy of re-investigation is ^{48}Ca . It has the largest energy release available (4.3 MeV), and there is little disagreement over its matrix element, which, however, tends to be small. The second largest Q-value (3.7 MeV) occurs in ^{150}Nd , which is favored over ^{48}Ca by the Coulomb factor.

The present data and theoretical expectations for the

two-neutrino mode in these isotopes is shown in Table 2. A comparison of these numbers with the ^{82}Se lifetime suggests that we may be on the verge of seeing the two-neutrino decay mode come in like gang-busters.

In conclusion, let me say that we have made great progress in the past five years, and I look forward to even more progress, both theoretically and experimentally in the next five years. Thank you for your kind attention.

Table 1: Experimental Data and Theoretical Predictions for Double Beta Decay in Four Isotopes

	$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	^{82}Se	^{130}Te	^{128}Te
Q_0 (keV)	2039.6 ± 0.9	2995 ± 6	2533 ± 4	868 ± 4
$T_{1/2}(\text{exptl})_{\text{yrs}}$	$>4\text{E}23$ (ov)	$1.3 \text{ E}20$ (Geo;Direct)	$(1-3) \text{ E}21$ Geo	$\text{E}24-25$ Geo
$T_{1/2}^{2\nu} \left \frac{M_{\text{GT}}^{2\nu}}{\langle \mu \rangle} \right ^2$	$7.7\text{E}18$	$2.3\text{E}17$	$2.1\text{E}17$	$1.2\text{E}21$
		3	3	
$\left \frac{M_{\text{GT}}^{2\nu}}{\langle \mu \rangle} \right ^2$	$2\text{E}(-2)$	$1\text{E}(-2)$	$1.3\text{E}(-2)$	$1.4\text{E}(-2)$
$T_{1/2}^{2\nu}$	$4\text{E}20$	$2.6\text{E}19$	$1.6\text{E}19$	$8.4\text{E}22$
$T_{1/2}^{\text{ov}} \left \frac{m}{\text{ev}} M_{\text{GT}}^{\text{ov}} (1-X_F) \right ^2$	$4.1\text{E}25$	$9.3\text{E}24$	$5.9\text{E}24$	$1.4\text{E}26$
$ M_{\text{GT}}^{\text{ov}} (1-X_F) ^2$	$2.5\text{E}1$	$1.7\text{E}1$	$3.8\text{E}1$	$3.7\text{E}1$
$T_{1/2}^{\text{ov}} \left(\frac{m\nu}{\text{ev}} \right)^2$	$1.6\text{E}24$	$5.6\text{E}23$	$1.6\text{E}23$	$3.8\text{E}24$

Table 2: Possibilities for New Experiments

	^{48}Ca	^{136}Xe	^{100}Mo	^{150}Nd
Q ₀	4271 ± 4	2479 ± 8	3034 ± 6	3367 ± 2
$T_{1/2}(\text{exptl})_{0\nu}^{2\nu}$	>4E(19) >2E(21)	>2E(19) >1E(21) Chairman P.	>1E(18) >1.7E(20) Ejiri at Sen.	>1.3E(19)
$T_{2\nu} \left \frac{M^{2\nu}}{\langle \mu \rangle} \right ^2$	2.5E(16)	2E(17)	1.1E(17)	8.4E(15)
$\left \frac{M^{2\nu}}{\langle \mu \rangle} \right ^2$	8.6E(-4) HSS	9E(-3) Klapdor...	5.3E(-2) V-F	6.4E(-2)
$T_{2\nu}$	3E(19)	2E(19)	2.1E(18)	1.3E(17)
$T_{0\nu} \left \frac{m_{\nu}}{\text{ev}} \right ^2 0\nu ^2$	4E(24)	5E(24)	6E(24)	1.2E(24)

References

1. G. Racah, Nuovo Cimento 14, 322 (1937).
2. E. Majorana, Nuovo Cimento 14, 171 (1937).
3. W. H. Furry, Phys. Rev. 56, 1184 (1939).
4. R. Davis, Phys. Rev. 97, 766 (1955).
5. For a review of this point see E. Takasugi, these proceedings.
6. M. Ido, T. Kotani, and E. Takasugi, Prog. Theoret. Phys.,

Supplement No. 83 (1985); W. C. Haxton and G. J. Stephenson, Jr., Progress in Particle and Nuclear Physics 12, 409 (1984); M. G. Shchepkin, Soviet Physics--Uspekhi, 27, 555 (1984); H. Primakoff and S. P. Rosen, Ann. Rev. Nucl. Sci. 31, 145 (1981); J. Vergados, Phys., Rep. 133, 1 (1986).

7. K. Grotz and H. V. Klapdor, Phys., Lett. 153B (1985) 1.
8. P. Vogel, these proceedings.
9. J. Schechter and J. W. F. Valle, Phys. Rev., D25, 2951 (1982); E. Takasugi, Phys. Lett 149B, 372 (1984); J. Nieves, *ibid* 147B, 375 (1984); and B. Kayser, S. Petcov and S. P. Rosen (to be published).
10. A. Halprin, M. Minkowski, H. Primakoff and S. P. Rosen, Phys. Rev. D13, 2567 (1976).
11. A. Halprin, S. Petcov, and S. P. Rosen, Phys. Lett. 125B, 335 (1983).
12. P. Langacker, Sathiapalan, and G. Steigman, Nucl. Phys.
13. See D. Caldwell, these proceedings, for a review.
14. M. Moe, these proceedings.
15. T. Kirsten, these proceedings.
16. O. Manuel, these proceedings