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THEORY OF AN UNTAMPED SUBCRITICAL SPHERE WITH A SURFACE SOURCE

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ABSTRACT

Snyder, Wilson, and Woodward have measured the neutron flux at the center of a subcritical 25 sphere with 28 and 25 fission chambers. The source was an outer close-fitting 25 shell activated with slow neutrons. The ratio 1 + M of counts with the sphere to without the sphere in place was recorded. Two spheres of 25 and one of normal uranium were used. The experimental and theoretical values of 1 + M are given as follows:

Sphere		73% 25		Normal Uranium
Detector		28	65% 25	28
Small Sphere	Exp.	1.13 ± .01	1.26 ± .01	.89 ± .03
	Theo.	1.06	1.23	.84
Medium Sphere	Exp.	1.21 ± .02	1.445 ± .02	---------
	Theo.	1.12	1.42	---------

The agreement is good for the 65% 25 but poor for the 28 detector. The result on the multiplication of the small 25 sphere was 1.076 ± .008 experimentally and 1.074 theoretically.

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THEORY OF AN UNTAMPED SUBCRITICAL SPHERE WITH A SURFACE SOURCE

INTRODUCTION

In this paper we wish to discuss more completely the theory of the experiments suggested by Frankel and Nelson[1] and performed by Snyder, Wilson, and Woodward[2]. In a large cavity of the Building X graphite column, a sphere of β-stage material (about 70% 25) was immersed in the thermal neutron flux. The sphere consisted of an outer shell of enriched 25, a removable core of active material and a central fission detector with 28 or 25 foils. The significant measured quantity was the ratio 1 + M of the counts of either detector with core to without core in place. These integral experiments, closely allied to the important problem of determining critical masses[1], serve as a test of our theoretical knowledge and furnish in addition information or criteria to fix more precisely the differential nuclear constants.

1. METHOD OF COLLISIONS

In the small-size spheres (radius about a half a neutron mean free path) used in the experiments, neutrons from a source on the surface of the sphere which have made many collisions should contribute little to the total flux at the center. Our method is to treat carefully the first two collisions and then to approximate the effect of the higher collisions.

In reality, the sphere had an extended shell source, a core with a hole partially filled with scattering matter, and spherical foils for detectors.

1) LAMS-130
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For practical reasons, we start with the most idealized conditions and then progressively remove the simplifying assumptions. We give a chart of various approximations used in our calculation and then consider in turn the more important.

- Shell
  - a. Surface source
  - b. Extended source
- Core
  - a. Solid
  - b. Hole
    - 1. Filled - - Partially filled
    - 2. Empty
- Detector
  - a. Center point
  - b. Off center in spherical foils

(A) Surface Source, Solid Core and Center Detection

The simplest case in the above chart is the one in which the shell source is of zero thickness, the core is solid to the center, and detection takes place in the center.

Direct Flux. On a sphere (radius a) q neutrons are emitted per sec. The flux at the center for no core is $q/4\pi a^2$. With core the attenuated direct beam will produce a flux F_o at the center.

$$F_o = qe^{-\sigma a}/4\pi a^2 \quad (1)$$

σ is the total transport cross-section (in reciprocal cms). Thus far, we have, as $1 + M = (4\pi a^2/q)(F_o + ...)$,

$$1 + M = e^{-\sigma a} \quad (2)$$

First Collisions. We now calculate the contribution from first collisions in the core. The neutron emitted at q travels along r' and is scattered at P to enter along r the detector d at the center. The flux at the center from first collisions will now be derived. The flux at P, nv_P will arise from the surface source.

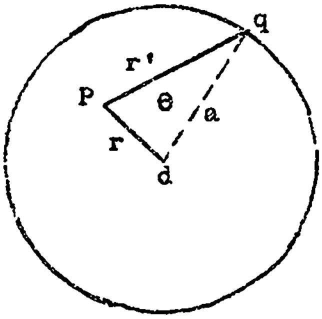

Fig. 1

$$nv_P = qe^{-\sigma r'}/4\pi r'^2. \quad (3)$$

This flux nv_P produces at P a neutron collision source dq_P

$$dq_P = nv_P\sigma(1 + f)\,d\underline{r}_P. \quad (4)$$

Here f is the number of neutrons emitted per collision and $d\underline{r}_P$ is the volume element at P. This collision source dq_P creates at d a flux

$$dq_P(e^{-\sigma r}/4\pi r^2) \quad (5)$$

The neutron flux F_1 due to first collisions is then

$$F_1 = \int dq_P \frac{e^{-\sigma r}}{4\pi r^2} = \int nv_P \frac{\sigma(1 + f)}{4\pi r^2} d\underline{r}_P\, e^{-\sigma r}$$
$$= \int q \frac{e^{-\sigma r'}}{4\pi r'^2} \frac{\sigma(1 + f)}{4\pi r^2} d\underline{r}_P\, e^{-\sigma r} \quad (6)$$

Introducing the variables suggested by Fig. 1, we have $d\underline{r}_P = 2\pi r^2 dr d\mu$; $\mu = \cos\theta$; $0 \leq r \leq a$; $-1 \leq \mu \leq 1$. Thus we get finally

$$F_1 = \frac{q\sigma(1 + f)}{8\pi} \int_0^a \int_{-1}^{+1} \frac{dr\, d\mu}{r'^2} e^{-\sigma(r + r')} \quad (7)$$

We evaluate this integral by expanding the exponential term. Physically the first term of the expansion means no attenuation from surface source to central detector. The presence of $\sigma r'$ represents attenuation over

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the source path qP and likewise σr is a sign of attenuation over the detector path Pd.

$$F_1 = \frac{q\sigma(1+f)}{8\pi}\int_0^a\int_{-1}^{+1}\frac{dr\,d\mu}{r'^2}\left(1-\sigma r-\sigma r'+\dots\right) \qquad (8)$$

We use $r'^2 = r^2 + a^2 - 2ar\mu$ and integrate over the angle variable μ.

$$F_1 = \frac{q\sigma(1+f)}{8\pi}\left\{\int_0^a \ln\frac{r+a}{|r-a|}\,\frac{dr}{ar}\,(1-\sigma r) - \sigma\int_0^a\Big[|r+a| - |r-a|\Big]\frac{dr}{ar} + \dots\right\} \qquad (9)$$

Setting $x \equiv r/a$

$$F_1 = \frac{q\sigma(1+f)}{8\pi a}\left\{\int_0^1 \ln\frac{1+x}{1-x}\,\frac{dx}{x} - \sigma a\int_0^1 \ln\frac{1+x}{1-x}\,dx - \sigma a\int_0^1 2dx + \dots\right\}$$

$$= \frac{q\sigma(1+f)}{8\pi a}\left\{\frac{\pi^2}{4} - \sigma a(2\ln 2 + 2) + \dots\right\} \qquad (10)$$

The contribution to $1 + M$ from F_1 is $F_1(4\pi a^2/q)$ or

$$1 + M = (4\pi a^2/q)\left[F_0 + F_1 + \dots\right]$$

$$1 + M = e^{-\sigma a} + \sigma a(1+f)\left\{\pi^2/8 - (1+\ln 2)\sigma a + \dots\right\} \qquad (11)$$

We have reached a stage where we can arrive at some simple conclusions. Consider the σa term of Eq. (11).

$$1 + M = 1 + \left\{(\pi^2/8)(1+f) - 1\right\}\sigma a = 1 + \left\{1.2337(1+f) - 1\right\}\sigma a \qquad (11')$$

We have set $e^{-\sigma a} = 1 - \sigma a$, since higher powers of σa have already been neglected in the treatment of the single scattering term and in the omission of multiple scattering, and since the expansion of the attenuation term evidently gives a better approximation by leading to canc[illegible]en

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positive and negative terms in the higher orders.

A simple interpretation can be given to Eq. (11'). We have two terms, the $1.2337\,(1 + f)\sigma a$, the flux scattered to the center by collisions; and $-\sigma a$, the flux thrown out by attenuation in a first collision. We note that a non-fissionable core ($f = 0$) does not imply $M = 0$. In fact $1 + M(f = 0) = 1 + .2337\sigma a > 1$. We will now give rough estimates of $1 + M(25)$ and $1 + M(28)$, the results to be expected for 25 and 28 detectors. The radius $a \sim 2$ cm, and mean free path ~ 4 cm or $\sigma a \sim .5$. For 25, $\sigma \sim 5$, $\sigma_f \sim 1.3$, $\sigma_r \sim .07$ barns. For 28, $\sigma \sim 5$, $\sigma_f \sim .3$, $\sigma_r \sim .13$, $\sigma_{in} \sim .9$ barns. σ_f and σ_r are the fission and radiative capture cross sections. $\sigma_{in}(28)$ is the inelastic scattering below the 28 threshold and therefore acts as capture. Taking ν the number of neutrons per fission process, as 2.44, we get

$$f(25) = \frac{1.44 \times 1.34 - .07}{5} = .37;\quad 1 + f(25) = 1.37$$

$$f(28) = \frac{1.44 \times .3 - .13 - .9}{5} = -.12;\quad 1 + f(28) = .88$$

We note that it is $\sigma_{in}(28)$ which makes $f(28)$ negative. Finally we have

$$1 + M(28) \sim 1.04;\quad 1 + M(f = 0) \sim 1.12$$
$$\text{and } 1 + M\,(25) \sim 1.34 \qquad (11'')$$

<u>Double Collisions.</u> Using the methods outlined above, we evaluate the flux F_2 due to double collisions.

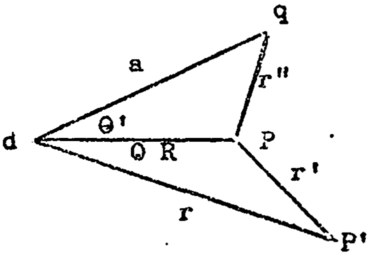

Fig. 2

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$$dF_2 = \sigma(1 + f)\int_0^a \int_{-1}^{+1} d(nv_{P'}) \frac{e^{-\sigma r}}{4\pi r^2}\, d\underline{r}_{P'} \qquad (12)$$

$$F_2 = \sigma^2(1 + f)^2\int_0^a\int_{-1}^{+1}\int_0^a\int_{-1}^{+1} \frac{q e^{-\sigma r'' - \sigma r' - \sigma r} 2\pi R^2 dR d\mu' \; 2\pi r^2 dr d\mu}{4\pi r''^2 \cdot 4\pi r'^2 \cdot 4\pi r^2} \qquad (13)$$

We used $d\underline{r}_{P'} = 2\pi r^2 dr d\mu$ and $d\underline{r}_P = 2\pi R^2 dR d\mu'$. Recalling $r''^2 = R^2 + a^2 - 2aR\mu'$ and $r'^2 = R^2 + r^2 - 2Rr\mu$, we easily perform the two angle integrations

$$F_2 = \frac{q\sigma^2(1 + f)^2}{16\pi}\int_0^1\int_0^1 \ln\frac{1 + y}{1 - y}\, \ln\frac{y + x}{|y - x|}\, dy\, \frac{dx}{x} + \;\ldots. \qquad (14)$$

We have given explicitly only the unattenuated first term of F_2. Further, $y \equiv R/a$ and $x \equiv r/a$. A numerical evaluation of the integral gives

$$F_2 = \frac{q\sigma^2(1 + f)^2}{4\pi} \times 1.15166 \qquad (15)$$

The total $1 + M$ up to $(\sigma a)^2$ terms is then

$$1 + M = 1 + \left\{(\pi^2/8)(1 + f) - 1\right\}\sigma a + \left\{.5 - (1 + \ln 2)(1 + f) + 1.15166(1 + f)^2\right\}(\sigma a)^2 \qquad (16)$$

Summarizing our final equation, we give the origin of the several terms (a) 1: direct unattenuated flux; (b) $(\pi^2/8)(1 + f)\sigma a$: unattenuated first collision; (c) $-\sigma a$: linear attenuation in direct flux; (d) $0.5(\sigma a)^2$: square attenuation in direct flux; (e) $-(1 + f)(\sigma a)^2$: linear attenuation over source path qP in first collision; (f) $-(\ln 2)(1 + f)(\sigma a)^2$: linear attenuation over detector path (Pd) in first collision; (g) $1.15166(1 + f)^2(\sigma a)^2$: unattenuated double collisions.

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(B) Surface Source, Filled Hole, and Center Detection

We travel one more step in the direction of actuality and assume that the detector space (radius b) has the same total cross section for scattering as the core, but no fission.

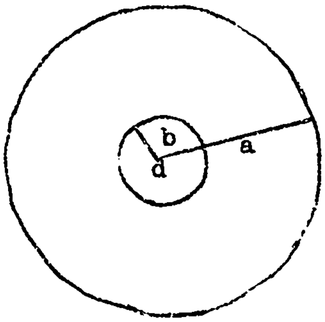

Fig. 3

The derivation of 1 + M proceeds as in section A. However, the regions of integration are different. F_0 is maintained. F_1 of Eq. (8) breaks up into two parts, $F_{1,c}$ and $F_{1,h}$. The former represents the first collision in the core and latter arises from the first collision in the hole $(0 \leq r \leq b)$. We write down the result

$$F_{1,c} = \frac{q\sigma(1+f)}{8\pi}\int_b^a\int_{-1}^{+1}\frac{dr\,d\mu}{r'^2}(1 - \sigma r - \sigma r' + \ldots)$$

$$= \frac{q\sigma(1+f)}{8\pi a}\left\{\int_{b/a}^1 \ln\frac{1+x}{1-x}\,\frac{dx}{x} - \sigma a\int_{b/a}^1 \ln\frac{1+x}{1-x}\,dx - \sigma a\int_{b/a}^1 2dx + \ldots\right\} \tag{3'}$$

$$F_{1,h} = \frac{q\sigma}{8\pi}\int_0^b\frac{dr\,d\mu}{r'^2}(1 - \sigma r - \sigma r' + \ldots)$$

$$= \frac{q\sigma}{8\pi a}\left\{\int_0^{b/a}\ln\frac{1+x}{1-x}\,\frac{dx}{x} - \sigma a\int_0^{b/a}\ln\frac{1+x}{1-x}\,dx - \sigma a\int_0^{b/a}2dx + \ldots\right\} \tag{3''}$$

The double collision gives rise to four types: $F_{2,c,c}$, $F_{2,c,h}$, $F_{2,h,c}$, and $F_{2,h,h}$. For instance $F_{2,h,c}$ implies a double collision flux at the center caused by a first collision in the hole followed by a second collision in core with consequent detection at the center. The final integrals are

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$$F_{2,c,c} = \frac{q\sigma^2(1+f)^2}{16\pi}\int_{b/a}^{1}\int_{b/a}^{1} \ln\frac{1+y}{1-y}\,\ln\frac{y+x}{|y-x|}\,dy\,\frac{dx}{x} \qquad (14')$$

$$F_{2,c,h} = \frac{q\sigma^2(1+f)}{16\pi}\int_{b/a}^{1}\int_{0}^{b/a} \ln\frac{1+y}{1-y}\,\ln\frac{y+x}{|y-x|}\,dy\,\frac{dx}{x} \qquad (14'')$$

$$F_{2,h,c} = \frac{q\sigma^2(1+f)}{16\pi}\int_{0}^{b/a}\int_{b/a}^{1} \ln\frac{1+y}{1-y}\,\ln\frac{y+x}{|y-x|}\,dy\,\frac{dx}{x} \qquad (14''')$$

$$F_{2,h,h} = \frac{q\sigma^2}{16\pi}\int_{0}^{b/a}\int_{0}^{b/a} \ln\frac{1+y}{1-y}\,\ln\frac{y+x}{|y-x|}\,dy\,\frac{dx}{x} \qquad (14'''')$$

(C) Surface Source, Empty Hole, and Center Detection

An equally important approximation is to assume that the hole has no scattering at all.

Then Eq. (1) for F_o becomes

$$F_o = qe^{-\sigma(a-b)}/4\pi a^2 \qquad (1')$$

Eq. (3') for $F_{1,c}$ has an additional term $\Delta F_{1,c}$ evaluated below. $F_{1,h}$ is zero. Similarly $F_{2,c,c}$ is retained, but $F_{2,c,h}$, $F_{2,h,c}$ and $F_{2,h,h}$ become zero.

$\Delta F_{1,c}$ originates in the attenuated paths that cross the empty hole. Paths terminating in a point P have an unattenuated length SR, the traverse across the hole. We find

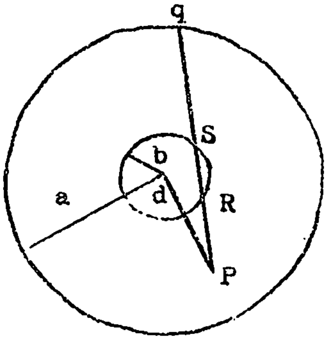

Fig. 4

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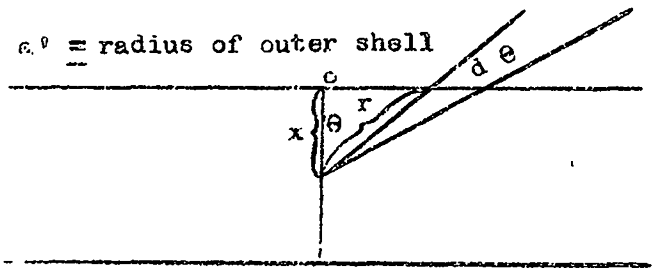
that

$$\Delta F_{1,c} = \frac{qc^2(1+f)}{4\pi}\ \frac{b}{a}\int_0^{\arcsin b/a}\sqrt{1-\frac{a^2}{b^2}\sin^2\theta}\left\{\frac{\pi}{2}-\theta-\arccos\left(\frac{a}{b}\sin\theta\right)\right\}d\theta \qquad (8''')$$

(D) Extended Source

In the previous sections we idealized our source as a surface. In reality the source is a shell of finite thickness undergoing fission in a cloud of slow neutrons. Neglecting the effect of curvature, we calculate the source distribution $Q(x)$ in a parallel plate of active material.

a' = radius of outer shell



a = radius of outer core

The number of slow neutrons $dQ(x)$ reaching a depth x at angle θ in solid angle $d\omega$ is for a uniform angular distribution of the slow neutrons,

$$dQ(x) \sim e^{-\sigma_s r}\, d\omega \sim e^{-\sigma_s x/\cos\theta} d(\cos\theta) = e^{-\sigma_s x/\mu} d\mu \qquad (17)$$

σ_s is the total cross section for slow neutrons in the shell.

Integrating Eq. (17), we get

$$Q(x) \sim \int_0^1 e^{-\sigma_s x/\mu}\, d\mu = \int_1^\infty e^{-\sigma_s xy}\, dy/y^2; \quad y \equiv 1/\mu \qquad (18)$$

These slow neutrons then cause fissions which are the source of fast neutrons. The integral of Eq. (18) is tabulated in a WPA release.

To include the effect of the extended source $Q(x)$, the results of the previous sections have to be integrated over $Q(x)$. Thus F_0 of Eq. (1')

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in Section C becomes

$$F_0 = \int_a^{a'} \frac{Q(x)\,dx\,e^{-\sigma r'}}{4\pi r'^2} \; ; \; r' = a' - x \qquad (18')$$

In the same manner we generalize the other expressions.

(E) Off-Center Detection

That the detection takes place in spherical fission foils off the center must be considered. Derivations of 1 + M lead to more complicated but basically the same expressions[3] as found above.

(F) Cross Section Averages

The slow neutrons bombard the enriched shell and create a fission source distributed in energy E as $\chi_f(E)$. These neutrons in turn by elastic, inelastic, and fission collisions become a new spectrum; the first collision spectrum which by a further collision changes into a second collision spectrum, etc.

Direct Flux Averages. The direct unattenuated flux requires an energy average of

$$\int_0^{\infty} \chi_f(E)\,\sigma_d(E)\,dE \equiv \overline{\sigma_d} \qquad (19)$$

Here σ_d is the detector cross section which is either $\sigma_f(28)$ or $\sigma_f(25)$ of the foils in the fission chamber.

Similarly, linear attenuation in the direct flux is given by

$$\int_0^{\infty} \chi_f \sigma \,\sigma_d \,dE \equiv \overline{\sigma\,\sigma_d} \qquad (19')$$

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3) See 2A' below.

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Square attenuation in the same way involves

$$\int_0^{\infty} \chi_f \sigma^2 \sigma_d dE \doteq \overline{\sigma^2 \sigma_d} \qquad (19'')$$

For a first collision, the spectrum becomes in scattering from E to E'.

$$\chi_f(E) \left\{ \sigma_e \delta(E - E') + \nu\sigma_f(E) \chi_f(E') + \sigma_{in}(E) \chi_{in}(E,E') \right\} \doteq \sigma_1 S_1(E,E') \qquad (20)$$

$\sigma_e \delta(E - E')$ is the elastic scattering to the same energy. $\nu\sigma_f(E)\chi_f(E')$ is the number of neutrons emitted in a fission collision at energy E'. $\sigma_{in} \chi_{in}(E,E')$ is the inelastic spectrum. If this spectrum $\sigma_1 S_1(E,E')$ is directly detected, the resulting average is

$$\iint \sigma_1 S_1(E,E') \; \sigma_d(E') dE dE' \doteq \overline{\sigma(1 + f_1)\sigma_d} \qquad (21)$$

For a first collision and a linear attenuation, the attenuation may precede or succeed the collision.

Attenuation followed by a collision requires the integral

$$\iint \sigma(E) \sigma_1 S_1(E,E') \sigma_d(E') dE dE' \doteq \overline{\sigma^2(1 + f_2)\sigma_d} \qquad (22)$$

On the other hand attenuation preceded by a first collision involves

$$\iint \sigma(E') \sigma_1 S_1(E,E') \sigma_d(E') dE dE' \doteq \overline{\sigma' \sigma(1 + f_3)\sigma_d} \qquad (23)$$

When two collisions occur, the spectrum becomes

$$\sigma_1 S_1(E,E') \left\{ \sigma'_e \delta(E' - E'') + \nu\sigma_f' \chi_f(E'') + \sigma_{in}(E') \chi_{in}(E',E'') \right\} \doteq \sigma_2^2 S_2(E,E'') \qquad (24)$$

E" is the final energy after the second collision. The average sought is

$$\iiint \sigma_2{}^2 S_2(E, E'')\sigma_d(E'')\,dE\,dE'\,dE'' = \overline{\sigma^2(1 + f_4)^2\sigma_d} \qquad (25)$$

To take account of these different cross sections Eq. (16) of section A must be rewritten:

$$1 + M = \frac{1}{\overline{\sigma_d}}\left\{ \overline{\sigma_d} + \left[\frac{\pi^2}{8}\overline{\sigma(1 + f_1)\sigma_d} - \overline{\sigma\,\sigma_d}\right] a + \left[.5\overline{\sigma^2\sigma_d} - \overline{\sigma^2(1 + f_2)\sigma_d} - \overline{\ln 2\sigma'\sigma(1 + f_3)\sigma_d} + 1.15166\,\overline{\sigma^2(1 + f_4)^2\sigma_d}\right] a^2 \right\} \qquad (16')$$

2. <u>DISCUSSION OF NUMERICAL DATA</u>

We have computed with two sets of nuclear constants which we call the old and the new. The old values were essentially those given in the handbook LA-140. The $\sigma_f(25)$ curve was made to go down through the Bretscher value at 4 Mev. Also we took $\sigma_c(25)/\sigma_f(25) = .16\ (1 - E/2)$, $\sigma_c(28)/\sigma_f(28) = .1$ and $\chi_{in}(E, E') \sim E'^{1/2}$ for $E' \leq 1.1$ and zero elsewhere. The new nuclear constants were revised to include Richard's recent fission spectrum and Manley's[4] last measurements on the 28 and 25 transport and inelastic cross sections. We also decided to flatten the $\sigma_f(25)$ curve beyond 0.6 Mev. Present evidence indicates that the Bretscher value at 4 Mev is low. All available data were used in arriving at the new constants. We assumed $\sigma_c(25)/\sigma_f(25) = .14(1 - E/2)$. Only the thermal value is experimentally known in the latter relation. The inelastic spectra for 25 and

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28 were deduced from Nanley's data. The 25 inelastic spectrum which gave a simple and adequate fit was $E^{1/2}$ from 0 to .4 Mev, flat up to 1.0 Mev and then zero. The 28 inelastic spectrum, similarly, was assumed to be $(E - .1)^{1/2}$ from .1 to .4 Mev, then flat up to 1 Mev and again zero elsewhere.

We give the results of the collision averages in Table I. The composition of the core was taken as 73% 25.

Table I

Core 73% 25

σ_d	$\sigma_f(28)$		$\sigma_f(25)$	
Set	Old	New	Old	New
$\overline{\sigma_d}$	.269	.382	1.292	1.341
$\overline{\sigma\sigma_d}$	1.133	1.652	6.302	6.122
$\overline{\sigma^2\sigma_d}$	4.800	7.205	31.87	28.58
$\overline{\sigma(1 + f_1)\sigma_d}$	1.047	1.450	8.516	8.814
$\overline{\sigma^2(1 + f_2)\sigma_d}$	4.838	6.562	41.22	40.60
$\overline{\sigma'\sigma(1 + f_3)\sigma_d}$	4.426	6.343	44.92	48.78
$\overline{\sigma^2(1 + f_4)^2\sigma_d}$	5.700	8.097	55.60	62.55

(A) Small Sphere (1.5")

The small sphere had a 73% 25 core, diameter 1.5" and a detector hole .315" in diameter. The shell was 44% 25 and .065" thick. The density of the core was 18.4 gm/cm^3.

Various estimates of 1 + M are entered in Table II. The remainder is the contribution of higher collisions than the second to 1 + M. These

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values were obtained by Richman.

A measurement with normal uranium as core and 28 as detector was also made. As the hole was about one-quarter filled, the final value was obtained by linearly interpolating one quarter way between the empty and filled hole values.

Table II. 1 + M for the Small Sphere

Core	25		Normal Uranium
Detector	28	65% 25	28
Surface source			
(a) Solid Core	1.084	1.313	
(b) Filled Hole	1.080	1.277	
Extended Source			
(a) Solid Core	1.073	1.289	.82
(b) Filled Hole	1.069	1.253	.85
(c) Empty Hole	1.045	1.203	.84
Remainder	.007	.010	.00
Final Value	1.058	1.226	.84
Exp. Value	1.13 ± .01	1.26 ± .01	.89 ± .03

A' <u>Multiplication in the Small Sphere</u>

The multiplication of the small 25 sphere was measured in the graphite block. The activity of an indium foil was volume integrated outside the sphere with the core and then without the core in position. The ratio of these two quantities gives the multiplication.

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The number of neutrons created per second within the solid core for Q neutron per second emitted from the surface is

$$\sigma \int_0^a nv(r) 4\pi r^2 dr = Q R_M \tag{26}$$

$nv(r)$ is the neutron flux at r and the multiplication as measured is $1 + R_M$.

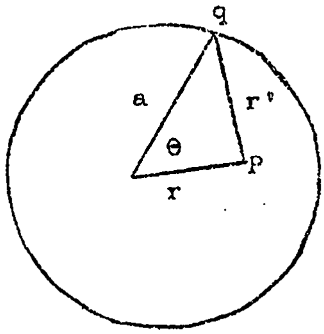

Fig. 6

For the direct flux $n_o v(r)$ to P, we have

$$n_o v(r) = \frac{Q}{2} \int_{-1}^{+1} \frac{e^{-\sigma r'}}{4\pi r'^2} d\mu$$

$$= \frac{Q}{8\pi} \left\{ \frac{1}{ar} \ln \frac{r + a}{|r - a|} - \frac{\sigma}{ar} \left[|r + a| - |r - a| + \dots \right] \right. \tag{27}$$

$$= \frac{Q}{4\pi a^2} \left\{ \frac{a}{2r} \ln \frac{a + r}{a - r} - \sigma a + \dots\dots \right\}$$

[redacted]

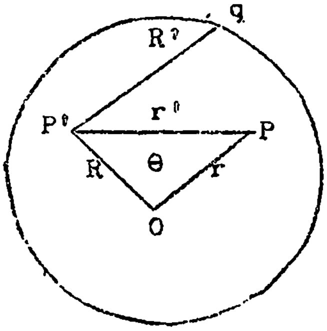

Fig. 7

For first collision to P,

$$n_1 v(r) = \int_0^a \int_{-1}^{+1} \sigma(1 + f)\ n_0 v(R)\ \frac{e^{-\sigma r'}}{4\pi r'^2}\ 2\pi R^2 dR d\mu \tag{28}$$

We shall need to calculate only the principal term of Eq. (28), which arises when attenuation over qP' and P'P are neglected.

$$n_1 v(r) = \frac{Q\sigma(1 + f)}{16\pi a} \int_0^a \ln\frac{a + R}{a - R}\ \ln\frac{R + r}{|R - r|}\ \frac{dR}{r} \tag{29}$$

To $(\sigma a)^2$ terms $nv(r) = n_0 v + n_1 v$ which, substituted in Eq. (26) gives

$$QR_M = \frac{Q\sigma f}{a^2} \int_0^a r^2 dr \left\{ \frac{a}{2r} \ln\frac{a + r}{a - r} - \sigma a + \frac{\sigma a(1+f)}{4} \int_0^a \ln\frac{a+R}{a-R}\ \ln\frac{R+r}{|R-r|}\ \frac{dR}{r} \right\} \tag{30}$$

Introducing $x \equiv r/a$ and $y \equiv R/a$, we get

$$R_M = \sigma\ fa \left\{ \int_0^1 x \ln\frac{1 + x}{1 - x}\ dx - \sigma a \int_0^1 x^2 dx \right.$$

$$\left. + \frac{\sigma a(1 + f)}{4} \int_0^1 \int_0^1 x \ln\frac{1 + y}{1 - y}\ \ln\frac{x + y}{|x - y|}\ dy\, dx \right\} \tag{31}$$

$$= \sigma fa \left[1/2 + \left\{ -1/3 + .3479\ (1 + f) \right\} \right] \sigma a$$

The hole and shell corrections and the contribution of higher terms than $(\sigma a)^2$ are here negligible. Final results are entered in Table III.

Table III

$1 + R_M$ for small 25 sphere

Theoretical (Old constants)	1.074
Theoretical (New constants)	1.083
Experimental	1.076 ± .008

(B) Medium Sphere (2")

The medium sphere was composed of the small sphere and an additional shell to bring the diameter to 2". The shell source was 65% 25 and .090" thick. Table IV summarizes our results for 1 + M.

Table IV. 1 + M for the Medium Sphere, 73% 25 Core

Detector	28	65% 25
Surface Source and Solid Core	1.144	1.486
Extended Source and Empty Hole	1.110	1.383
Remainder	.014	.033
Final Value	1.124	1.416
New Final Value	1.120	1.469
Exp. Value	1.21 ± .02	1.4[illegible] + .02

[redacted]

The entry, New Final Value, was computed on the basis of the new constants. The extra work entailed in computing a filled hole value does not compensate for the added accuracy. Moreover, the hole is less important for the medium sphere and it should be a fair approximation to consider the hole as empty.

The 1 + M of Table II and III were computed for center detection. The off-center results were not appreciably different.

We observe that the agreement for both spheres with the 25 detector is good. Too close a check is not to be expected because of the uncertainty introduced by the amount and distribution of matter in the hole. The results for the 28 detector are rather poor. We estimate that if the 25 and 28 inelastic scattering to below the 28 threshold were reduced 30 per cent, the existing discrepancies would be explained.

[redacted]