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...epresentation for ENDF

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New Fission Neutron Spectrum Representation for ENDF

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NEW FISSION NEUTRON SPECTRUM REPRESENTATION FOR ENDF

by

David G. Madland

ABSTRACT

A new representation of the prompt fission neutron spectrum is proposed for use in the Evaluated Nuclear Data File (ENDF). The proposal is made because a new theory exists by which the spectrum can be accurately predicted as a function of the fissioning nucleus and its excitation energy. Thus, prompt fission neutron spectra can be calculated for cases where no measurements exist or where measurements are not possible. The mathematical formalism necessary for application of the new theory within ENDF is presented and discussed for neutron-induced fission and spontaneous fission. In the case of neutron-induced fission, expressions are given for the first-chance, second-chance, third-chance, and fourth-chance fission components of the spectrum together with that for the total spectrum. An ENDF format is proposed for the new fission spectrum representation, and an example of the use of the format is given.

I. INTRODUCTION

The fission neutron spectrum representation summarized in the following is based upon recent new developments in the theory of prompt fission neutron spectra due to Madland and Nix.[1-6] A new theory exists which is based upon standard nuclear-evaporation theory and accounts for the physical effects of the motion of the fission fragments, the distribution of fragment excitation energy, and the energy dependence of the process inverse to neutron emission, namely, compound-nucleus formation.

The simplest form of the new theory is the constant compound-nucleus cross-section approximation[5,6] by which the nuclear level-density parameter is adjusted slightly to simulate the energy dependence of the cross section. In this case, an accurate closed-form expression for the spectrum is obtained which is easily evaluated using any modern computer facility. In addition, a closed-form expression is obtained for the integral of the spectrum over arbitrary energy range. The closed-form expressions for the spectrum and the integral of the spectrum mean that relatively simple computer coding requirements exist for their implementation and that relatively fast computer evaluation times are involved in their use. The practical consequences for computation are obvious in, for example, the use of the spectrum in Monte-Carlo calculations or the use of the spectrum integral in constructing multigroup spectra. For reasons such as these, we use the constant cross-section approximation in this proposal. This approximation is identical to that described and recommended at the recent Workshop on Evaluation Methods and Procedures at Brookhaven National Laboratory.[5]

In Sec. II we present the mathematical formulation and discuss the cases of neutron-induced first-chance fission and spontaneous fission. The complexities introduced by the onset of multiple-chance fission processes and the consequent additional mathematical formalism are presented in Sec. III. An ENDF format for the new fission spectrum representation and an example of the use of the format are given in Sec. IV, where an expression for the integral of the spectrum over an arbitrary energy range is also given. A concluding discussion is presented in Sec. V. The current methods to evaluate the constants of the new spectrum are presented in App. A and details concerning the use of the average prompt neutron multiplicity $\overline{\nu}_p$ in the multiple-chance fission spectra of Sec. III are given in App. B.

The reader who is concerned only with incident neutron energies below the second-chance fission threshold (approximately 6 MeV) or with spontaneous fission may omit Sec. III and App. B.

II. FIRST-CHANCE FISSION AND SPONTANEOUS FISSION

The expression $f(E \to E')$ for the spectrum of fission neutrons of energy E' due to first-chance fission induced by neutrons of energy E, and the expression $f(E')$ for the spectrum of fission neutrons of energy E' due to spontaneous fission, are identical in form and differ only in the values of the three constants involved. This form is given in the constant cross-section approximation[5,6] by

$$f(E \to E') = \frac{1}{2}\left[g(E',E_f^L) + g(E',E_f^H)\right] \quad , \tag{1}$$

where

$$g(E',E_f) = \frac{1}{3(E_f T_m)^{1/2}} \left[u_2^{3/2} E_1(u_2) - u_1^{3/2} E_1(u_1) + \gamma(3/2,u_2) - \gamma(3/2,u_1)\right] \quad . \tag{2}$$

In Eq. (2) the values of u_1 and u_2 are given by

$$u_1 = \left(\sqrt{E'} - \sqrt{E_f}\right)^2/T_m \quad ,$$
$$u_2 = \left(\sqrt{E'} + \sqrt{E_f}\right)^2/T_m \quad , \tag{3}$$

$$E_1(x) = \int_x^\infty \frac{e^{-u}}{u}\, du \tag{4}$$

is the exponential integral, and

$$\gamma(a,x) = \int_o^x u^{a-1}\, e^{-u}\, du \tag{5}$$

is the incomplete gamma function.

The exponential integral and incomplete gamma functions are generally available as program library functions. Moreover, as stated in Sec. I, the evaluation times of these functions are sufficiently small so as to not prohibit their use in large computer program loops. A closed-form expression for the integral of the spectrum given by Eqs. (1) and (2) exists for an arbitrary finite integration range. This expression, also involving the exponential integral and incomplete gamma function, is given in Sec. IV.

The three constants appearing in Eqs. (1) and (2) are E_f^L, E_f^H, and T_m, which are, respectively, the average kinetic energy per nucleon of the average light fission fragment, the average kinetic energy per nucleon of the average heavy fission fragment, and the maximum temperature of the fission-fragment residual-nuclear-temperature distribution. The constants E_f^L and E_f^H are to a good approximation independent of the incident neutron energy E, whereas $T_m = T_m(E)$ for neutron-induced fission and T_m = constant for spontaneous fission. Of course, all three of the constants depend on certain fission-related quantities and on certain parameters of the compound fissioning nucleus. Methods and formulas to obtain values of the three constants are discussed in Refs. 1-6 and in App. A.

Examples of the dependence of the spectrum given by the constant cross-section approximation of Eqs. (1) and (2) upon both the fissioning nucleus and its excitation energy are shown in Figs. 1 and 2. Figure 1 shows how the spectrum at high energy increases and at low energy decreases as the charge of the fissioning nucleus increases, for thermal-neutron-induced fission. Figure 2 shows how the spectrum at high energy increases and at low energy decreases as the kinetic energy of the incident neutron increases, for the first-chance fission of ^{235}U.

A comparison of the prediction of the constant cross-section approximation given by Eqs. (1) and (2) with experimental data and with the energy-dependent cross-section calculation[3-6] is shown in Fig. 3. The figure shows that the constant cross-section calculation given by the dashed curve agrees very well with

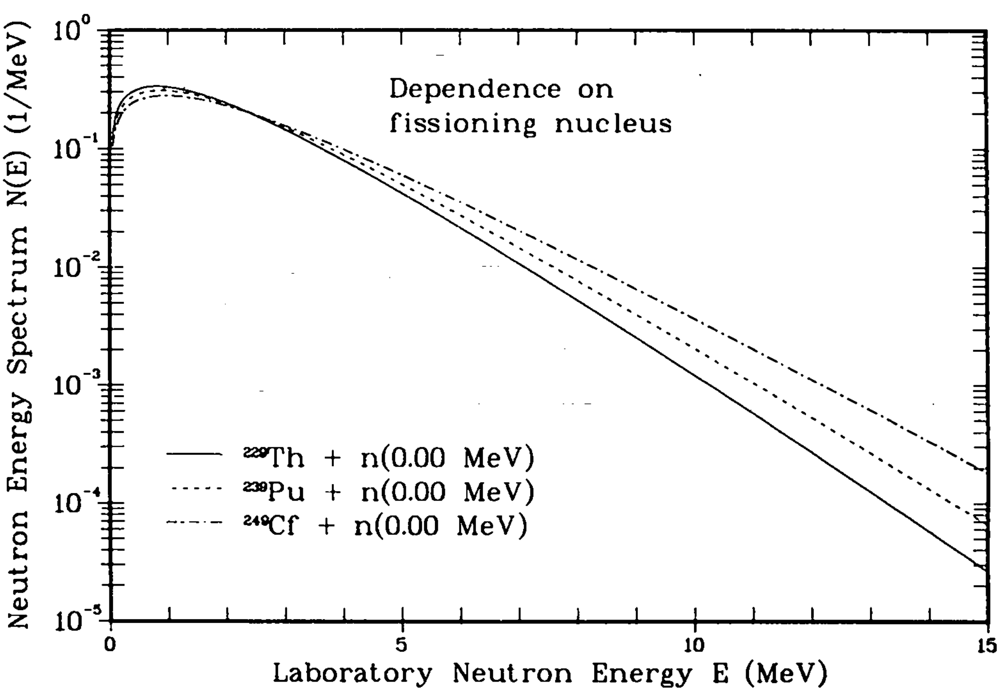

Fig. 1. Dependence of the prompt fission neutron spectrum on the fissioning nucleus, for thermal-neutron-induced fission. The calculations are performed using Eqs. (1) and (2).

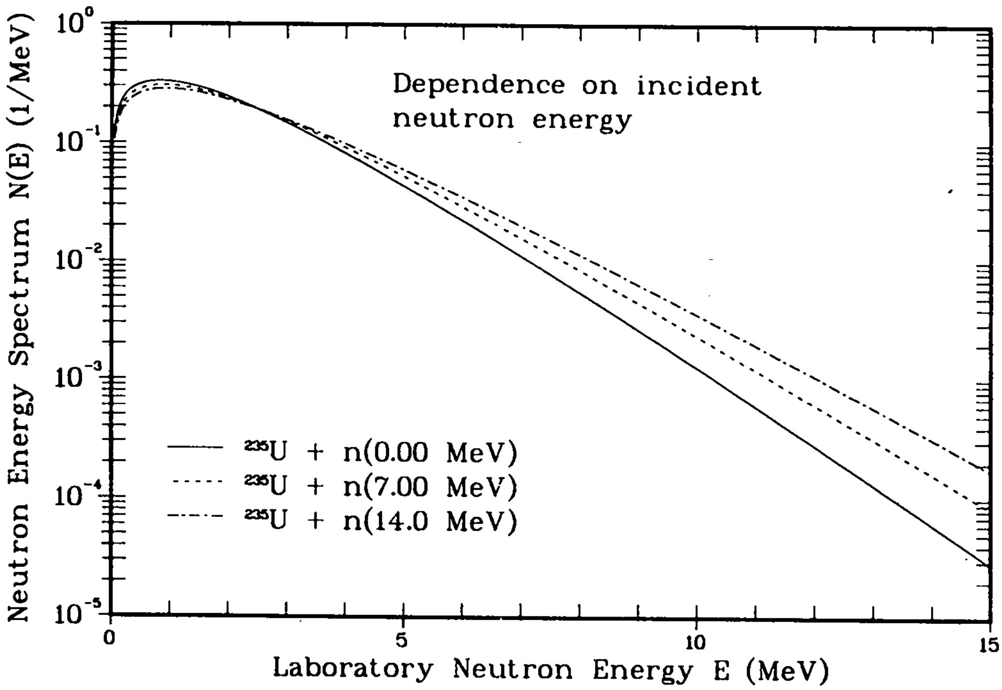

Fig. 2. Dependence of the prompt fission neutron spectrum on the kinetic energy of the incident neutron, for the fission of ^{235}U. *The calculations are performed using Eqs. (1) and (2) under the assumption of first-chance fission only.*

the energy-dependent cross-section calculation given by the solid curve and with the experiment[7] over the entire energy range shown. The value of the adjusted nuclear level-density parameter as used in the constant cross-section calculation is given by a_{eff} = A/(10 MeV).

For neutron-induced fission we note that the first-chance fission spectrum (MT = 19) is identical to the total fission spectrum (MT = 18) for incident neutron energies E less than the second-chance fission threshold. Thus, for E < 6 MeV both MT = 18 and MT = 19 are given by Eqs. (1) and (2) together with the values of E_f^L and E_f^H and a tabulation $T_m(E)$. Readers who are concerned only with first-chance fission or spontaneous fission may omit the next section and proceed directly to Sec. IV.

III. MULTIPLE-CHANCE FISSION

In order to consider second-chance, third-chance, and fourth-chance fission, in addition to the first-chance fission already discussed, the following notation and corresponding ENDF identification are given:

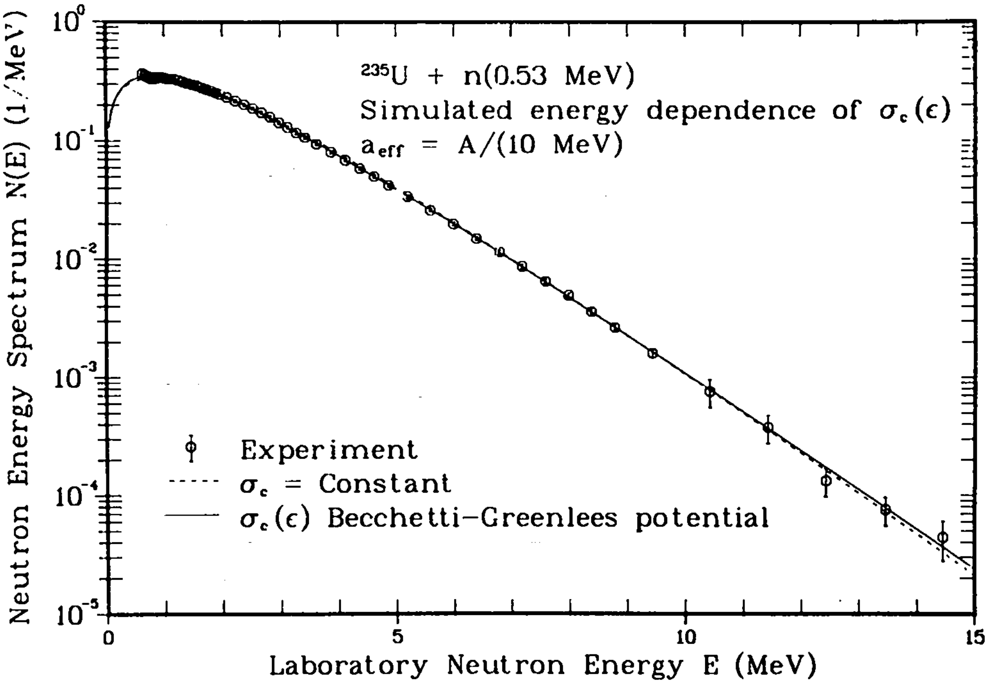

Fig. 3. Prompt fission neutron spectra for the fission of ^{235}U *induced by 0.53-MeV neutrons, illustrating the simulated energy-dependence of the compound-nucleus cross section* $\sigma_c(\varepsilon)$*. The dashed curve shows the constant cross-section approximation given by Eqs. (1) and (2) using the effective nuclear level-density parameter* a_{eff} = *A/(10 MeV) to simulate the energy dependence of* $\sigma_c(\varepsilon)$*. The solid curve shows the result obtained by using energy-dependent cross sections in the calculation of Refs. (3-6). The experimental data are those of Johansson and Holmqvist (Ref. 7).*

File 5 (MF = 5)

$F(E \rightarrow E')$	= total fission spectrum	MT = 18
$F_1(E \rightarrow E')$	= first-chance fission spectrum	MT = 19
$F_2(E \rightarrow E')$	= second-chance fission spectrum	MT = 20
$F_3(E \rightarrow E')$	= third-change fission spectrum	MT = 21
$F_4(E \rightarrow E')$	= fourth-chance fission spectrum	MT = 38

File 3 (MF = 3)

$\sigma(E)$	=	total fission cross section	MT = 18
$\sigma_1(E)$	=	first-chance fission cross section	MT = 19
$\sigma_2(E)$	=	second-chance fission cross section	MT = 20
$\sigma_3(E)$	=	third-chance fission cross section	MT = 21
$\sigma_4(E)$	=	fourth-chance fission cross section	MT = 38

B_1 = binding energy of neutron in the compound system for target isotope ZA

= Q(ZA) MT = 102

B_2 = binding energy of neutron in the compound system for target isotope ZA - 1

= Q(ZA - 1) MT = 102

B_3 = binding energy of neutron in the compound system for target isotope ZA - 2

= Q(ZA - 2) MT = 102

B_4 = binding energy of neutron in the compound system for target isotope ZA - 3

= Q(ZA - 3) MT = 102

where in all cases E and E' are defined as in Sec. II.

In addition, the following evaporation spectra (LF = 9 Secondary Energy Distribution Law already existing in ENDF-102) are required to describe the spectrum of neutrons emitted prior to fission in second-chance, third-chance, and fourth-chance fission:

$$H_2(E \rightarrow E') = \left(E'/\theta_2^2\right) \exp\left(-E'/\theta_2\right)$$
$$\theta_2 = \theta_2(E) \quad , \tag{6}$$

$$H_3(E \rightarrow E') = \left(E'/\theta_3^2\right) \exp\left(-E'/\theta_3\right)$$
$$\theta_3 = \theta_3(E) \quad , \tag{7}$$

$$H_4(E \rightarrow E') = \left(E'/\theta_4^2\right) \exp\left(-E'/\theta_4\right)$$
$$\theta_4 = \theta_4(E) \quad , \tag{8}$$

where the subscripts 2, 3, 4 refer to second-chance, third-chance, and fourth-chance fission, respectively, and in each case the temperature θ is a tabulation $\theta = \theta(E)$.

Finally, we adopt the following notation for the average prompt neutron multiplicity in each succeeding fission within the multiple-chance fission sequence:

$\nu_1(E)$ = average prompt neutron multiplicity $\overline{\nu}_p(E)$ corresponding to neutrons of energy E incident on target isotope ZA,

$\nu_2(E)$ = average prompt neutron multiplicity $\overline{\nu}_p(E)$ corresponding to neutrons of energy E incident on target isotope ZA − 1,

$\nu_3(E)$ = average prompt neutron multiplicity $\overline{\nu}_p(E)$ corresponding to neutrons of energy E incident on target isotope ZA − 2, and

$\nu_4(E)$ = average prompt neutron multiplicity $\overline{\nu}_p(E)$ corresponding to neutrons of energy E incident on target isotope ZA − 3.

Using the preceding notation and corresponding ENDF identification, we construct the first-chance, second-chance, third-chance, and fourth-chance fission components of the prompt fission neutron spectrum and the total spectrum:

$$F_1(E\to E') = \{\sigma_1(E)\nu_1(E)f_1(E\to E')\}/D(E) \quad , \tag{9}$$

$$F_2(E\to E') = \{\sigma_2(E)[H_2(E\to E') + \nu_2(E - 2\theta_2 - B_2) \times f_2(E\to E')]\}/D(E) \quad , \tag{10}$$

$$F_3(E\to E') = \{\sigma_3(E)[H_2(E\to E') + H_3(E\to E') + \nu_3(E - 2\theta_2 - B_2 - 2\theta_3 - B_3) \times f_3(E\to E')]\}/D(E) \quad , \tag{11}$$

$$F_4(E\to E') = \{\sigma_4(E)[H_2(E\to E') + H_3(E\to E') + H_4(E\to E') + \nu_4(E - 2\theta_2 - B_2 - 2\theta_3 - B_3 - 2\theta_4 - B_4)f_4(E\to E')]\}/D(E), \text{ and} \tag{12}$$

$$F(E\to E') = F_1(E\to E') + F_2(E\to E') + F_3(E\to E') + F_4(E\to E'), \text{ where} \tag{13}$$

$$D(E) = \sigma_1(E)\nu_1(E) + \sigma_2(E)\left[1 + \nu_2(E - 2\theta_2 - B_2)\right]$$

$$+ \sigma_3(E)\left[2 + \nu_3(E - 2\theta_2 - B_2 - 2\theta_3 - B_3)\right]$$

$$+ \sigma_4(E)\left[3 + \nu_4(E - 2\theta_2 - B_2 - 2\theta_3 - B_3\right.$$

$$\left. - 2\theta_4 - B_4)\right], \text{ and where} \tag{14}$$

$f_1(E \to E')$ is given by Eqs. (1) and (2) together with constants $E^L_{f_1}$ and $E^H_{f_1}$ and the tabulation $T_{m_1}(E)$,

$f_2(E \to E')$ is given by Eqs. (1) and (2) together with constants $E^L_{f_2}$ and $E^H_{f_2}$ and the tabulation $T_{m_2}(E - 2\theta_2)$,

$f_3(E \to E')$ is given by Eqs. (1) and (2) together with constants $E^L_{f_3}$ and $E^H_{f_3}$ and the tabulation $T_{m_3}(E - 2\theta_2 - 2\theta_3)$, and

$f_4(E \to E')$ is given by Eqs. (1) and (2) together with constants $E^L_{f_4}$ and $E^H_{f_4}$ and the tabulation $T_{m_4}(E - 2\theta_2 - 2\theta_3 - 2\theta_4)$.

The evaporation spectra $H_2(E \to E')$, $H_3(E \to E')$, and $H_4(E \to E')$ are given by Eqs. (6)-(8), respectively, where θ_2, θ_3, and θ_4 are tabulated as $\theta_2(E)$, $\theta_3(E - 2\theta_2)$, and $\theta_4(E - 2\theta_2 - 2\theta_3)$. Finally, ν_1, ν_2, ν_3, and ν_4 are either calculated directly or are obtained from ENDF as discussed in App. B. Equations (9), (10), (11), (12), and (13) give, using Eq. (14), the files MT = 19, MT = 20, MT = 21, MT = 38, and MT = 18, respectively.

The probabilities $p_k(E)$ that the distribution functions $f_i(E \to E')$ and $H_j(E \to E')$ are used at energy E are obtained from Eqs. (9)-(12) by inspection. They are given by

<u>MT = 19</u>

$$p_1^{(19)}(E) = \sigma_1(E)\nu_1(E)/D(E) \quad , \tag{15}$$

MT = 20

$$p_1^{(20)}(E) = \sigma_2(E)/D(E) \tag{16}$$

$$p_2^{(20)}(E) = \sigma_2(E)\nu_2(E - 2\theta_2 - B_2)/D(E) \quad , \tag{17}$$

MT = 21

$$p_1^{(21)}(E) = \sigma_3(E)/D(E) \tag{18}$$

$$p_2^{(21)}(E) = p_1^{(21)}(E) \tag{19}$$

$$p_3^{(21)}(E) = \sigma_3(E)\nu_3(E - 2\theta_2 - B_2 - 2\theta_3 - B_3)/D(E), \text{ and} \tag{20}$$

MT = 38

$$p_1^{(38)}(E) = \sigma_4(E)/D(E) \tag{21}$$

$$p_2^{(38)}(E) = p_3^{(38)}(E) = p_1^{(38)}(E) \tag{22}$$

$$p_4^{(38)}(E) = \sigma_4(E)\nu_4(E - 2\theta_2 - B_2 - 2\theta_3 - B_3 - 2\theta_4 - B_4)/D(E) \quad , \tag{23}$$

where

$$p_1^{(19)}(E) + p_1^{(20)}(E) + p_2^{(20)}(E) + p_1^{(21)}(E) + p_2^{(21)}(E) + p_3^{(21)}(E)$$

$$+ p_1^{(38)}(E) + p_2^{(38)}(E) + p_3^{(38)}(E) + p_4^{(38)}(E) = 1 \quad . \tag{24}$$

Note that $p_1^{(19)}(E) \equiv 1$ for all values of E less than the second-chance fission threshold (approximately 6 MeV).

To conclude this section, we note that an approximation exists by which the above equations can be greatly simplified, but at the expense of the physical content, and therefore, accuracy of the calculated multiple-chance fission neutron spectrum. The approximation consists in the use of the following equations:

$$\nu_2(E - 2\theta_2 - B_2) = \nu_1(E) - 1 \quad ,$$

$$\nu_3(E - 2\theta_2 - B_2 - 2\theta_3 - B_3) = \nu_1(E) - 2 \quad \text{, and} \tag{25}$$

$$\nu_4(E - 2\theta_2 - B_2 - 2\theta_3 - B_3 - 2\theta_4 - B_4) = \nu_1(E) - 3 \quad .$$

We do not recommend the use of Eq. (25) in the calculation of multiple-chance fission spectra.

IV. ENDF FORMAT

In this section we propose an ENDF format for the new fission neutron spectrum representation. We refer to page numbers of the October 1979 version of the ENDF-102 Manual[8] to identify text insertion points.

<u>Page 5.4</u> Before "Note: Distribution laws are not..." insert the following:

LF = 12, <u>Energy-Dependent Fission Neutron Spectrum (Madland and Nix)</u>

$$f(E \rightarrow E') = \frac{1}{2}\,[g(E',EFL) + g(E',EFH)] \quad ,$$

where

$$g(E',EF) = \frac{1}{3(EF \cdot TM)^{1/2}}\,[u_2^{3/2}E_1(u_2) - u_1^{3/2}E_1(u_1) + \gamma(3/2,u_2) - \gamma(3/2,u_1)] \quad ,$$

$$u_1 = (\sqrt{E'} - \sqrt{EF})^2/TM \quad ,$$

$$u_2 = (\sqrt{E'} + \sqrt{EF})^2/TM \quad .$$

EFL and EFH are constants, and TM depends on the incident neutron energy E. $E_1(x)$ is the exponential integral and $\gamma(a,x)$ is the incomplete gamma function. The integral of this spectrum between zero and infinity is one. The value of the integral for a finite integration range is given in Sec. 5.4.10.

Page 5.6 After the fifth line "(LF = 11.)," insert the following:

EFL, EFH are constants used in the energy-dependent fission neutron spectrum (Madland and Nix), LF = 12.

TM is the maximum temperature parameter, TM(E), of the energy-dependent fission neutron spectrum (Madland and Nix), LF = 12.

Page 5.8 Before "Note that no formats have been described...," insert the following:

LF = 12, Energy-Dependent Fission Neutron Spectrum (Madland and Nix)

$[MAT,5,MT/0.0,0.0,0,LF = 12,\ NR,NP/E_{int}/p(E)]TAB1$

$[MAT,5,MT/EFL,EFH,0,0,NR,NE/E_{int}/TM(E)]TAB1$

Page 5.11 Delete "with LF = 11 preferred" in the third and fourth lines and replace with a period.

Page 5.12 Delete the first sentence of the second line.

Page 5.14 After the expression for the average energy using LF = 11, insert the following:

12 $\quad \frac{1}{2}(EFL + EFH) + \frac{4}{3}\,TM \quad .$

After page 5.14, insert Sec. 5.4.10.

5.4.10 Additional Procedures for LF = 12, Energy-Dependent Fission Neutron Spectrum (Madland and Nix)

1. Integral Over Finite Energy Range [a,b]

Set $\alpha = \sqrt{TM}$, $\beta = \sqrt{EF}$,

$$A = \left(\sqrt{a} + \beta\right)^2/\alpha^2 \quad ,$$

$$B = (\sqrt{b} + \beta)^2/\alpha^2 \quad ,$$

$$A' = (\sqrt{a} - \beta)^2/\alpha^2 \quad , \text{ and}$$

$$B' = (\sqrt{b} - \beta)^2/\alpha^2 \quad .$$

Then, the integral is given by one of the following three expressions depending on the region of integration in which a and b lie.

<u>Region I ($a \geq EF$, $b > EF$)</u>

$$3(EF \cdot TM)^{1/2} \int_a^b g(E', EF)dE' =$$

$$\left[\left(\frac{2}{5}\,\alpha^2 B^{5/2} - \frac{1}{2}\,\alpha\beta B^2\right)E_1(B) - \left(\frac{2}{5}\,\alpha^2 A^{5/2} - \frac{1}{2}\,\alpha\beta A^2\right)E_1(A)\right] .$$

$$- \left[\left(\frac{2}{5}\,\alpha^2 B'^{5/2} + \frac{1}{2}\,\alpha\beta B'^2\right)E_1(B') - \left(\frac{2}{5}\,\alpha^2 A'^{5/2} + \frac{1}{2}\,\alpha\beta A'^2\right)E_1(A')\right]$$

$$+ \left[(\alpha^2 B - 2\alpha\beta B^{1/2})\gamma(3/2,B) - (\alpha^2 A - 2\alpha\beta A^{1/2})\gamma(3/2,A)\right]$$

$$- \left[(\alpha^2 B' + 2\alpha\beta B'^{1/2})\gamma(3/2,B') - (\alpha^2 A' + 2\alpha\beta A'^{1/2})\gamma(3/2,A')\right]$$

$$- \frac{3}{5}\,\alpha^2\left[\gamma(5/2,B) - \gamma(5/2,A) - \gamma(5/2,B') + \gamma(5/2,A')\right]$$

$$- \frac{3}{2}\,\alpha\beta\left[e^{-B}(1 + B) - e^{-A}(1 + A) + e^{-B'}(1 + B') - e^{-A'}(1 + A')\right] \quad .$$

<u>Region II ($a < EF$, $b \leq EF$)</u>

$$3(EF \cdot TM)^{1/2} \int_a^b g(E', EF)dE' =$$

$$\left[\left(\frac{2}{5}\,\alpha^2 B^{5/2} - \frac{1}{2}\,\alpha\beta B^2\right)E_1(B) - \left(\frac{2}{5}\,\alpha^2 A^{5/2} - \frac{1}{2}\,\alpha\beta A^2\right)E_1(A)\right]$$

$$- \left[\left(\frac{2}{5}\,\alpha^2 B'^{5/2} - \frac{1}{2}\,\alpha\beta B'^2\right)E_1(B') - \left(\frac{2}{5}\,\alpha^2 A'^{5/2} - \frac{1}{2}\,\alpha\beta A'^2\right)E_1(A')\right]$$

$$+ \left[(\alpha^2 B - 2\alpha\beta B^{1/2})\gamma(3/2,B) - (\alpha^2 A - 2\alpha\beta A^{1/2})\gamma(3/2,A)\right]$$

$$- \left[(\alpha^2 B' - 2\alpha\beta B'^{1/2})\gamma(3/2,B') - (\alpha^2 A' - 2\alpha\beta A'^{1/2})\gamma(3/2,A')\right]$$

$$- \frac{3}{5}\,\alpha^2\left[\gamma(5/2,B) - \gamma(5/2,A) - \gamma(5/2,B') + \gamma(5/2,A')\right]$$

$$- \frac{3}{2}\,\alpha\beta\left[e^{-B}(1 + B) - e^{-A}(1 + A) - e^{-B'}(1 + B') + e^{-A'}(1 + A')\right] \quad .$$

<u>Region III (a < EF, b > EF)</u>

$$3(EF\cdot TM)^{1/2} \int_a^b g(E',EF)dE' =$$

$$\left[\left(\frac{2}{5}\,\alpha^2 B^{5/2} - \frac{1}{2}\,\alpha\beta B^2\right)E_1(B) - \left(\frac{2}{5}\,\alpha^2 A^{5/2} - \frac{1}{2}\,\alpha\beta A^2\right)E_1(A)\right]$$

$$- \left[\left(\frac{2}{5}\,\alpha^2 B'^{5/2} + \frac{1}{2}\,\alpha\beta B'^2\right)E_1(B') - \left(\frac{2}{5}\,\alpha^2 A'^{5/2} - \frac{1}{2}\,\alpha\beta A'^2\right)E_1(A')\right]$$

$$+ \left[(\alpha^2 B - 2\alpha\beta B^{1/2})\gamma(3/2,B) - (\alpha^2 A - 2\alpha\beta A^{1/2})\gamma(3/2,A)\right]$$

$$- \left[(\alpha^2 B' + 2\alpha\beta B'^{1/2})\gamma(3/2,B') - (\alpha^2 A' - 2\alpha\beta A'^{1/2})\gamma(3/2,A')\right]$$

$$- \frac{3}{5}\,\alpha^2\left[\gamma(5/2,B) - \gamma(5/2,A) - \gamma(5/2,B') + \gamma(5/2,A')\right]$$

$$- \frac{3}{2}\,\alpha\beta\left[e^{-B}(1 + B) - e^{-A}(1 + A) + e^{-B'}(1 + B') + e^{-A'}(1 + A') - 2\right] \quad .$$

The expression for Region III would be used to calculate a normalization integral I for the finite integration constant U, if a physical basis existed by which U could be well determined.

2. Multiple-Chance Fission

The use of the LF = 12 fission neutron spectrum together with the LF = 9 neutron evaporation spectrum to represent multiple-chance fission spectra has been completely specified including all possible multiple-chance fission cases up through fourth-chance fission.* In this reference, equations are given for the fission spectra MT = 19, MT = 20, MT = 21, and MT = 38 as well as for MT = 18. In addition, equations are given for the required probabilities $p_k(E)$ as a function of File 3(MF = 3) multiple-chance fission cross sections and average prompt neutron multiplicities $\overline{\nu}_p(E)$, which can either be calculated or obtained from File 1 (MF = 1), MT = 456 for the successive isotopes involved in the multiple-chance fission process. The $p_k(E)$ are defined such that for any value of E their sum over all energetically allowed multiple-chance fission spectrum components is unity. This violates the sum rule given in Sec. 5.1 which stipulates that the sum is unity for any single multiple-chance fission spectrum component.

In closing this section, we give in Table I an example of the proposed ENDF format for the new fission neutron spectrum.

V. DISCUSSION

A new fission neutron spectrum representation has been proposed for use in ENDF. The spectrum is based on recent new theoretical results which address the dependence of the prompt fission neutron spectrum upon the fissioning nucleus and the incident neutron energy, for first-chance as well as multiple-chance fission. The same theory describes the spontaneous fission neutron spectrum. The new fission spectrum representation is a closed expression and its integral over an arbitrary energy range is also of closed form. The basic spectrum is characterized by two constants and an energy-dependent temperature parameter, all of which can be calculated using existing simple relations as given in App. A.

There are approximately forty ENDF/B-V actinide files that contain prompt fission neutron spectra. Of these forty cases only a few (≈ 5) are based on actual measurements of the spectrum for the nucleus in question. The remaining cases (≈ 35) are for the most part described by Maxwellian distributions in which the magnitude as well as the physical origin of the Maxwellian temperature parameter is questionable. These cases are good examples of where the spectrum representation we propose can be put to good use.

*David G. Madland, "New Fission Neutron Spectrum Representation for ENDF," Los Alamos National Laboratory report LA-9285-MS, ENDF-321 (February 1982).

TABLE I

EXAMPLE OF NEW FISSION NEUTRON SPECTRUM FORMAT

9.92990+ 4	2.96430+ 2	0	0	1		05000	5	19	2001
0.00000+ 0	0.00000+ 0	0	12	1		235000	5	19	2002
23	2					5000	5	19	2003
1.00000- 5	1.00000+ 0	5.50000+ 6	1.00000+ 0	6.00000+ 6	9.80000-	15000	5	19	2004
6.50000+ 6	9.30000- 1	7.00000+ 6	8.60000- 1	7.50000+ 6	8.10000-	15000	5	19	2005
8.00000+ 6	7.70000- 1	9.00000+ 6	7.50000- 1	1.00000+ 7	7.50000-	15000	5	19	2006
1.10000+ 7	7.00000- 1	1.15000+ 7	7.00000- 1	1.20000+ 7	6.50000-	15000	5	19	2007
1.25000+ 7	6.00000- 1	1.30000+ 7	5.50000- 1	1.40000+ 7	5.00000-	15000	5	19	2008
1.50000+ 7	4.90000- 1	1.60000+ 7	4.50000- 1	1.65000+ 7	4.40000-	15000	5	19	2009
1.70000+ 7	4.00000- 1	1.75000+ 7	3.60000- 1	1.80000+ 7	3.20000-	15000	5	19	2010
1.90000+ 7	2.60000- 1	2.00000+ 7	2.30000- 1			5000	5	19	2011
1.05250+ 6	5.43493+ 5	0	0	1		285000	5	19	2012
28	5					5000	5	19	2013
1.00000- 5	9.59475+ 5	1.00000+ 0	9.59476+ 5	1.00000+ 2	9.59478+	55000	5	19	2014
1.00000+ 3	9.59498+ 5	1.00000+ 4	9.59696+ 5	1.00000+ 5	9.61681+	55000	5	19	2015
2.50000+ 5	9.64980+ 5	5.00000+ 5	9.70453+ 5	7.50000+ 5	9.75896+	55000	5	19	2016
1.00000+ 6	9.81308+ 5	1.25000+ 5	9.86691+ 5	1.50000+ 6	9.92045+	55000	5	19	2017
1.75000+ 6	9.97369+ 5	2.00000+ 6	1.00267+ 6	2.50000+ 6	1.01318+	65000	5	19	2018
3.00000+ 6	1.02358+ 6	4.00000+ 6	1.04407+ 6	5.00000+ 6	1.06417+	65000	5	19	2019
6.00000+ 6	1.08390+ 6	7.00000+ 6	1.10327+ 6	8.00000+ 6	1.12231+	65000	5	19	2020
9.00000+ 6	1.14103+ 6	1.00000+ 7	1.15945+ 6	1.20000+ 7	1.19544+	65000	5	19	2021
1.40000+ 7	1.23037+ 6	1.60000+ 7	1.26434+ 6	1.80000+ 7	1.29742+	65000	5	19	2022
2.00000+ 7	1.32968+ 6					5000	5	19	2023
		0	0	0		05000	5	0	2024

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APPENDIX A

EVALUATION OF E_f^L, E_f^H, and T_m

The average kinetic energy per nucleon E_f^L of the average light fission fragment and the average kinetic energy per nucleon E_f^H of the average heavy fission fragment are given by

$$E_f^L = \frac{A_H}{A_L} \frac{\langle E_f^{tot} \rangle}{A} \quad , \tag{A-1}$$

and

$$E_f^H = \frac{A_L}{A_H} \frac{\langle E_f^{tot} \rangle}{A} \quad , \tag{A-2}$$

where A is the mass number of the compound nucleus undergoing fission, A_L and A_H are the average mass members of the light and heavy fragments, respectively, and $\langle E_f^{tot} \rangle$ is the total average fission-fragment kinetic energy.

We obtain the values of A_L and A_H from the experimental measurements of Unik, et al.[9] or from the review by Hoffman and Hoffman.[10] In cases where experimental measurements do not exist we interpolate using the data contained in Table I and Figs. 1 and 2 of Unik, et al.[9]

For the values of $\langle E_f^{tot} \rangle$ we use the experimental results of Unik, et al.,[9] which are appropriate to thermal-neutron-induced fission or spontaneous fission. Hoffman and Hoffman[10] include additional measurements of $\langle E_f^{tot} \rangle$ in their review. For actinide nuclei at low excitation energy that are not included in this compilation, one can either interpolate between the values for nearby nuclei or use the results of a least-squares adjustment by Unik, et al.[9]; namely,

$$\langle E_f^{tot} \rangle = 0.13323(Z^2/A^{1/3}) - 11.64 \quad \mathrm{MeV} \quad , \tag{A-3}$$

where (Z,A) refers to the fissioning compound nucleus. For actinide nuclei at high excitation energy the results of a least-squares adjustment by Viola[11] are more appropriate, namely,

$$\langle E_f^{tot} \rangle = 0.1071(Z^2/A^{1/3}) + 22.2 \text{ MeV} \quad . \tag{A-4}$$

The maximum temperature T_m of the fission-fragment residual nuclear temperature distribution is given by

$$T_m = (\langle E^* \rangle / a_{eff})^{1/2} \quad , \tag{A-5}$$

where, currently, the effective nuclear level-density parameter a_{eff} is given by

$$a_{eff} = A/(10 \text{ MeV}) \quad . \tag{A-6}$$

The total average fission-fragment excitation energy $\langle E^* \rangle$ is given by

$$\langle E^* \rangle = \langle E_r \rangle + B_n + E_n - \langle E_f^{tot} \rangle \quad . \tag{A-7}$$

In this equation $\langle E_r \rangle$ is the average energy release, B_n and E_n are the separation energy and kinetic energy of the neutron inducing fission, and $\langle E_f^{tot} \rangle$ is the total average fission-fragment kinetic energy discussed above.

For spontaneous fission, B_n and E_n are set to zero. We obtain values of $\langle E_r \rangle$ and B_n using the experimental or derived systematic masses of Wapstra and Bos[12] when they exist and otherwise the mass formula of Myers[13] or Moller and Nix.[14]

The average energy release $\langle E_r \rangle$ is then given by the difference between the ground-state mass of the fissioning compound nucleus and the sum of the average ground-state masses of two sets of average complementary fission fragments. Each set consists of seven mass and change divisions that are centered about the central fragment (Z_L, A_L) or (Z_H, A_H). The averaging is performed with a relative weighting of two for the central fragment compared to the other fragments for both the light and heavy fragment sets. The heavy fragment set is depicted in Fig. 4 for the case of the compound nucleus ^{236}U. The average or central fragment mass numbers A_L and A_H used in the averaging have been discussed above. The average or central fragment charge numbers Z_L and Z_H are obtained using Figs. 1 and 2 of Unik, et al.[9]

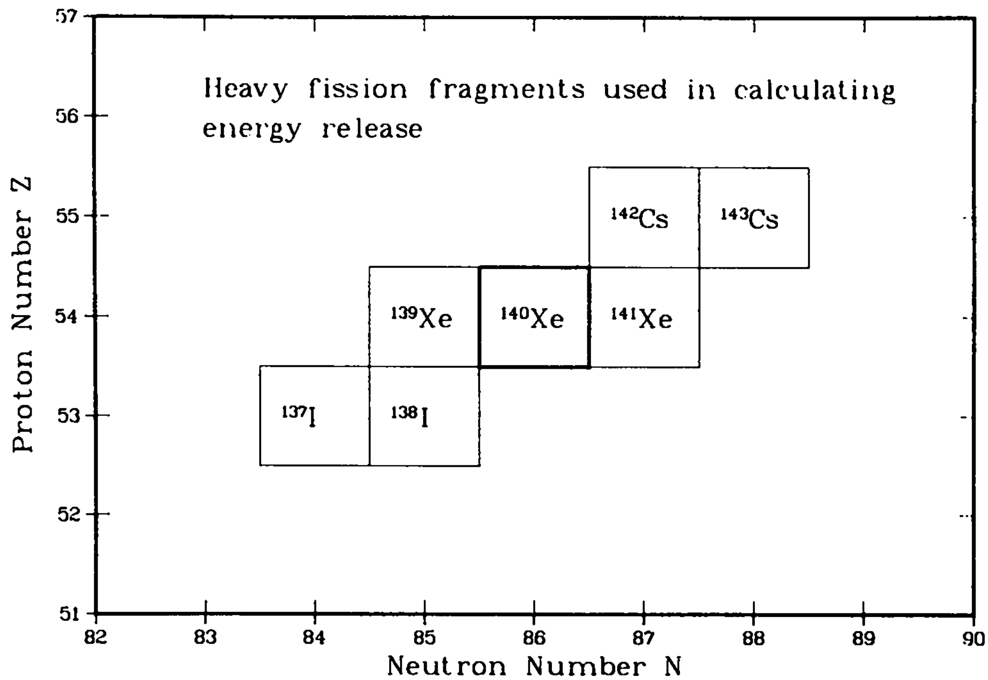

Fig. 4. Heavy fission fragments used in calculating the average energy release $\langle E_r \rangle$ *of the compound nucleus* ^{236}U. *The average or central fragment is* ^{140}Xe.

APPENDIX B

AVERAGE PROMPT NEUTRON MULTIPLICITY $\bar{\nu}_p(E)$

The multiple-chance fission formalism presented in Sec. III requires the average prompt neutron multiplicity $\bar{\nu}_p(E)$ as a function of E for up to four fissioning compound nuclei, namely, the ZA + 1, ZA, ZA − 1, and ZA − 2 compound systems. In ENDF isotope identification notation these are, respectively, the ZA, ZA − 1, ZA − 2, and ZA − 3 "target" isotopes.

The $\bar{\nu}_p(E)$ can be easily calculated using the formulae of Refs. 5 and 6 or they can be obtained from ENDF File 1 (MF = 1), MT = 456 for the appropriate isotopes.

Since ratios of $\bar{\nu}_p(E)$ values are involved in the multiple-chance fission formalism, the variation of $\bar{\nu}_p(E)$ with Z and A is more significant than the absolute magnitude of $\bar{\nu}_p(E)$. The dependence of $\bar{\nu}_p(E)$ on Z and A is given by the theory of Refs. 5 and 6, particularly for those isotopes for which no experimental

$\overline{\nu}_p(E)$ data exist or are possible. If, on the other hand, the $\overline{\nu}_p(E)$ are obtained from MF = 1, MT = 456 for the required isotopes, the question of the correct variation of $\overline{\nu}_p(E)$ with Z and A immediately arises. This is because there are no four-member isotopic sequences in ENDF where all members have experimentally based $\overline{\nu}_p(E)$ representations. Moreover, there are only a few such three-member isotopic sequences. In addition, use of the existing ENDF $\overline{\nu}_p(E)$ representations in the multiple-chance fission formalism requires their extrapolation to negative values of E. This is due to the fact that compound nuclei occurring in the multiple-chance fission sequence fission at excitation energies below the neutron binding energy.

For the above reasons we recommend that the $\overline{\nu}_p(E)$ values used in the multiple-chance fission formalism of Sec. III be calculated according to the theory of Refs. 5 and 6.

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