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TITLE: ASYMPTOTIC DERIVATION OF THE MODIFIED TIME-DEPENDENT SP₂ EQUATIONS AND NUMERICAL CALCULATIONS

AUTHOR(S): UNCHEOL SHIN, WARREN F. MILLER, JR., JIM E. MOREL

SUBMITTED TO: AMERICAN NUCLEAR SOCIETY WINTER MEETING-1993 November 14-19, 1993 San Francisco, California

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Asymptotic Derivation of the Modified Time-Dependent SP: Equations and Numerical Calculations

Uncheol Shin and Warren F. Miller, Jr.
Los Alamos National Laboratory and
University of California at Berkeley

and

Jim E. Morel
Los Alamos National Laboratory
Los Alamos, NM 87545

Introduction

Converting the independent variables of the transport equation to dimensionless parameters, asymptotic analyses can be performed to show that, for an important class of problems, the diffusion equation is an asymptotic limit of the transport equation.\(^1\) A recent paper by Larsen, McGhee, and Morel\(^2\) provides a broadened view of the result discussed above. It deals with the steady-state transport equation and shows that the Simplified Spherical-Harmonics (SPs) equations are robust high-order asymptotic approximations of the transport equation in a physical regime in which the conventional diffusion equation is the leading-order approximation. According to the reported numerical results for the steady-state cases,\(^3\) for many problems, low-order SP-equations capture most (Gamino\(^3\) reports "greater than 80\(^6\)") of the transport corrections to the diffusion approximation. And Larsen\(^3\) shows that, in nearly all cases, the SP2 results are significantly more accurate than diffusion results in the steady-state problems.

In this paper, we deal with the time-dependent case and find that if one neglects the time derivative term of the second moment of angular flux, $\partial \psi_i/\partial t$, in the time-dependent SP_i equations (we will call this equation as the Modified Time-Dependent SP_i

Equations), this equation has the same asymptotic approximations of the time-dependent transport equation up to the third-order in a physical regime in which the time-dependent diffusion equation is the leading-order approximation. Also, we formulate the time-dependent P₂ equations for one-group, slab geometry problems (in slab geometry, the SP₂ equations are exactly same as P₂ equations) and compare numerical time-dependent diffusion, P₂ and discrete-ordinates (S₁₀) solutions in several classes of problems. Because the time-dependent SP₂ equations can be transformed to the same form of equation as the time-dependent diffusion equation, the SP₂ results are obtained with almost the same computational effort as the diffusion results. In addition, we compare the numerical solutions of the time-dependent P₂ and modified P₂ equations with those of the diffusion and S₁₀ equations in the slab geometry

Time-Dependent P. Equations in the Slab Geometry

If we eliminate the time variable in the time-dependent P_1 equations using an implicit scheme, and formulate an equation in the scalar flux, ψ_0 , only, the result is in the form of a conventional diffusion equation. With this equation, we can make the matrix equation $\underline{\mathbf{A}} \cdot \underline{\mathbf{\Psi}}^{n+1} = \underline{\mathbf{S}}^n$ at each time step, n, where $\underline{\mathbf{A}}$ is tri-diagonal symmetric matrix. From the numerical results, we find that, in the vast majority of cases, the time-dependent P_2 solutions are more accurate than the conventional diffusion solutions. Even in the optically-thin regimes (very small total cross-sections), the time-dependent P_2 solutions are quite close to the S_{16} solutions (see Figure 1.) These results provide the motivation to consider the multi-dimensional case

Asymptotic Derivation of the Modified Time-Dependent SP: Equations

One-group, multi-dimensional transport equation with isotropic scattering is,

$$\frac{1}{v}\frac{\partial}{\partial t}\Psi + \Omega \cdot \nabla \Psi + \sigma_i \Psi - \sigma_i \Phi + O$$

If we consider the asymptotic scaling 1:

$$\sigma_{i} \Rightarrow \frac{\sigma_{i}}{\epsilon}, \sigma_{a} \Rightarrow \epsilon \sigma_{a}, \sigma_{r} \Rightarrow \frac{\sigma_{i}}{\epsilon} - \epsilon \sigma_{a}, Q \Rightarrow \epsilon Q, v \Rightarrow \frac{v}{\epsilon}$$

where, $\varepsilon <<1$ and expand,

$$\psi = \psi_0 + \varepsilon \psi_1 + \varepsilon^2 \psi_2 + \cdots$$
, $\varphi = \varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \cdots$;

apply these to the time-dependent transport equation, and equate coefficients of different powers of ϵ , the result is

$$O(\varepsilon) \qquad \frac{1}{v} \frac{\partial}{\partial t} \phi_0 - D \nabla^2 \phi_0 + \sigma_u \phi_0 = Q \text{, where } D = \frac{1}{3\sigma_0}$$

time-dependent diffusion equation

$$O(\varepsilon^2) \qquad \frac{1}{\nu} \frac{\partial}{\partial t} \phi_1 - D \nabla^2 \phi_1 + \sigma_a \phi_1 = 0$$

$$O(\varepsilon^3) \qquad \qquad (\frac{1}{v\sigma_1}\frac{\partial}{\partial t} + \frac{4}{15\sigma_2^2}\nabla^2)D\nabla^2\phi_0 + \frac{1}{v}\frac{\partial}{\partial t}\phi_2 - D\nabla^2\phi_2 + \sigma_2\phi_2 = 0$$

Modified Time-Dependent SP, Equation in General Geometry

If one neglects the time derivative term of ψ_i , $\frac{\partial}{\partial t}\psi_j$, in the time-dependent SP_i equations, and reforms them with only the terms of ψ_i (= ϕ), one can get;

$$(\frac{1}{v\sigma_{\star}}\frac{\partial}{\partial t} - \frac{4}{15\sigma_{\star}^{2}}\nabla^{2})(\frac{1}{v}\frac{\partial}{\partial t}\phi + \sigma_{\star}\phi - Q) + \frac{1}{v}\frac{\partial}{\partial t}\phi - D\nabla^{2}\phi + \sigma_{\star}\phi = Q$$

Applying the same scaling and expansion to this equation, we find the Modified Time-Dependent SP₂ Equation has the same asymptotic approximations of the transport equation up to the third-order in a physical regime in which the time-dependent diffusion equation is the leading-order approximation. In other words, the modified time-dependent SP₂ equations contain higher-order asymptotic corrections to the time-dependent diffusion equation than do the SP₂ equations without modification. Also we compare numerical solutions of the time-dependent P₂ and modified P₂ equations, and the results show that the

modified time-dependent P₂ equations give even better answers than the time-dependent P₂ equations in many cases (see Figure 1.)

Conclusion

We have shown that the Modified Time-Dependent SP₂ Equations contain higher-order asymptotic corrections to the time-dependent diffusion equation and in the slab geometry, in most of cases, the time-dependent P₂ solutions are more accurate than the diffusion solutions. This implies that in general geometry, the time-dependent (modified) SP₂ equations could give a lot more accurate results than the time-dependent diffusion equation with almost the same computational efforts, and could give relatively inexpensive solutions to the time-dependent S_N equations. The study for the time-dependent SP₂ equations in more general cases, such as in 2-D multiplying medium, multigroup for energy, with anisotropic scattering and delayed neutron precursors, is being performed by the authors to validate the advantages of the time-dependent (modified) SP₂ equations in more realistic problems

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Figure 1. In the Homogeneous Slab of $C_t=0.1$, C_s / $C_t=0.9$, with constant source S=1.0 at t=0

