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HEAT TRANSFER AND THERMAL STRESSES IN NUCLEAR REACTOR SHELLS

by

Franklin P. Durham

REACTORS--RESEARCH AND POWER

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REACTORS--RESEARCH AND POWER

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ABSTRACT

A method is presented for determining heat transfer rates and thermal stresses from the gamma-ray energy absorption of nuclear reactor shells for plane, cylindrical, and spherical geometries. Criteria for minimizing thermal stresses are developed, along with the corresponding external cooling rates necessary to minimize the thermal stress. Design charts are presented for rapid determination of approximate thermal stresses and heat transfer rates, along with a numerical example illustrating the use of the charts.

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1. Introduction

The problem of gamma-ray heating must be considered in the design of many nuclear reactors. It is especially important in cases where the gamma heating may cause thermal stresses in the external shell of the reactor if this shell is a structural member and subject to internal pressures which also result in stresses of large magnitude. Thus, while thermal stresses are relieved by local yielding or creep under high temperatures, severe yielding may cause structural failure under repeated cyclic operation. From the standpoint of reactor design it is desirable to know the magnitudes of these thermal stresses and to minimize them to an acceptable level.

To arrive at values of thermal stresses resulting from gamma-ray heating requires the solution of three separate, but related, problems. The first of these problems is the determination of the magnitude and distribution of the energy associated with the absorption of gamma-rays. The second related problem is the determination of the heat transfer rates and temperature distributions based on the physical properties of the material and the manner in which heat is being removed from the material. The third related problem is the determination of thermal stresses resulting from the temperature distribution obtained from the heat transfer solution.

The purpose of this paper is to integrate these problems for several simple geometries with emphasis on the heat transfer and thermal stress aspects from the standpoint of design.

2. Notation

a = Inside radius of sphere or cylinder

A = Area

b = Outside radius of sphere or cylinder

c = Thickness of slab

C = Constant

E = Modulus of elasticity

F(r) = Function defined by $\int_a^r I(r)rdr$

F(x) = Function defined by $\int_o^x I(x)dx$

G(r) = Function defined by $\int_a^r S(r)r^2dr$

h = Convective heat transfer coefficient

I(r) = Total gamma heating per unit volume cylinder

I(x) = Total gamma heating per unit volume in slab

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k = Thermal conductivity

K_1 = Gamma-ray source strength per unit area for plane geometry

K_2 = Constant related to gamma-ray source strength for cylinder by Eq. (3)

K_3 = Constant related to gamma-ray source strength for cylinder by Eq. (5)

L = Length

$q(r)$ = Rate of heat flow by conduction in sphere or cylinder

$q(x)$ = Rate of heat flow by conduction in slab

r = Radius

$S(r)$ = Total gamma heating per unit volume in sphere

t = Temperature

t_a = Temperature at inner face of wall

t_b = Temperature at outer face of wall

t_m = Mean temperature

x = Distance from face of slab

W = Total gamma-ray energy rate at any location

α = Thermal coefficient of expansion

θ = Temperature difference, $t - t_a$

θ_m = Temperature difference, $T_m - t_a$

μ = Gamma-ray absorption coefficient

σ_t = Thermal stress, tensile when positive

3. Gamma-Ray Energy Absorption

The problem of gamma-ray energy absorption is complex and depends on the source energy, geometry, and its angular and spatial distribution, as well as on the geometry and physical properties of the absorbing medium. For a reactor shell the problem can be broken down into several parts:

1. The self-shielding characteristics of the fuel and moderator in the reactor core, and the resulting emergent radiation from prompt and delayed fission gamma-rays and from gamma-rays resulting from neutron capture in the fuel and moderator.

2. The absorption of this gamma energy in the vessel wall.

3. The neutron leakage from the reactor core, both fast and thermal.

4. The absorption of gamma energy emitted from thermal neutron capture in the vessel wall.

5. The absorption of gamma energy emitted from inelastic scattering of fast neutrons in the vessel wall.

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6. If the vessel wall is very thick or if the wall material is a good moderator, the energy absorbed by elastic scattering of fast neutrons must also be considered, along with the gamma energy emitted by capture of the fast neutrons that have been thermalized.

The total gamma-ray heating is then obtained as the sum of the contributions from the individual parts as functions of the source strength or intensity, gamma-ray energy spectrum, and the spatial coordinates involved.

In general it is necessary to consider several gamma-ray energy levels since a single energy level will not adequately represent the emitted energy. The different energy levels have different absorption probabilities in a given material, thus complicating the problem further. As a result of the large number of variables involved, even the most complicated analytical solutions must be based on certain simplifying assumptions, and the errors resulting from these assumptions may be of considerable magnitude when compared to experimental results.

For the purposes of this paper it will be sufficient to assume that a first order approximation to the solution of the gamma-ray heating problem is that of simple exponential absorption.

For a plane geometry this is:

$$I(x) = K_1 \mu e^{-\mu x} \tag{1}$$

where $I(x)$ is the gamma heating rate per unit volume

K_1 is the total effective source strength per unit area

μ is the mean absorption coefficient for the gamma-ray energy levels involved.

For a cylindrical geometry, Eq. (1) becomes

$$I(r) = K_2 \frac{\mu e^{-\mu r}}{r} \tag{2}$$

where $I(r)$ is the gamma-heating rate per unit volume

K_2 is a constant related to the source strength per unit area at the surface $r = a$ by

$$\frac{\text{Source}}{\text{Area}} = K_2 \frac{e^{-\mu a}}{a} \tag{3}$$

For a spherical geometry, Eq. (1) becomes

$$S(r) = K_3 \frac{\mu e^{-\mu r}}{r^2} \tag{4}$$

where $S(r)$ is the gamma-heating rate per unit volume

K_3 is a constant related to the source strength per unit area at the surface $r = a$ by

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$$\frac{\text{Source}}{\text{Area}} = K_3 \frac{e^{-\mu a}}{a^2} \qquad (5)$$

4. Steady-State Heat Transfer

Solutions for one-dimensional, steady-state heat transfer can be made with relative simplicity for plane, cylindrical, and spherical geometries. These solutions require the knowledge of the magnitude and distribution of the gamma-ray energy absorption. In this section it will be assumed that the functions describing the magnitude and distribution of this energy are known.

Case 1: Heat Transfer in a Slab of Finite Thickness and Infinite Extent.

The steady-state heat conduction equation in one-dimension is

$$q(x) = -kA \frac{dt}{dx} \qquad (6)$$

For an element of the slab of thickness dx the difference between the heat flow through the face at x and the heat flow through the face at x + dx is

$$\frac{dq(x)dx}{dx} = kA \frac{d^2t}{dx^2} dx \qquad (7)$$

where the thermal conductivity is taken as the mean value for the temperature range to be covered.

The gamma heating generated in the element of the slab of thickness dx is

$$\frac{dW}{dx} dx = AI(x)dx \qquad (8)$$

where I(x) is the gamma heating from all sources per unit volume and Adx is the volume of the element.

For steady-state operation the energy balance on the element is

Heat conducted through face at x - Heat conducted through face at x + dx = Heat generated in width dx.

From Eqs. (7) and (8) this may be written as

$$-kA \frac{d^2t}{dx^2} = AI(x) \qquad (9)$$

Integrating this expression with respect to x gives

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$$\frac{dt}{dx} = -\frac{1}{k}\int_0^x I(x)dx + C_1 \tag{10}$$

where C_1 is a constant to be determined. A second integration gives an expression for the temperature distribution

$$t = -\frac{1}{k}\int_0^x F(x)dx + C_1x + C_2 \tag{11}$$

where $F(x) = \int_0^x I(x)dx$ and C_2 is a constant.

At the face where $x = 0$, $t = ta$, and C_2 is found to be

$$C_2 = ta \tag{12}$$

If the slab is cooled on both sides there will be some intermediate location x_o where the heat transfer by conduction is zero and hence the temperature gradient dt/dx is zero. For this condition from Eq. (10) and the definition of $F(x)$

$$C_1 = \frac{1}{k}\ F(x_o) \tag{13}$$

If the slab is insulated on one side, that is, at $x = c$, then x_o in Eq. (13) becomes c.

Now let a temperature difference be defined by $\theta = t - ta$. (14)

Combining Eqs. (11) - (14) results in

$$\theta = -\frac{1}{k}\int_0^x F(x)dx + \frac{1}{k}F(x_o)x \tag{15}$$

This form for the temperature distribution is convenient in the determination of thermal stresses since temperature differences, rather than temperatures, are necessary in thermal stress solutions.

It will be shown later that for minimum thermal stresses the temperature difference θ should be zero at the face $x = c$. (By definition θ is zero at the face $x = 0$.) The value of x_o for this optimum case can be determined from Eq. (15) as the value that satisfies the relation

$$F(x_{om}) = \frac{\int_0^c F(x)dx}{c} \tag{16}$$

where x_{om} is the value of x_o for minimum thermal stress.

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The arbitrary choice of a distance x_o from which the heat flows in opposite directions is also desirable in determining the external cooling required to minimize thermal stresses. The location of x_o automatically determines the cooling rates at both faces of the slab. At the face $x = 0$ the rate of heat transfer is, from Eqs. (6),(10) and (13),

$$q(o) = AF(x_o) \tag{17}$$

While at the face $x = c$, the rate of heat transfer is

$$q(c) = A\left[F(c) - F(x_o)\right] \tag{18}$$

The surface temperatures at these faces may be determined from Eqs. (17) and (18) and Newton's law of cooling, which at $x = o$ is

$$q(o) = h_a A(t_a - t_{fa}) \tag{19}$$

and at $x = c$ is

$$q(c) = h_c A(t_c - t_{fc}) \tag{20}$$

where h_a and h_c are the combined heat transfer coefficients and t_{fa} and t_{fc} are the external medium temperatures at the slab faces.

Combining Eqs. (17) and (19) and Eqs. (18) and (20),

$$t_a = \frac{F(x_o)}{h_a} + t_{fa} \tag{21}$$

$$t_c = \frac{F(c) - F(x_o)}{h_c} + t_{fc} \tag{22}$$

Case 2: Heat Transfer in a Hollow Cylinder of Infinite Extent.

The steady-state heat conduction equation in one-dimension is

$$q(r) = -2\pi kLr\frac{dt}{dr} \tag{23}$$

For an element of the cylinder of thickness dr the difference between the heat flow through the cylinder at r and the heat flow through the cylinder at r + dr is

$$\frac{dq(r)}{dr}\,dr = 2\pi kL\left(r\frac{d^2t}{dr^2} + \frac{dt}{dr}\right)dr \tag{24}$$

The gamma heating generated in the element of the cylinder of thickness dr is

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$$\frac{dW}{dr}\,dr = 2\pi L I(r) r\,dr \tag{25}$$

where I(r) is the total gamma heating per unit volume and $2\pi Lrdr$ is the volume of the element.

For steady-state operation the heat balance on the element is, from Eqs. (24) and (25),

$$-k\left(r\frac{d^2t}{dr^2} + \frac{dt}{dr}\right) = I(r)r \tag{26}$$

Integration of this expression gives

$$\frac{dt}{dr} = -\frac{1}{kr}\int_a^r I(r)r\,dr + \frac{C_1}{r} \tag{27}$$

where C_1 is a constant to be determined. A second integration between the limits of a and r gives directly

$$\theta = -\frac{1}{k}\int_a^r \frac{F(r)}{r}\,dr + C_1 \ln\frac{r}{a} \tag{28}$$

where $F(r) = \int_a^r I(r)r\,dr$.

As before, a location r_o is chosen where the heat transfer by conduction is zero and the temperature gradient dt/dr is zero. From Eq. (22) and the definition of F(r)

$$C_1 = \frac{1}{k}F(r_o) \tag{29}$$

When C_1 in Eq. (28) is replaced by the above value,

$$\theta = -\frac{1}{k}\int_a^r \frac{F(r)}{r}\,dr + \frac{1}{k}F(r_o)\ln\frac{r}{a} \tag{30}$$

The optimum value of r_o for minimum thermal stresses will be that which gives θ equal zero at the boundary r = b. This optimum value can be found from Eq. (30) as that which satisfies the relation

$$F(r_{om}) = \frac{\int_a^b \frac{F(r)}{r}\,dr}{\ln b/a} \tag{31}$$

where r_{om} is the value of r_o for minimum thermal stress.

[redacted]

As before, the location of r_o automatically determines the cooling rates for both surfaces of the cylinder. At the surface r = a the rate of heat transfer is, from Eqs. (23), (27), and (29)

$$q(a) = -2\pi L F(r_o) \quad (32)$$

At the surface r = b, the rate of heat transfer is

$$q(b) = 2\pi L \left[F(b) + F(r_o)\right] \quad (33)$$

The surface temperatures may be found in the same manner as in the previous section and are

$$t_a = \frac{F(r_o)}{ah_a} + t_{fa} \quad (34)$$

$$t_b = \frac{F(b) - F(r_o)}{bh_b} + t_{fb} \quad (35)$$

Case 3: Heat Transfer in a Hollow Sphere.

The steady-state heat conduction equation in one-dimension is

$$q(r) = -4\pi k r^2 \frac{dt}{dr} \quad (36)$$

For an element of the sphere of thickness dr the difference between the heat flow through the cylinder at r and the heat flow through the cylinder at r + dr is

$$\frac{dq(r)}{dr} dr = -4\pi k\left(r^2 \frac{d^2t}{dr^2} + 2r \frac{dt}{dr}\right) \quad (37)$$

The gamma heating generated in the element of the cylinder of thickness dr is

$$\frac{dW}{dr} dr = 4\pi S(r) r^2 dr \quad (38)$$

where S(r) is the total gamma heating per unit volume and $4\pi r^2 dr$ is the volume of the element.

For steady-state operation, the heat balance on the element is, from Eqs. (37) and (39),

$$-k\left(r^2 \frac{d^2t}{dr^2} + 2r \frac{dt}{dr}\right) = S(r) r^2 \quad (39)$$

Integration of this expression gives

$$\frac{dt}{dr} = -\frac{1}{kr^2}\int_a^r S(r) r^2 dr + \frac{C_1}{r^2} \quad (40)$$

where C_1 is a constant to be determined. A second integration between the limits of a and r gives the temperature difference

$$\theta = -\frac{1}{k}\int_a^r \frac{G(r)}{r^2}\,dr - C_1\left(\frac{1}{r} - \frac{1}{a}\right) \tag{41}$$

where $G(r) = \int_a^r S(r)r^2 dr$

As before, a location r_o is chosen where the heat transfer is zero and, from Eq. (40),

$$C_1 = \frac{1}{k}G(r_o) \tag{42}$$

The final solution is then

$$\theta = -\frac{1}{k}\int_a^r \frac{G(r)}{r^2}\,dr + \frac{1}{k}G(r_o)\left(\frac{1}{a} - \frac{1}{r}\right) \tag{43}$$

The optimum value of r_o for minimum thermal stress, determined as before, is that which satisfies the relation

$$G(r_{om}) = \frac{ab}{b - a}\int_a^b \frac{G(r)}{r^2}\,dr \tag{44}$$

where r_{om} is the value of r_o for minimum thermal stress.

At the surface r = a the rate of heat transfer is, from Eqs. (36), (40), and (42),

$$q(a) = -4\pi G(r_o) \tag{45}$$

and at the surface r = b the rate of heat transfer is

$$q(b) = 4\pi\left[G(b) - G(r_o)\right] \tag{46}$$

The corresponding surface temperatures are

$$t_a = \frac{F(r_o)}{a^2 h_a} + t_{fa} \tag{47}$$

$$t_b = \frac{G(b) - G(r_o)}{b^2 h_b} + t_{fb} \tag{48}$$

[redacted]

5. Thermal Stresses

The thermal stresses of interest in vessel walls can, in general, be determined from the simple equation

$$\sigma_{th} = \frac{E\alpha}{1 - \nu} (t_m - t) \qquad (49)$$

where E is the modulus of elasticity

α is the linear coefficient of thermal expansion

ν is Poisson's ratio

t_m is the mean temperature of the material

t is the temperature at the point in question.

The equation states that the thermal stress at any point is proportional to the difference between the mean temperature of the material and the temperature at the point. This may be visualized as follows: The product αt_m gives the average or mean expansion of the bulk of the material. The product αt gives the expansion that would have taken place in the vicinity of t if there were no restraint from the surrounding bulk of the material. The quantity $\alpha(t_m - t)$ then gives the amount of restraint at the point t, since the material is continuous and planes are assumed to remain plane. The factor $(1 - \nu)$ is introduced to account for the effect of restraint in one direction transverse to the direction of the calculated stress.

The equation assumes that the material is stressed below the elastic limit throughout. If the elastic limit is slightly exceeded, or if creep takes place, the thermal stresses are relieved in these regions and reduced to a much lower value than that calculated from Eq. (49). In cases where temperature cycling takes place, plastic cyclic flow will not be appreciable if the stress as calculated by Eq. (49) does not exceed twice the yield strength for a ductile material whose yield strength in compression is equal to the yield strength in tension. A comprehensive discussion of the limitations imposed by thermal stresses is given in References 1 and 2.

The difference between the maximum and minimum temperature of the material will be a minimum for a given energy absorption when the material is cooled in such a way that the temperatures on both surfaces are equal. Since the mean temperature, t_m, lies between the maximum and minimum temperature in the material, the magnitude of the largest thermal stress existing in the material for a given energy absorption will also be a minimum when the temperatures at both surfaces are equal.

For the purposes of this paper, it is convenient to replace the temperatures in Eq. (44) by the appropriate values of the temperature difference θ. The equation then becomes, with no loss in generality,

[redacted]

[redacted]

$$\sigma_{th} = \frac{E\alpha}{1 - \nu} (\theta_m - \theta) \tag{50}$$

The mean temperature t_m depends upon the geometry of the material. The three geometries of interest here are the infinite slab, infinite hollow cylinder, and hollow sphere.

Case 1. Slab of Finite Thickness and Infinite Extent.

The mean temperature for this case is simply

$$t_m = \frac{\int_o^c t dx}{c} \tag{51}$$

or, in terms of the temperature difference θ,

$$\theta_m = \frac{\int_o^c \theta dx}{c} \tag{52}$$

Case 2. Infinite Hollow Cylinder.

The mean temperature for circumferential stress in this case is itself a function of the radius r and as given in Reference 1 is

$$t_m = \frac{r^2 + a^2}{r^2(b^2 - a^2)} \int_a^b tr dr + \frac{1}{r^2} \int_a^r tr dr \tag{53}$$

Since the maximum thermal stresses occur at the surfaces r = a and r = b, some simplification of Eq. (53) can be obtained by evaluating the equation at these limits. When this is done and written in terms of the temperature difference, the result is

$$\theta_m = \frac{2}{b^2 - a^2} \int_a^b \theta r dr \tag{54}$$

While the above equation applies rigorously only at the surfaces r = a and r = b, it may be used for approximate determination of internal stresses.

Case 3: Hollow Sphere.

The mean temperature in this case is a function of the radius r and is given in Reference 3 as

$$t_m = \frac{2r^3 + a^3}{r^3(b^3 - a^3)} \int_a^b tr^2 dr + \frac{1}{r^3} \int_a^b tr^2 dr \tag{55}$$

[redacted]

As in the case of the cylinder the maximum thermal stresses of interest occur at the surfaces $r = a$ and $r = b$. When Eq. (55) is evaluated at these limits and the results are expressed in terms of the temperature difference θ,

$$\theta_m = \frac{3}{b^3 - a^3}\int_a^b r^2 dr \tag{56}$$

6. Application of Theory

Because of the complex nature of rigorous gamma-heating solutions, the integral functions involved in the heat transfer and thermal stress equations must be solved in general by numerical or graphical means. However, a simple analytical solution for a plane geometry is possible, based on the assumption of exponential gamma-ray absorption given by Eq. (1). This equation will be used to illustrate the method of determining the heat transfer rates and thermal stresses, followed by a numerical example.

For a plane geometry with exponential absorption given by Eq. (1), the heating function $F(x)$ in Eq. (11) becomes

$$F(x) = K_1(1 - e^{-\mu x}) \tag{57}$$

The related functions necessary for the temperature distribution function of Eq. (15) are

$$F(x_o) = K_1(1 - e^{-\mu x_o}) \tag{58}$$

$$\int_o^x F(x_o)dx = K_1(x + \frac{e^{-\mu x}}{\mu} - \frac{1}{\mu}) \tag{59}$$

Eq. (15) then becomes

$$\theta = \frac{K_1}{k\mu}\ (1 - \mu x e^{-\mu x_o} - e^{-\mu x}) \tag{60}$$

The optimum value of the thickness x_o for minimum thermal stress is found from Eq. (16) to be

$$X_{om} = \frac{1}{\mu}\ell n\ \frac{\mu c}{1 - e^{-\mu c}} \tag{61}$$

Figure 1 is a graphical presentation of Eq. (61), normalized by dividing both sides by the total slab thickness c.

From Eqs. (17), (58), and (61), rate of heat transfer at the slab face $x = 0$ for

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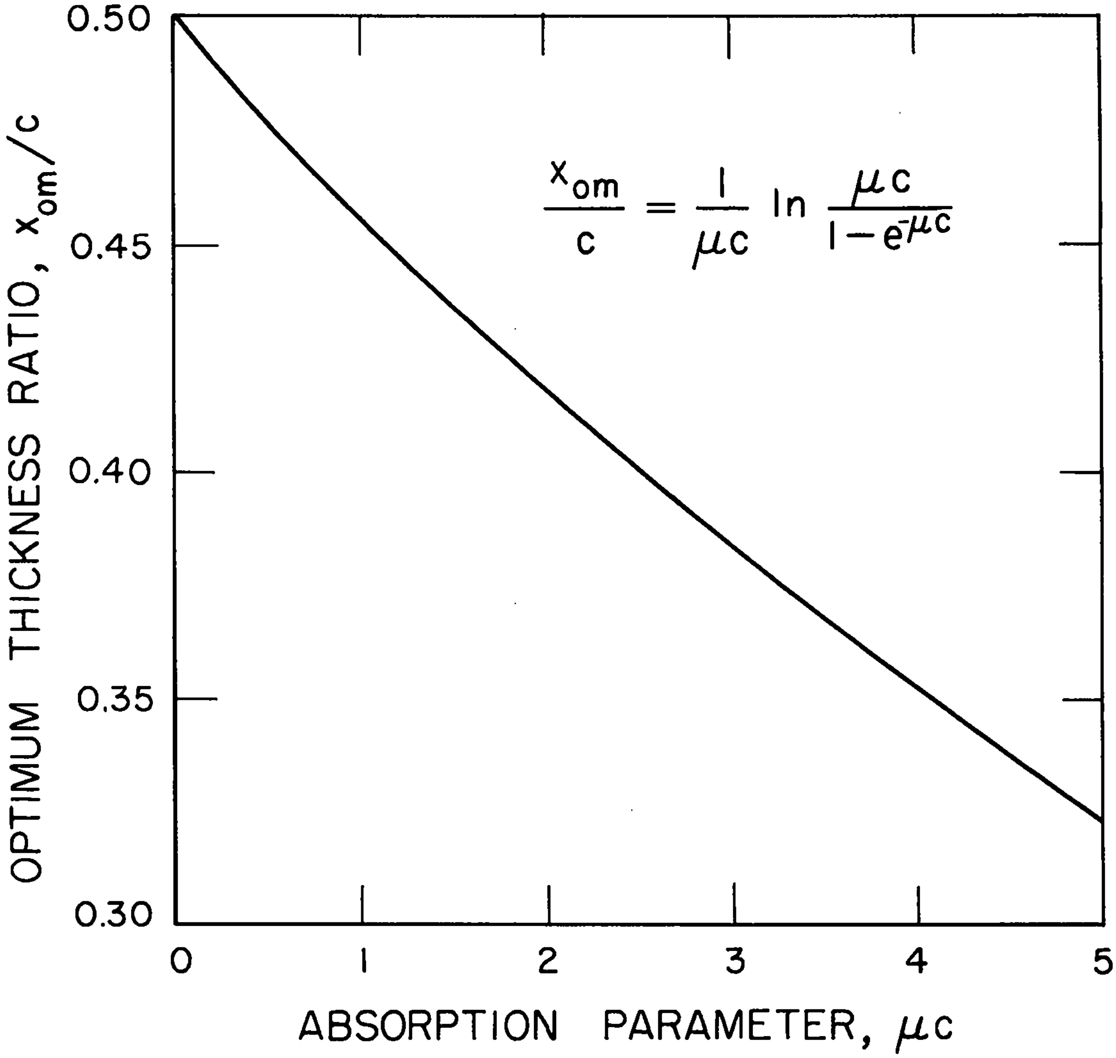

Fig. 1. Thickness ratio for optimum cooling.

[redacted]

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minimum thermal stresses is given as a fraction of the total source strength as

$$\frac{q(o)}{K_1 A} = 1 - \frac{1}{\mu c} + \frac{e^{-\mu c}}{\mu c} \tag{62}$$

From Eqs. (16), (57), and (61), the rate of heat of heat transfer at the slab face $x = c$ for minimum thermal stress is given as a fraction of the total source strength as

$$\frac{q(c)}{K_1 A} = \frac{1}{\mu c} - (1 + \frac{1}{\mu c}) e^{-\mu c} \tag{63}$$

Figure 2 is a graphical presentation of Eqs. (62) and (63) and shows the fraction of the total gamma energy crossing the boundaries as heat.

The mean temperature function for minimum thermal stress from Eqs. (52), (60), and (61) is

$$\theta_m = \frac{K_1}{2k\mu} \left[1 - \frac{2}{\mu c} + (1 + \frac{2}{\mu c}) e^{-\mu c} \right] \tag{64}$$

For optimum cooling and minimum thermal stresses the value of θ at both external surfaces ($x = o$, and $x = c$) is zero and the thermal stresses at these surfaces, which are tensile, are given by

$$\sigma_{max} = \frac{E\alpha}{1 - \nu} \theta_m \tag{65}$$

Thus the thermal stresses at the surface are proportional to the function expressed in Eq. (64). Figure 3 is a plot of this function versus the absorption parameter μc.

Example:

Determine the maximum thermal stresses and optimum cooling in a nuclear reactor shell that is constructed of steel 5 in. thick and having a yield strength of 50,000 psi. The reactor rating is 30 megawatts and the inner surface area of the shell is 18 sq ft and the mean metal temperature is 600^{o}F.

Assume that the self-shielding factor for gamma-rays in the core is 0.8 and the self-shielding factor for neutrons not causing fission in the core is 0.2. Also assume that the average energy of all gamma-rays is approximately 3 Mev.

Solution:

The necessary physical properties are determined from References 4 and 5 to be

$k = 23$ B/hr-ft-F

$\mu = 0.7$/inch

$E = 28\ (10)^6$ psi

$\alpha = 7\ (10)^{-6}/^{o}F$

$\nu = 0.3$

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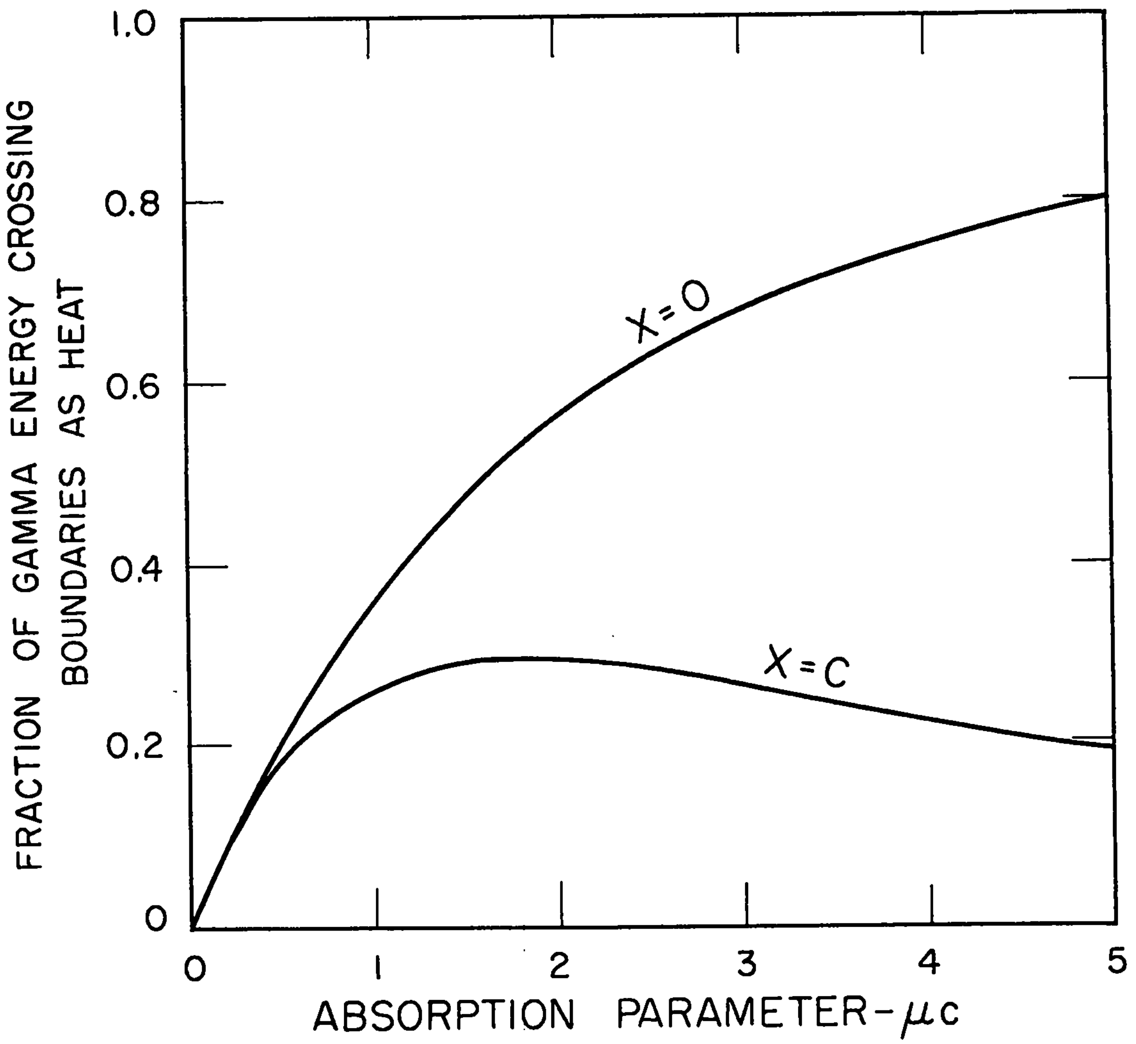

Fig. 2. Optimum cooling rates at external slab faces.

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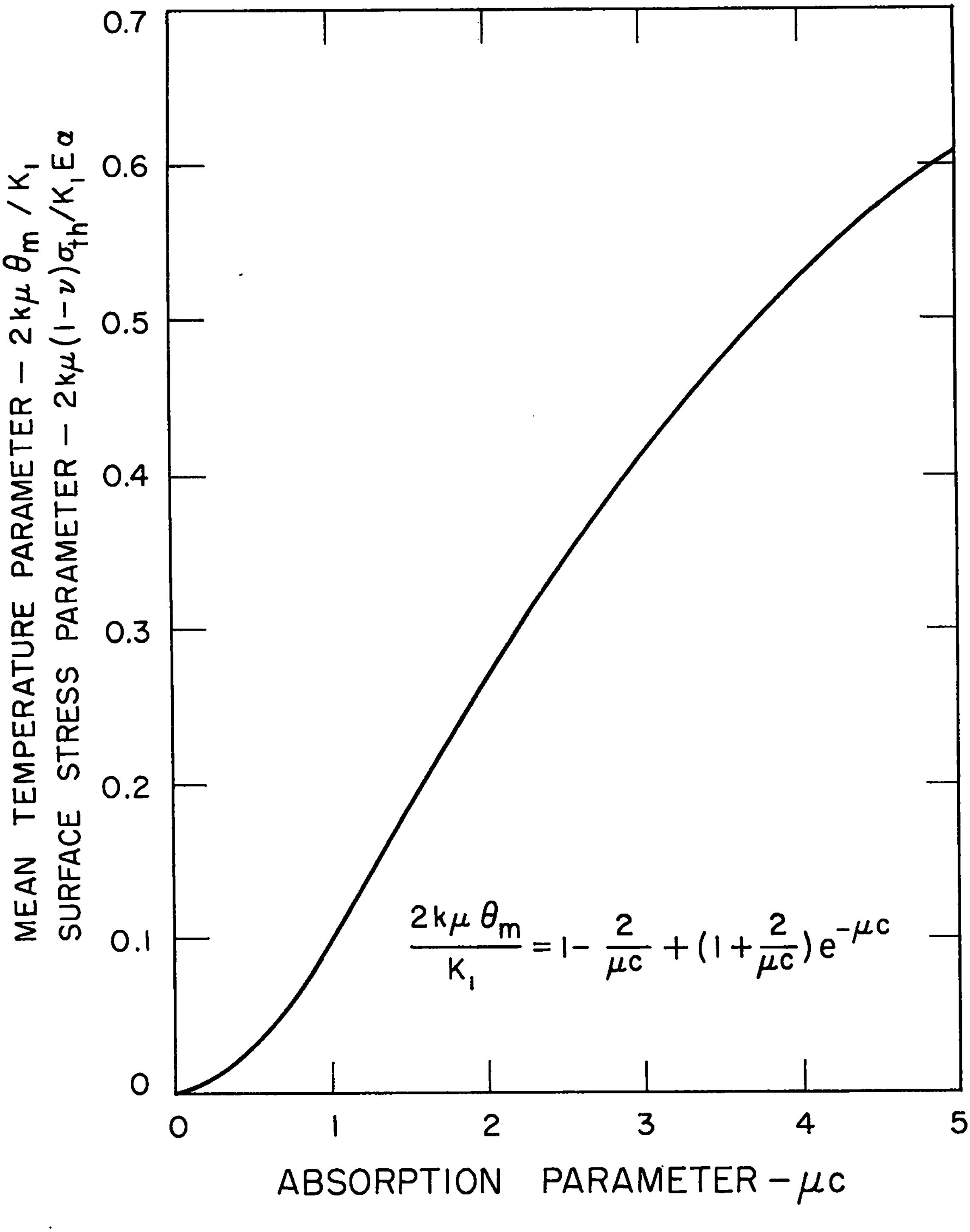

Fig. 3. Mean temperature and surface stress for optimum cooling.

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The source strength is determined approximately as follows:

The useful energy per fission is taken from Reference 6 to be 189 Mev and the prompt and delayed gamma-ray energy plus capture gammas is taken to be 12 Mev per fission. Since the self-shielding factor of the core is assumed to be 0.8, the gamma energy crossing the interface between the core and shell is

$$0.2\ (12) = 2.4 \text{ Mev per fission}$$

Of the 1.5 neutrons per fission that do not enter into the fission reaction 0.8 are assumed to escape from the core. Of these, approximately 0.3 are thermal neutrons captured in iron emitting 7 Mev gamma energy per capture. Thus the maximum energy to be recovered from these neutrons is

$$0.8\ (1.5)\ (0.3)\ 7 = 2.5 \text{ Mev per fission}$$

The total source strength is then 4.9 Mev per fission which is equivalent to 0.026 of the total energy. or 0.78 megawatts.

Converting this to British thermal units per hour and dividing by the surface area gives

$$K_1 = 1.47\ (10)^5 \text{ B/hr-ft}^2$$

The value of the absorption parameter is $\mu c = 0.7(5) = 3.5$

From Fig. 2 the optimum cooling rates are

$$q\frac{(o)}{A} = 1.47(10)^5(0.72) = 1.06(10)^5 \text{ B/hr-ft}^2$$

$$q\frac{(c)}{A} = 1.47(10)^5(0.24) = 0.35(10)^5 \text{ B/hr-ft}^2$$

From Fig. 3 for an absorption parameter of 3.5 and the constants that have been determined,

$$\sigma_{th} = \frac{1.47(10)^5\ 28(10)^6\ 7(10)^{-6}(0.475)}{2(23)\ 12(0.7)\ (1 - 0.3)} = 50{,}600 \text{ psi}$$

7. References

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