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LA REPORT 101

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June 27, 1944 This document contains [redacted] pages

STATISTICAL FLUCTUATIONS IN THE WATER BOILER AND THE DISPERSION OF NEUTRONS EMITTED PER FISSION

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ABSTRACT

Short time fluctuations in neutron intensity of the water boiler were investigated with the boiler running at critical. A theoretical deviation is given connecting these fluctuations with the quantity $\overline{\nu^2} - \overline{\nu}$, that is, the average of the square of the number of neutrons per fission minus the average number of neutrons per fission. Experimental results for the fluctuations of the boiler are given.

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STATISTICAL FLUCTUATIONS IN THE WATER BOILER AND THE DISPERSION OF NEUTRONS EMITTED PER FISSION

I INTRODUCTION

It was theoretically predicted and experimentally confirmed as soon as the water boiler at Site Y was in operation that there were fluctuations of the neutron intensity which were larger than an ordinary Poisson fluctuation when the boiler was run at critical.

Closer examination of the theoretical formulae for such a fluctuation gave hopes of determining a quantity which was needed to calculate the probability of predetonation of a gadget. The quantity in question is the quantity $\overline{\nu^2} - \bar{\nu}$, i.e., the average of the square of the number of neutrons per fission minus the average number of neutrons per fission.

In the following, a theoretical derivation is given for the fluctuations of the boiler investigated. This shows the connection between the fluctuations of the boiler and the quantity $\overline{\nu^2} - \bar{\nu}$. Experimental arrangements and results for the fluctuations of the boiler are also presented. It should be clearly understood that this may not be interpreted as a measurement of the quantity $\overline{\nu^2} - \bar{\nu}$, since some further experimental work regarding a certain constant, namely, the prompt multiplication of the boiler, is still lacking.

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II. DERIVATION OF FORMULA

If we let c be the number of counts for a specified time interval and a bar denote averages then we shall proceed to obtain expressions for $\bar{c}$ and $\overline{c^2}$ and later combine them in such a fashion as to give $\left[(\overline{c^2}) - (\bar{c})^2\right]/\bar{c}$ which can be shown to involve directly the quantity $\overline{\nu^2} - \bar{\nu}$. In the case of a Poisson distribution of c the quantity $\left[(\overline{c^2}) - (\bar{c})^2\right]/\bar{c}$ is unity.

It has been shown by Serber and de Hoffmann that the effect due to delayed neutrons can be neglected if the "gate-width", i.e., time over which counts are taken, is small compared to the average delay period. In the following it is assumed that this condition is fulfilled and we may neglect the effect due to delays.

The fact that a neutron born at one place may not produce the same subsequent effect on the boiler as one born at another place has a small effect in our case. This is due to the symmetry of the boiler and also to its small size so that a neutron has a good chance of traversing the whole sphere during its lifetime. Calculations of Feynman have shown that this geometrical factor when taken account of amounts to only a few percent and can really be neglected.

Feynman has further carried out the derivation with a continuum of velocities of neutrons and the result obtained is identical with that when only one type of neutron is assumed. Because of the above information concerning delays, geometry and velocities, we shall in the following give a simplified derivation treating all neutrons in the boiler as being equally effective.

If we introduce a large number of neutrons into the boiler, then since a number die, the rate of change of neutrons will just be proportional to the number present, i.e.,

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$$\dot{N} = -\alpha N, \text{ i.e., } N = N_0 e^{-\alpha t} \qquad (1)$$

where α is the rate of decay. Thus if at time zero one neutron is introduced into the boiler then the expected number of neutrons present at time t is $e^{-\alpha t}$.

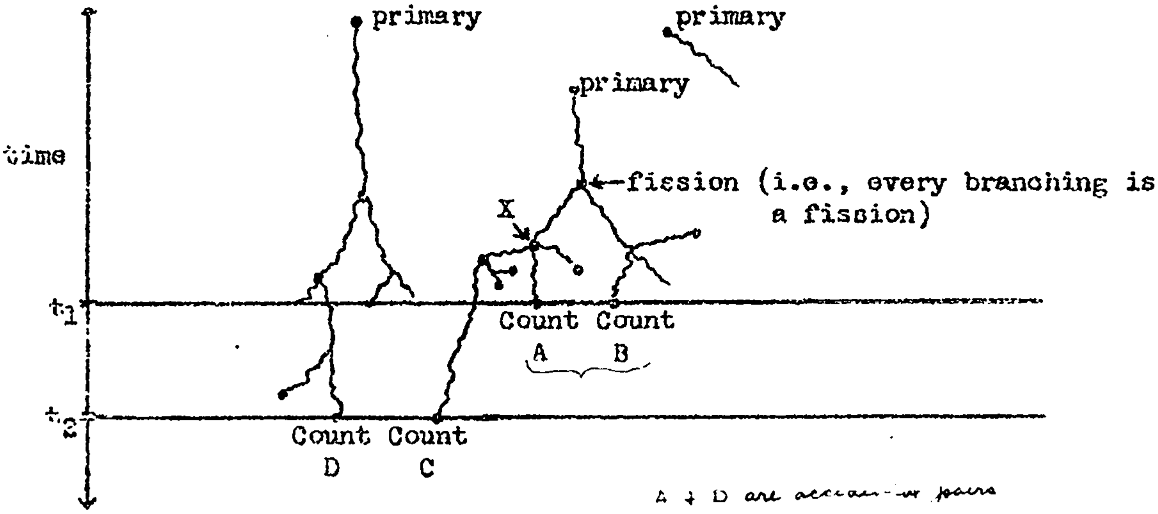
If one neutron is known to be present in the boiler, then let the probability of its producing a fission in time dt be dt/τ. Further, let the probability that any neutron present, whether an initial or a fission neutron, gives a count in the chamber during the time dt be $(\epsilon/\tau)dt$. Let S be the number of fissions per second. Then we may say that $S\tau$ neutrons are present in the boiler on the average and hence $S\epsilon$ counts are registered per second. Thus ϵ = number of counts per fission in the boiler and

$$\bar{c} = S\epsilon t \qquad (2)$$

where t is the gate-width, defined as being the time over which counts are taken.

Next let us select two arbitrary times t_1 and t_2 with corresponding intervals dt_1 and dt_2 and let us ask what the expected number of pairs of counts is, i.e., a count at t_1 in dt_1 and also a count at t_2 in dt_2.

If we remember that there are a certain number of primary neutrons born every second due to such things as spontaneous fission, cosmic rays, γ-n reactions in the BeO tamper, etc., then we may draw the following schematic picture:



A + D are accidental pairs
A + C are coupled p[illegible]

This shows us that we may have two different kinds of pairs of counts at t_1 and t_2. The first type we shall call "accidental" pairs. These are due to a count registering at t_1 due to one primary source neutron and a count registering at t_2 due to an entirely separate family from another independent primary neutron. The pair of counts A and D represents such a situation. The second type we shall call "coupled" pairs. These are pairs of counts which in the last analysis can be traced back to some common ancestor, i.e., one fission. Such a situation would be represented by A and C. The common ancestor is X.

Thus let us express mathematically the expected number of pairs in dt_1 and dt_2 due to accidental and coupled pairs. The expected number due to accidental pairs is merely the probability of having a count in dt_1 which is $S\epsilon dt_1$ multiplied by the probability of having a count in dt_2 which is $S\epsilon dt_2$. We may multiply in these probabilities since* we are dealing with independent events

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* For an expansion of this argument, see Appendix II.

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if we talk about accidental pairs. So the expected number of accidental pairs $= S^2\epsilon^2(dt_1)(dt_2)$. ϵ = counts per fission

Now let us treat the number of expected coupled pairs. Let us assume that the common ancestor, i.e., fission, occurred at time T, which can be anywhere from $T = -\infty$ to $T = t_1$, the time at which the first count registers. The expected number of coupled pairs is then made up of the following independent probabilities:

1) The probability that such a common fission occurred at time t in the interval dt which is

$$S\,dt$$

2) The probability that a certain number of neutrons which we shall call ν be liberated in this fission. Let this be

$$P_\nu$$

3) The expected number of neutrons at t_1 present due to the ν neutrons created at time t

$$\nu \; e^{-\alpha(t_1 - T)}$$

4) The probability that each of the neutrons present at time t_1 will register a count in dt_1

$$\frac{\epsilon}{\tau}\,dt_1$$

5) The expected number of neutrons present at t_2 due to the ν neutrons created at time t and remembering that there are only $\nu - 1$ left to deal with which can produce the second count

$$(\nu-1)\; e^{-\alpha(t_2 - t_1)}$$

6) The probability that each of the neutrons present at time t_2 will register a count in dt_2

$$\frac{\epsilon}{\tau}\,dt_2$$

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Lastly we have to remember that we shall have to do this for all t from $-\infty$ to t_1 and all possible values of ν, i.e., sum over ν, so that we may write:

(Expected number of coupled pairs)

$$= \sum_\nu \int_{-\infty}^{t_1} S\ dt\ p_\nu\ \nu\, e^{-\alpha(t_1 - T)}\ \frac{\epsilon}{\tau} dt_1\ (\nu - 1) e^{-\alpha(t_2 - t_1)} \frac{\epsilon}{\tau} dt_2 \qquad (3)$$

$$= \frac{S\epsilon^2}{2\alpha\tau^2} e^{-\alpha(t_1 - t_2)}\ dt_1\ dt_2 \sum_\nu p_\nu\ (\nu^2 - \nu).$$

Let us note that $\sum_\nu p_\nu\ (\nu^2 - \nu)$ is merely $\overline{\nu^2} - \bar{\nu}$ so that we may write

(Expected number of pairs in dt_1 and dt_2)

$$= S^2\epsilon^2\ dt_1\ dt_2 + \frac{S\epsilon^2}{2\alpha\tau^2} e^{-\alpha(t_1 - t_2)}\ dt_1\ dt_2\ (\overline{\nu^2} - \bar{\nu}). \qquad (4)$$

Next let us examine what the expected number of pairs over an interval of time t (i.e., our gate-width) is:

(Expected number of pairs in interval t)

$$= \int_{t_2=0}^{t_2=t} \int_{t_1=0}^{t_1=t_2} \text{(expected number of pairs in } dt_1 \text{ and } dt_2\text{)}. \qquad (5)$$

Making use of Eq. (4) and integrating as indicated, we get

(Expected number of pairs in interval t)

$$= \frac{S^2\epsilon^2 t^2}{2} + \frac{S\epsilon^2(\overline{\nu^2} - \bar{\nu})t}{2\alpha^2\tau^2}\left(1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right) \qquad (6)$$

Let us further note that if we get c counts per interval of time, then the expected number of pairs in that interval is merely $\overline{c(c-1)}/2$. Remembering that, by Eq. (2), $S\epsilon t = \bar{c}$, we may write

$$\frac{\overline{c^2} - \bar{c}}{2} = \frac{(\bar{c})^2}{2} + \frac{(\overline{\nu^2} - \bar{\nu})\bar{c}}{2\alpha^2\tau^2}\left(1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right) \qquad (7)$$

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Multiplying both sides of Eq. (7) by $1/\bar{c}$ and transposing $\bar{c}$ and $\overline{c^2}$, we get:

$$\frac{\overline{c^2} - (\bar{c})^2}{\bar{c}} = 1 + \frac{\epsilon(\overline{\nu^2} - \bar{\nu})}{(\alpha\tau)^2}\left(1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right) \tag{8}$$

It remains now to show what α means in terms of more conventional quantities of the boiler.

Let us recall that one primary neutron introduced into the boiler means that $e^{-\alpha t}$ will be the expected number of neutrons present at time t due to that primary neutron. Then there is $e^{-\alpha t}\,dt/\tau$ fission expected during a time dt. Hence integrating over all t the total number of fissions generated is $1/\alpha\tau$, each producing ν neutrons. Hence 1 neutron produces $\nu/\alpha\tau$ progeny, and thus $\nu/\alpha\tau$ is the multiplication of a source which is usually written as $1/\nu_p$. Thus we may write $(\alpha\tau) = \nu\nu_p$ and hence

τ

$$\frac{\overline{c^2} - (\bar{c})^2}{\bar{c}} = 1 + \frac{\epsilon(\overline{\nu^2} - \bar{\nu})}{(\bar{\nu})^2(\nu_p)^2}\left(1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right) \tag{9}$$

For simplicity's sake we shall introduce the quantity $Y = \frac{\epsilon(\overline{\nu^2} - \bar{\nu})}{(\bar{\nu})^2(\nu_p)^2}\left(1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right)$ i.e.,

$$\left[\overline{c^2} - (\bar{c})^2\right]/\bar{c} = 1 + Y \tag{10}$$

III. DISCUSSION OF FORMULA AND EXPERIMENTAL CONSIDERATIONS

We note first of all that for large values of the quantity (αt) i.e., for long t the bracket $\left[1 - (1 - e^{-\alpha t})/\alpha t\right]$ tends towards one. Thus if $e^{-\alpha t} \ll 1$ then:

$$Y = \frac{\epsilon(\overline{\nu^2} - \bar{\nu})}{(\bar{\nu})^2(\nu_p{}^2)} \tag{11}$$

[redacted]

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Hence if we knew ϵ and ν_p we could obtain the required quantity $(\overline{\nu^2} - \bar{\nu})$ assuming $\bar{\nu}$ which is known from experiment to be about 2.5. The gate-width dependence given in Eq. (9) might be affected by the fact that the lifetime of a thermal neutron in the tamper is comparable to $\alpha = \nu(\nu_p/\tau)$. However, since the gate-width is long compared to either of the quantities ν_p or τ in the limiting case, the limiting formula (11) will be correct. As is shown later[1)] some film data helped to confirm that the quantity $(Y + 1)$ does fall off markedly with smaller gate-width.

In all measurements it can be seen that a high ϵ will help to determine Y since it will help to make the fluctuation term large compared to the Poisson term of value one.

Let us now examine the quantity $(\overline{\nu^2} - \bar{\nu})$ more closely. If ν were always 2.5, a physical impossibility, then $\overline{\nu^2} - \bar{\nu} = 3.75$. If ν were to divide equally between two and three then $\overline{\nu^2} - \bar{\nu} = 4.0$. Then the mathematical minimum of our $\overline{\nu^2} - \bar{\nu}$ is 3.75 and the lowest one physically possible is 4.0. It can be seen that if ν divided say between 2, 3 and 4 in such a way as to give $\bar{\nu} = 2.5$ the quantity $\overline{\nu^2} - \bar{\nu}$ would vary between about 4.1 and 4.4. If on the other hand ν divided consistently between 2 and 8 in such a fashion as to give $\bar{\nu} = 2.5$ then $\overline{\nu^2} - \bar{\nu} = 6.4$. A Poisson distribution of ν would lead to a value of $\overline{\nu^2} - \bar{\nu} = (\bar{\nu})^2$, i.e., 6.25.

These figures are merely quoted to show that it seems hopeless to conclude anything about the way ν actually divides merely from a knowledge of the

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1) See Fig. 4

[redacted]

quantity $\overline{\nu^2} - \bar{\nu}$. However, this is not the goal of the experiment since we originally set out to determine merely the quantity $\overline{\nu^2} - \bar{\nu}$ (which is needed for the probability of predetonation.)

The experiments performed were therefore set up with a view toward obtaining a value for the quantity Y so that if at a future time the quantity ν_p can be measured successfully a value for the quantity $\overline{\nu^2} - \bar{\nu}$ might be obtained.

IV. EXPERIMENTAL ARRANGEMENTS

Two distinct experimental methods were used which shall be called the "film" and "electrical" method respectively. They are described in the following:

1. THE FILM METHOD: It should first be noted that the largest share of credit for the successful use of this method is due to Brixner and Mack of the Ordnance Division. Due to their help in lending us their equipment as well as actively working with us, it was possible to use high speed photographic methods.

The experimental setup was as follows: a BF_3 chamber was so arranged as to have as large an efficiency as was easily obtainable with existing equipment on the boiler. The BF_3 chamber used had an active volume of 520 cm^3 and a pressure of 60.1 cm of Hg. It was placed just outside the BeO tamper with blocks of paraffin all around it. The efficiency of this arrangement ϵ, i.e., the number of counts in the BF_3 chamber per fission, in the boiler was determined by obtaining the ratio of counts in the BF_3 chamber to that of a 25 chamber in the center of the sphere. This ratio was found to be 1.45×10^{3}. The efficiency of the 25 chamber[2] is 2.54×10^{-7} counts/fission which makes the efficiency of the

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2) See forthcoming report on the operation of the water boiler.

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BF_3 chamber

$$\epsilon = 2.54 \times 10^{-7} \times 1.45 \times 10^{-3} = 3.69 \times 10^{-4} \text{ counts/fission}$$

Pulses from the BF_3 chamber were fed into a standard Nuckolls pre-amplifier and amplifier and then into a Higinbotham Discriminator Scalar. The rise time of this system is estimated to be of the order of 5-6 microseconds.

The boiler itself was run at critical and the intensity so adjusted that the scalar recorded about 500 counts per second.

The scalar was then tapped off at the output of the first stage and the signal fed to the vertical plates of a blue screen 5 inch DuMont Oscillograph. On the vertical horiz plates a specially built linear sweep was imposed and adjusted to a frequency of 250 cycles. The screen was photographed by means of a continuously moving film, with the film moving vertically. This is shown schematically in Fig. 1. The developed film showed a pattern like the sample shown in Fig. 3.

This pattern can be interpreted easily since the first stage of the scalar represents a flip-flop circuit, and consequently its output changes from a maximum to a minimum and back to a maximum with successive pulses. Thus a shift of the height of the line on the scope or film respectively will indicate that a pulse has registered. It should be clearly understood that with the use of this scheme the length of time between "breaks" (i.e., shifts in line-height) is the time elapsed between the appearance of two consecutive pulses and not the duration of a single pulse.

It is important that the sweep return very fast in order that pulses may not be lost between the end of one and the beginning of another sweep. Indeed the sweep used, which was designed by Higinbotham and Sands, proved to have a return time of only several microseconds. Since a single pulse in between two

[redacted]

sweeps is easily detected because of the shift in line-height, and further, since two pulses within that time are rather unlikely because of the limitations due to the resolving time of the amplifier, this sweep was found entirely adequate. Both the sweep and incoming signal were put directly on the respective sets of plates of the oscilloscope in order to by-pass the comparatively long time constants of the internal scope amplifiers. The incoming pulse height was regulated by means of a volume control between scalar and oscilloscope. It was so adjusted as to make the pulse height about a quarter of the distance between consecutive sweeps on the film, as can be seen in Fig. 3. This volume control had to be varied of course when a different film speed was used. Usually the film was run at a speed of about 100 ft/min through a General Radio Co. Oscillograph Camera.

The developed film is most easily read on a Recordak Microfilm Reader. Then the number of counts in say 4 milliseconds, i.e., corresponding to one sweep across the film, can be counted. In order to obtain random 4 millisecond intervals it is merely necessary to move the film through the viewing apparatus stopping at random and counting the particular 4 millisecond interval. This procedure of course can be repeated easily for 8, 12, 16 or more millisecond intervals by merely counting 2, 3, 4 or more consecutive sweeps.

The most obvious advantage of this method is the fact that one and the same data, i.e., strip of film, can be analyzed for several "gate-times" as described. This enables one to get some idea of the dependence of $\left[\overline{c^2} - (\bar{c})^2\right]/\bar{c}$ on the gate-time while one is sure that experimental conditions have not changed since we are merely analyzing identical data in various ways.

Unfortunately the longer the gate-time is taken the fewer pieces of information can we get off one individual film. Films which we took were 100 ft.

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long and for the smaller gate-times this meant about 100-300 data per film. But for instance, for the 160 x 10^{-5} sec gate-width analysis not more than 20 or so pieces of data can be taken from one film in a sufficiently random manner.

Even though we might use several films, it would still take many feet of film and a great many man hours to count the films to get a reasonable number of higher gate-widths.

The electrical method was therefore used to obtain higher gate-width data.

2. <u>THE ELECTRICAL METHOD</u>

The arrangement was the same as in Method 1 up to and including the discriminator scalar unit. The signal, however, was taken off the output of the discriminator then fed through a gate circuit and then fed into another Higginbotham discriminator scalar unit as shown in Fig. 2. The gate circuit which was designed and built by C.P. Baker was so designed as to let pulses pass through it starting from the time the switch was closed till a predetermined time later, e.g., 283 milliseconds.

Thus by closing the switch and reading the scalar, one can obtain the number of counts for a 283 millisecond interval.

The boiler was run at critical and at an intensity of about 1000 counts/sec in the BF_3 chamber.

The true length of time of gate was determined by feeding 1000 cycles from a standard audio-oscillator into the input of the gate circuit and noting the number of counts recorded on the scalar. This also enabled one to make a check of constancy. Over a 12 hour run the gate reproduced to better than 2 percent.

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V. EXPERIMENTAL RESULTS

On Fig. 4 there are plotted the data obtained from the film measurements. Counting was carried out over different gate-widths and all taken from three 100 ft. films. Since the average counting rate $\bar{c}$ was not the same on all three films it was necessary to compute the $Y + 1$ for a particular gate-width from each film and then combine them properly so as not to get fictitious fluctuation due to variations in $\bar{c}$.

If on one film a number of gates m was counted for one particular gate-time it can be shown that the computed $Y + 1$ from this film should be multiplied[3)] by the factor $m/(m - 1)$ to give the correct value of $Y + 1$. The data from several films were then combined by taking the mean correct $Y + 1$, weighed according to the respective m's. The probable error in the quantity obtained can be shown to be $\sqrt{2/e}$ where $e = \Sigma m$. This probable error is indicated on Fig. 4 in the usual manner.

A single point at a high gate-time was determined by the electrical method. 2200 individual gates were taken over a period of some 12 hours. These 2200 gates were then broken up into 76 small sets of from 10 to 50 gates in such a way that the mean $\bar{c}$ did not change violently during one set. Again this was done to avoid a fictitious increase of the quantity $Y + 1$ due to violent variations in $\bar{c}$. These sets were combined in the same fashion as described for combination of data from different films. The result was that for a gate-width of 283 milliseconds

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3) See Appendix I

[redacted]

$$Y + 1 = 5.17 \pm 0.16$$

and hence

$$Y_{(283)} = 4.17 \pm 0.16$$

The mean deviation of the 76 individual Y's from their mean was computed and found to be in complete agreement with the deviation expected statistically. This instills confidence that the value of Y obtained does not contain fluctuations from other than statistical causes.

A rough estimate of the correction due to the geometrical factor was made with the use of some previous data on distributions of neutrons and source-effectiveness in the boiler. The result was 1.01, i.e., negligible as predicted.

VI. DISCUSSION OF RESULTS

The results from films indicate a distinctive falling off of Y towards zero for small values of the gate-width t. From the rough data obtained it would probably be stretching a point to say that it follows the theoretically derived form. As an indication of the theoretical gate-width dependence a curve has been drawn in on Fig. 4, by assuming[4] an $\alpha = 135\ sec^{-1}$. A closer fit to experimental data can be obtained by choosing a different α. However it was not believed that our data were good enough to obtain anything like an experimental value of α. Nevertheless the data obtained sufficiently confirmed our idea of the properties of Y. It allowed us to choose a gate-time, namely 283 milliseconds, which we believed to be sufficiently long to be close to the asymptotic value for Y and yet

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4) This value obtained by using $\nu_p = 7.7 \times 10^{-3}$ and $\tau/\nu = 57 \times 10^{-6}$ sec as estimated by Christy.

[redacted]

[redacted]

be able to neglect the effect of delayed neutrons. This assumption could be made because of the considerations of long gate-width already explained on page 10.

It has already been pointed out that the quantity ν_p has not as yet been measured. Theoretically we can, however, at least arrive at a rough value for ν_p. This is so since we may write $\nu_p = 1 - K_p$ and we may say that at critical

$$\nu_p = 1 - K_p = \gamma f$$

where f is the fraction of neutrons that are delayed, and γ is the factor to account for the fact that they are born at lower energies, and hence have a better chance of leading to fission since they are less apt to leak out. Using data from the Chicago laboratories γ was calculated to be about 1.29. We found f reported as around 0.006. Fermi was kind enough to inform us that this value of f was a very rough value and might well be 0.007 or 0.008. Also rough preliminary experiments done on our boiler indicate that our γf lies somewhere in that region.

If merely for the sake of argument we would use a value of γf of about .008, we would for instance arrive at a value of 4.5 for the quantity $\overline{\nu^2} - \overline{\nu}$ [5].

It is hoped that a measurement of ν_p may be made and a supplementary report then issued, treating the quantity $\overline{\nu^2} - \overline{\nu}$ rather than just the fluctuations of the boiler itself.

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5) This rough estimate was $\epsilon = 3.69 \times 10$[illegible] counts per fission, as shown on page 12.

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APPENDIX I. Derivation of the Correction Factor $m/(m-1)$

In order to compute $Y + 1$ we have to compute the quantity

$$\overline{c^2} - (\bar{c})^2, \text{ i.e., } (1/m) \sum_i (c_i - \bar{c})^2$$

since this is merely a mathematical identity. Experimentally however we measure the quantity

$$(1/m) \sum (c_i - \frac{1}{m} \sum_i c_i)^2$$

Thus we shall examine the relation between the measured quantity and the required one. Now

$$(1/m) \sum (c_i - \frac{1}{m} \sum_i c_i)^2 = (1/m) \sum (c_i - \bar{c} + \bar{c} - \frac{1}{m} \sum_i c_i)^2$$
$$= (1/m) \sum_i (c_i - \bar{c})^2 - \left[(1/m) \sum (c_i - \bar{c})\right]^2$$

Thus we have to evaluate the expected value of $(1/m) \sum_i (c_i - c)$ to obtain the most probable difference between the two quantities. Now

$$\left[(1/m) \sum (c_i - \bar{c})\right]^2 = (1/m^2) \sum_i \sum_j (c_i - \bar{c})(c_j - \bar{c})$$

In this sum the terms with $i \neq j$ are just as likely to be positive as negative and therefore on the average cancel out, leaving only the terms with $i = j$ and hence the expected value of this quantity is:

$$(1/m^2) \sum_i (c_i - \bar{c})^2$$

Hence we see that we may expect that

$$(1/m) \sum \left[c_i - (1/m) \sum_i c_i\right]^2 = \left[(1/m) - (1/m^2)\right] \sum_i (c_i - \bar{c})^2$$
$$= \left[(m-1)/m\right](1/m) \sum_i (c_i - \bar{c})^2$$

or consequently

$$\left[\text{Expected true value of } (Y+1)\right] = \left[m/(m-1)\right]\left[\text{measured value}\right].$$

[redacted]

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APPENDIX II. Probability of Accidental Pairs

The fact that the probability of accidental pairs may be expressed as the product of the individual probabilities of single counts (in spite of the coupling that might exist between some of those single counts) may be made clear as follows:

Let the probability of a count at t_1 arising from a fission at time t be $F(t_1-t)$ and the probability of a fission per unit time be P. Then the probability of a count in dt_1 at t_1 is

$$\int_{-\infty}^{t_1} P\,F(t_1-t)\,dt$$

and the probability of having a count in dt_1 at t_1 arising from one fission and also a count in dt_2 at t_2 from another fission, is the product of two such expressions since the two fissions are independent events. Thus the probability of an accidental pair in dt_1, dt_2 is the product of the probabilities of counts in dt_1 and dt_2.

FIGURE NO.1

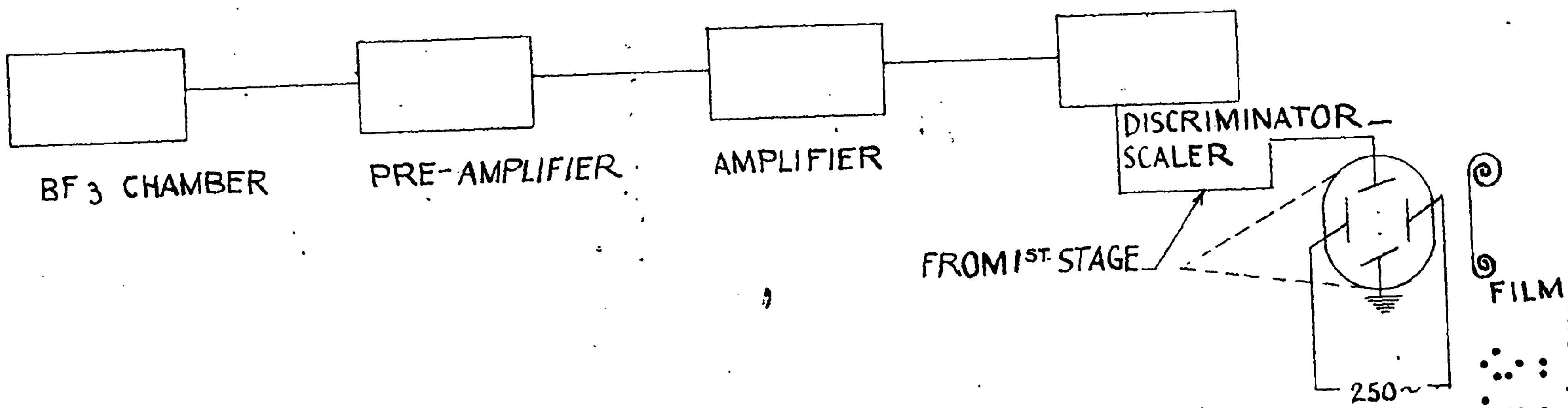
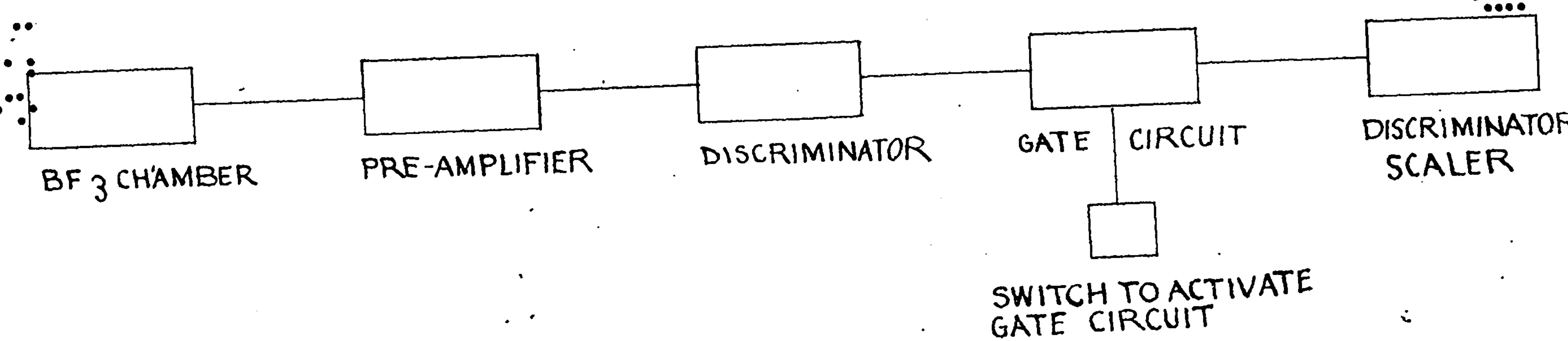
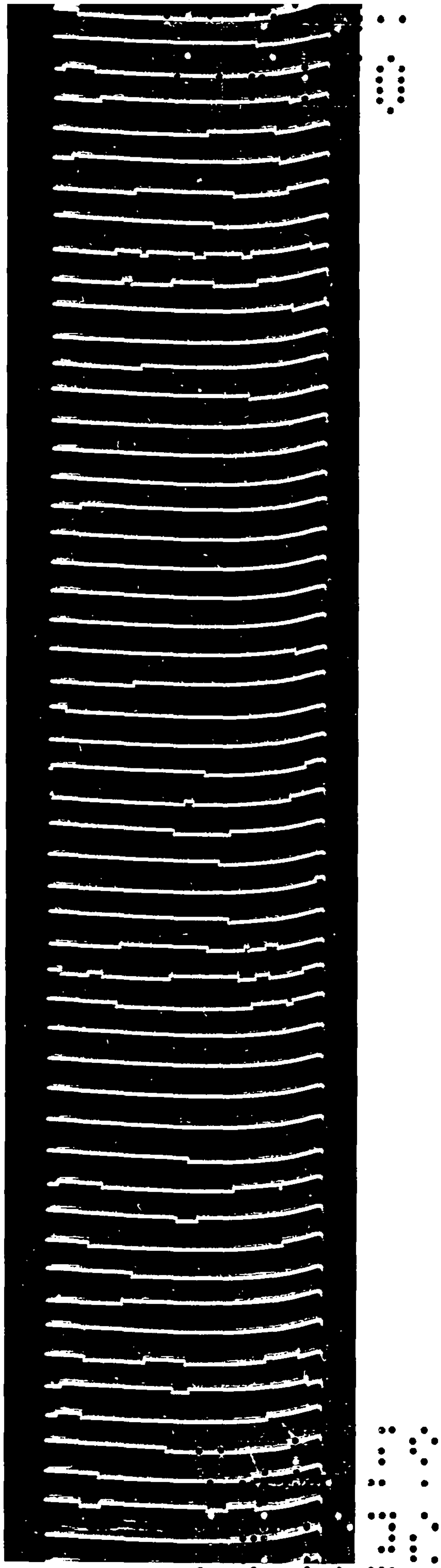


FIGURE NO.2





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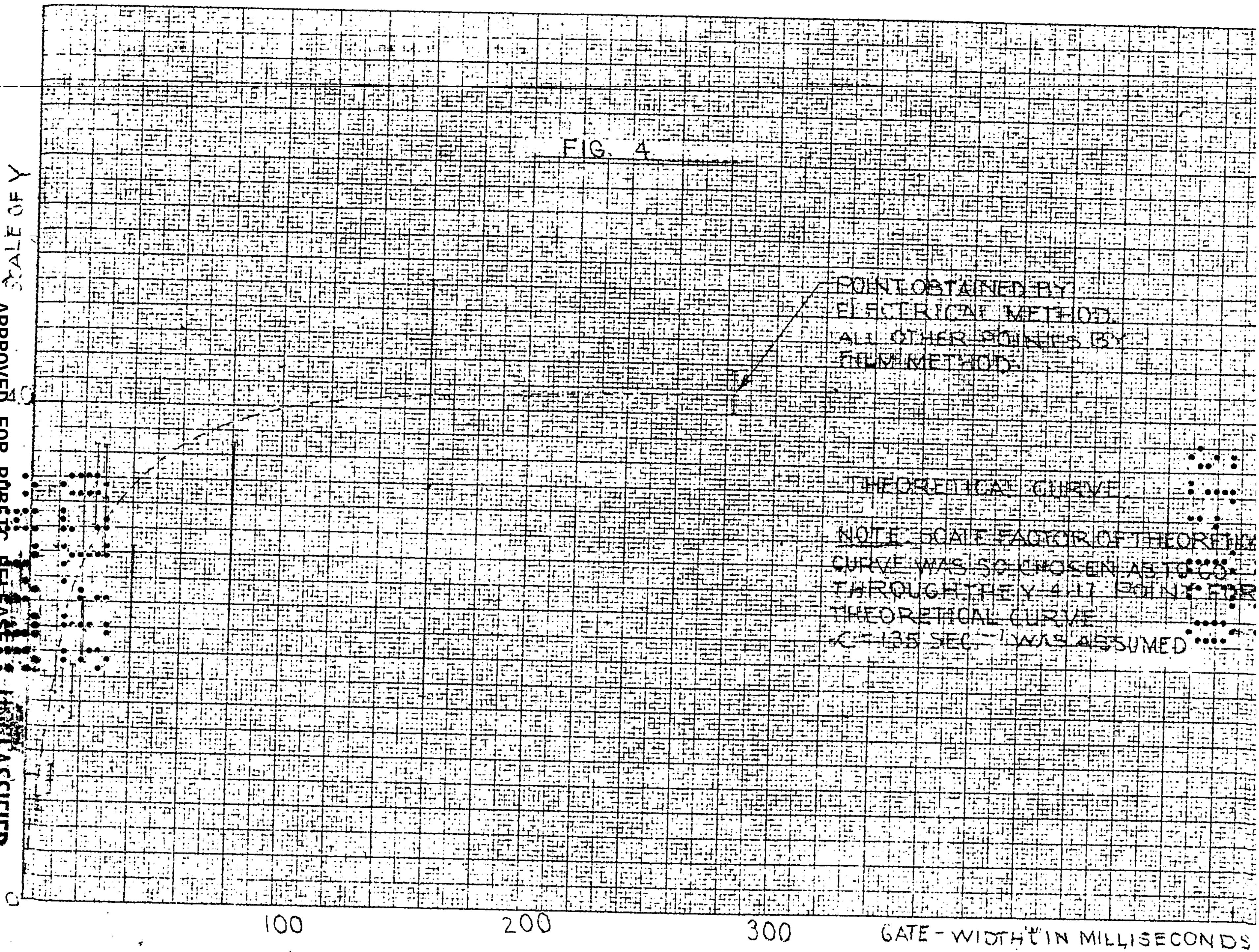
SCALE OF Y
10
0
100
200
300
GATE-WIDTH IN MILLISECONDS
POINT OBTAINED BY
ELECTRICAL METHOD.
ALL OTHER POINTS BY
FILM METHOD.
THEORETICAL CURVE
NOTE: SCALE FACTOR OF THEORETI
CURVE WAS SO CHOSEN AS TO GO
THROUGH THE Y 4.17 POINT. FOR
THEORETICAL CURVE
WAS ASSUMED

FIG. 4.

DOCUMENT ROOM

REC. FROM LD

DATE 10/31/44

REC. ____ NO. REC. 1

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