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SUBMITTED TO: AMERICAN NUCLEAR SOCIETY WINTER MEETING-1993
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Asymptotic Derivation of the Modified Time-Dependent SP_2 Equations and Numerical Calculations

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Introduction

Converting the independent variables of the transport equation to dimensionless parameters, asymptotic analyses can be performed to show that, for an important class of problems, the diffusion equation is an asymptotic limit of the transport equation.[1] A recent paper by Larsen, McGhee, and Morel[2] provides a broadened view of the result discussed above. It deals with the steady-state transport equation and shows that the Simplified Spherical-Harmonics (SP_N) equations are robust high-order asymptotic approximations of the transport equation in a physical regime in which the conventional diffusion equation is the leading-order approximation. According to the reported numerical results for the steady-state cases,[3,4] for many problems, low-order SP_N equations capture most (Gamino[5] reports "greater than 80%") of the transport corrections to the diffusion approximation. And Larsen[6] shows that, in nearly all cases, the SP_2 results are significantly more accurate than diffusion results in the steady-state problems.

In this paper, we deal with the time-dependent case and find that if one neglects the time derivative term of the second moment of angular flux, $\partial\psi_2/\partial t$, in the time-dependent SP_2 equations (we will call this equation as the Modified Time-Dependent SP_2

Equations), this equation has the same asymptotic approximations of the time-dependent transport equation up to the third-order in a physical regime in which the time-dependent diffusion equation is the leading-order approximation. Also, we formulate the time-dependent P_2 equations for one-group, slab geometry problems (in slab geometry, the SP_2 equations are exactly same as P_2 equations) and compare numerical time-dependent diffusion, P_2 and discrete-ordinates (S_{16}) solutions in several classes of problems. Because the time-dependent SP_2 equations can be transformed to the same form of equation as the time-dependent diffusion equation, the SP_2 results are obtained with almost the same computational effort as the diffusion results. In addition, we compare the numerical solutions of the time-dependent P_2 and modified P_2 equations with those of the diffusion and S_{16} equations in the slab geometry.

Time-Dependent P_2 Equations in the Slab Geometry

If we eliminate the time variable in the time-dependent P_2 equations using an implicit scheme, and formulate an equation in the scalar flux, ψ_0, only, the result is in the form of a conventional diffusion equation. With this equation, we can make the matrix equation $\underline{\underline{\mathbf{A}}} \cdot \underline{\boldsymbol{\psi}}^{n+1} = \underline{\mathbf{S}}^n$ at each time step, n, where $\underline{\underline{\mathbf{A}}}$ is tri-diagonal symmetric matrix. From the numerical results, we find that, in the vast majority of cases, the time-dependent P_2 solutions are more accurate than the conventional diffusion solutions. Even in the optically-thin regimes (very small total cross-sections), the time-dependent P_2 solutions are quite close to the S_{16} solutions (see Figure 1.) These results provide the motivation to consider the multi-dimensional case.

Asymptotic Derivation of the Modified Time-Dependent SP_2 Equations

One-group, multi-dimensional transport equation with isotropic scattering is,

$$\frac{1}{v}\frac{\partial}{\partial t}\psi + \Omega \cdot \nabla\psi + \sigma_t\psi = \sigma_s\phi + Q$$

If we consider the asymptotic scaling[1] :

$$\sigma_t \Rightarrow \frac{\sigma_t}{\varepsilon}\ ,\ \sigma_a \Rightarrow \varepsilon\sigma_a\ ,\ \sigma_s \Rightarrow \frac{\sigma_t}{\varepsilon} - \varepsilon\sigma_a\ ,\ Q \Rightarrow \varepsilon Q\ ,\ v \Rightarrow \frac{v}{\varepsilon}$$

where, $\varepsilon \ll 1$ and expand,

$$\psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \cdots\ ,\ \ \phi = \phi_0 + \varepsilon\phi_1 + \varepsilon^2\phi_2 + \cdots\ ;$$

apply these to the time-dependent transport equation, and equate coefficients of different powers of ε ; the result is

$O(\varepsilon)$ $$\frac{1}{v}\frac{\partial}{\partial t}\phi_0 - D\nabla^2\phi_0 + \sigma_a\phi_0 = Q\ ,\ \text{where}\ \ D = \frac{1}{3\sigma_t}$$

: time-dependent diffusion equation

$O(\varepsilon^2)$ $$\frac{1}{v}\frac{\partial}{\partial t}\phi_1 - D\nabla^2\phi_1 + \sigma_a\phi_1 = 0$$

$O(\varepsilon^3)$ $$\left(\frac{1}{v\sigma_t}\frac{\partial}{\partial t} + \frac{4}{15\sigma_t^2}\nabla^2\right)D\nabla^2\phi_0 + \frac{1}{v}\frac{\partial}{\partial t}\phi_2 - D\nabla^2\phi_2 + \sigma_a\phi_2 = 0$$

Modified Time-Dependent SP_3 Equation in General Geometry

If one neglects the time derivative term of ψ_2, $\frac{\partial}{\partial t}\psi_2$, in the time-dependent SP_3 equations,[6] and reforms them with only the terms of $\psi_0(\equiv\phi)$, one can get:

$$\left(\frac{1}{v\sigma_t}\frac{\partial}{\partial t} - \frac{4}{15\sigma_t^2}\nabla^2\right)\left(\frac{1}{v}\frac{\partial}{\partial t}\phi + \sigma_a\phi - Q\right) + \frac{1}{v}\frac{\partial}{\partial t}\phi - D\nabla^2\phi + \sigma_a\phi = Q$$

Applying the same scaling and expansion to this equation, we find the Modified Time-Dependent SP_3 Equation has the same asymptotic approximations of the transport equation up to the third-order in a physical regime in which the time-dependent diffusion equation is the leading-order approximation. In other words, the modified time-dependent SP_3 equations contain higher-order asymptotic corrections to the time-dependent diffusion equation than do the SP_3 equations without modification. Also we compare numerical solutions of the time-dependent P_3 and modified P_3 equations, and the results show that the modified time-dependent P_2 equations give even better answers than the time-dependent P_2 equations in many cases.(see Figure 1.)

Conclusion

We have shown that the Modified Time-Dependent SP_2 Equations contain higher-order asymptotic corrections to the time-dependent diffusion equation and in the slab geometry, in most of cases, the time-dependent P_2 solutions are more accurate than the diffusion solutions. This implies that in general geometry, the time-dependent (modified) SP_2 equations could give a lot more accurate results than the time-dependent diffusion equation with almost the same computational efforts, and could give relatively inexpensive solutions to the time-dependent S_N equations. The study for the time-dependent SP_2 equations in more general cases, such as in 2-D multiplying medium, multigroup for energy, with anisotropic scattering and delayed neutron precursors, is being performed by the authors to validate the advantages of the time-dependent (modified) SP_2 equations in more realistic problems

References

1. E.W. Larsen, J.E. Morel, and W.F. Miller,Jr., "Asymptotic Solutions of Numerical Transport Problems in Diffusive Regimes," *Comp. Phys.* **69**(#2), 283-324 (April, 1987).

2. E.W. Larsen, J.M. McGhee, and J.E. Morel, "The Simplified P_N Equations as an Asymptotic Limit of the Transport Equation," *Trans. Am. Nucl. Soc.* **66**,231 (1992).

3. Y.H. Liu and E.M. Gelbard, "Accuracy of Nodal Transport and Simplified P_3 Fluxes in Benchmark Tests," *Trans. Am. Nucl. Soc.* **52**, 430 (1986).

4. A.M. Mui, Y.I. Kim, and D.R. Harris, "Modified P_3 Transport Improvements for Reactor Diffusion Calculations," *Trans. Am. Nucl. Soc.* **55**, 584 (1987).

5. R.G. Gamino, "Three-Dimensional Nodal Transport Using the Simplified P_L Method," Proc. ANS Topical Mtg., *Advance in Mathematics, Computations, and Reactor Physics*, April 29~May 2, 1991, Pittsburgh, Vol. 2, Sec. 7.1, p. 3-1 (1991).

6. D. Tomašević and E.W. Larsen, "The Simplified P_2 Correction to the Multidimensional Diffusion Equation," *Trans. Am. Nucl. Soc.* **66**, 232 (1992).

Figure 1. In the Homogeneous Slab of σ_t=0.1, σ_s / σ_t=0.9, with constant source S=1.0 at t=0