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LOS ALAMOS SCIENTIFIC LABORATORY
OF THE UNIVERSITY OF CALIFORNIA ○ LOS ALAMOS NEW MEXICO

INFINITE CONDUCTIVITY THEORY OF THE PINCH

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OF THE UNIVERSITY OF CALIFORNIA LOS ALAMOS NEW MEXICO

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INFINITE CONDUCTIVITY THEORY OF THE PINCH

Work done by:

R. Garwin
A. Rosenbluth
M. Rosenbluth

Report written by:

M. Rosenbluth

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ABSTRACT

With the assumption of infinite conductivity a simple model may be constructed for the dynamic construction of a current carrying plasma, i.e., the pinch effect. The magneto-hydrodynamic equations of this model are discussed and solved. It is also shown that the infinite conductivity model can be derived from a picture of particles orbiting without collisions in the fields set up by their motions.

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I. INTRODUCTION

The perhapsatron is a device designed to study the compression and heating of ionized plasma by electromagnetic means with the production of thermonuclear reactions as the ultimate goal. It is a long cylinder or torus with an applied axial electric field. This induces an axial current flow. The current interacts with its own magnetic field in such a way as to cause radial contraction--the so-called pinch effect.

The theoretical problems in this device are very difficult. In particular the problem of determining the conductivity of ionized materials in strong electric and magnetic fields is very difficult. In this paper we will assume infinite conductivity. This leads to a thin surface layer of current whose value is determined by inductive effects. The best justification for this approximation is that the inductive-limited current is small compared to the current calculated, for example, from the conductivity proportional to $T^{3/2}$ law, which is as reasonable a guess as one can make at the conductivity. Thus it seems fair to assume that it is primarily the inductance which limits the current. This condition is not well satisfied for small electric fields, for very confined pinches, for poorly ionized material, or if a steady-state is reached. In these cases the conductivity is not effectively infinite. However, for most cases of interest it should be a fair approximation during the implosive phase.

In Section II we will calculate the inductive equations for the current.

In Section V we will calculate in some detail the properties of the surface layer of current in the limit of no collisions. We will in particular show that strong fields exist at the surface layer. A circumferential magnetic field is set up by longitudinal motion of the electrons near the surface. This magnetic field reflects the electrons, but not the heavy ions. These, however, can not escape because when they attempt to escape a strong space-charge radial electric field draws them back. Finally we find that the crossed electric and magnetic fields give the electrons just the correct drift velocity to produce a current sufficient to reflect all the particles. Thus the conclusion is reached that the surface layer acts exactly like a hydrodynamic piston in reflecting all incident particles.

In Sections III and IV we discuss the hydrodynamic equations which result from reflection of the particles by the magnetic wall. This is a complex problem because the gas is not many mean-free paths thick. If it were, then a standard hydrodynamic calculation would be correct. Some calculations on this model have been made. If no collisions at all occur then particles reflected from the surface layer move radially inward, through the axis, and radially outward until they strike the far wall. Some consequences of this model are also discussed.

II. INDUCTIVE EQUATIONS

A cross-section of the torus is shown in Fig. 1.

The inner and outer circles are the copper conductor enclosing the torus. The shaded region represents the plasma. At the left we see the leads for the applied voltage which apply a potential V across the gap. We will now integrate the Maxwell Equation

$$\text{curl } \vec{\mathcal{E}} = -\frac{1}{c}\dot{\vec{H}} \tag{1}$$

around the dotted integration path. The x's represent H into the paper, the dots H out of the paper. The path runs along the inside of the copper, goes halfway around the torus, comes into the outer surface of the plasma, goes around this surface and closes itself at the midpoint of the potential gap.

Integrating the above equation we obtain

$$\oint \vec{E} \cdot d\vec{s} = -\frac{1}{c}\frac{\partial}{\partial t}\iint H \, dS \tag{2}$$

Note: We are using the electrostatic system of units throughout. Thus emf is in units of 300 volts, H is in gausses, I in $\frac{1}{3 \times 10^9}$ amperes.

Let us first evaluate the line integral. By symmetry the line integrals at the short ends cancel if we integrate to the center of the potential gap, thus eliminating the radial component of the fields at the gap. In fact these integrals are both zero.

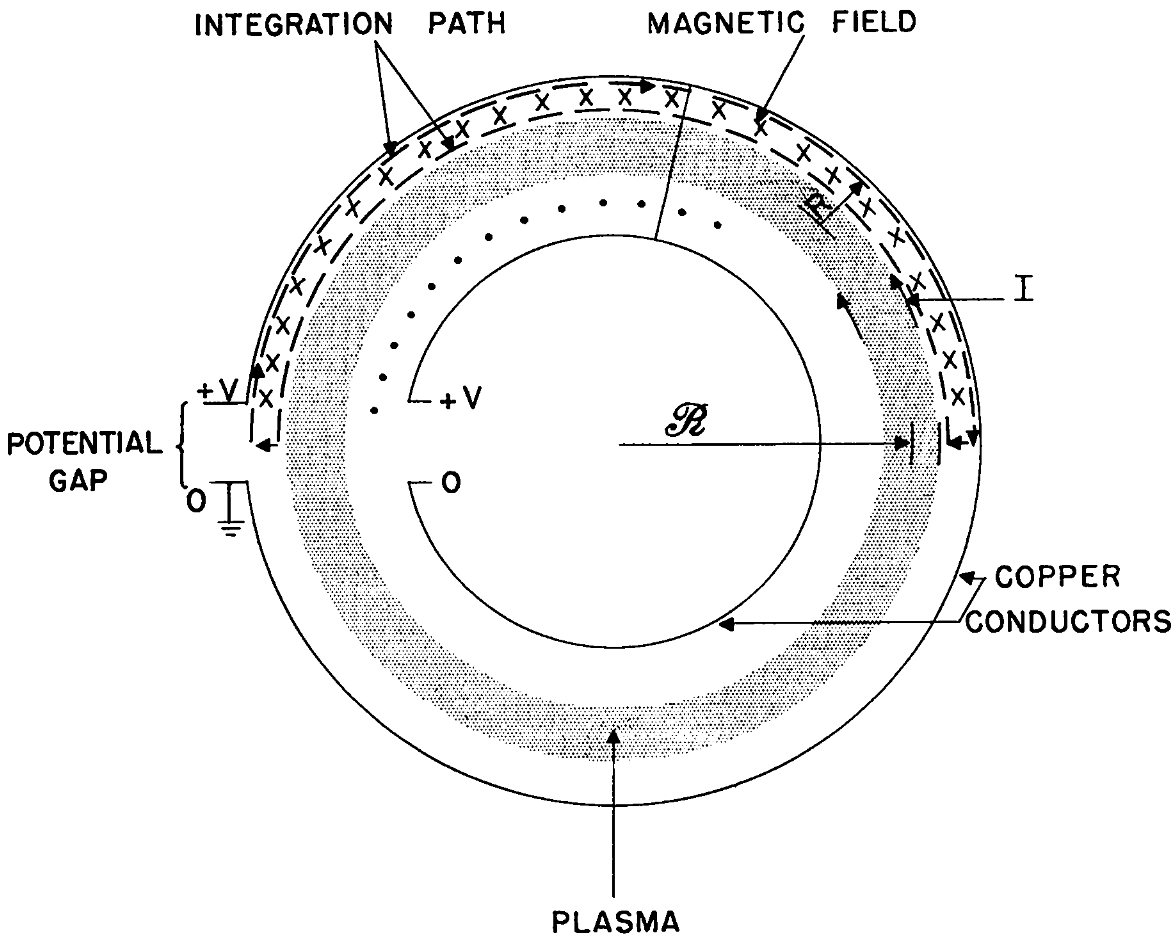

Fig. 1 Cross-section of the torus.

Integrating halfway across the gap we get a contribution 1/2. The rest of the integration around the copper gives nothing since the tangential electric field vanishes at a conductor surface. Finally if we call the field at the plasma surface $\mathcal{E}_s$ we obtain

$$\oint \vec{\mathcal{E}} \cdot d\vec{s} = \frac{V}{2} + \pi \mathcal{R}\, \mathcal{E}_s. \tag{3}$$

To evaluate the right hand side of the equation we use the Maxwell Equation

$$\text{curl } \vec{H} = \frac{4\pi \vec{j}}{c} . \tag{4}$$

This tells us that $H = \frac{2I}{cR}$ where I is the current carried by the plasma and R is the minor radius of a point in the torus. The integral of H may be written as

$$\iint H\, ds = \int dL \int_{R_o}^{\bar{R}} dR\, \frac{2I}{cR} = \pi \mathcal{R} \int_{R_o}^{\bar{R}} dr\, \frac{2I}{cR} \tag{5}$$

where R_o is the minor radius of the gas and $\bar{R}$ the minor radius of the copper.

We thus finally obtain from (2)

$$-\frac{V}{2\pi\mathcal{R}} = \mathcal{E}_s + \frac{2}{c^2}\, \ln \frac{\bar{R}}{R_o}\ \dot{I} \tag{6}$$

We will use the symbol $\mathcal{E}_o$ for $-\frac{V}{2\pi\mathcal{R}}$. It is the effective electric field per unit length of the torus. We still have one boundary condition to fulfill--that the electric field in the moving frame of the plasma must vanish, again because the plasma is assumed in this model to be a perfect conductor. So

$$\mathcal{E}' = \vec{\mathcal{E}} + \left(\frac{\vec{v}}{c} \times \vec{H}\right) = \mathcal{E}_s + \frac{\dot{R}_o H}{c}$$

or

$$\mathcal{E}_s = -\frac{2\dot{R}_o I}{c^2 R_o} \tag{7}$$

Substituting (7) into (6) we finally obtain

$$\mathcal{E}_o = \frac{\partial}{\partial t}\left[\frac{2I}{c^2}\ln\frac{\vec{R}}{R_o}\right]$$

or

$$I \ln\frac{\bar{R}}{R_o} = \frac{c^2}{2}\int_0^t \mathcal{E}_o \, dt \tag{8}$$

It will be noted that equation (8) is of the correct form for a purely inductive system. It tells us how to relate the current and plasma radius to the applied voltage.

III. SNOW-PLOW MODEL

We must now consider the hydrodynamic equations. They are linked to the inductive eouation by the requirement that the material pressure at the surface must balance the magnetic pressure; in other words, that the momentum transfer to the gas is balanced by the force which the magnetic field produces on the current.

Thus we have

$$P_s = \frac{H_o^2}{8\pi} = \frac{I^2}{2\pi c^2 R_o^2} \tag{9}$$

where P_s is the material pressure which serves as the boundary condition for the hydrodynamic problem.

As we have mentioned before several hydrodynamic models are possible. There is the usual hydrodynamic model where (9) serves as a boundary condition for a standard type implosion. There is the free-particle model where the atoms are assumed to bounce freely back and forth between the incoming magnetic walls. Some standard hydrodynamic implosions have been run by A. Rosenbluth and will be discussed later. Petschek and Goad are considering the free-particle model.

However, the simplest hydrodynamic model, the only one not requiring complex equations and machine integrations, is the snow-plow model. In this we assume that all material which has been swept up by the magnetic piston is piled up in a very thin layer at the piston and travels in with it. This of course gives no information about compressions but is otherwise reasonably accurate until the arrival of the back-shock from the center. We now write the momentum equation for the snow-plow.

$$\frac{d}{dt} M \dot{R}_o = -2 \pi R_o P_s \tag{10}$$

where M is the mass per unit length swept up by the snow-plow. In more detail

$$\frac{d}{dt} \left\{ \pi \rho_o \left[\bar{R}^2 - R_o^2 \right] \dot{R}_o \right\} = - 2 \pi R_o P_s \tag{11}$$

Here ρ_o is the initial gas density. We have assumed that initially the outer radius of the gas is at the radius of the outer conductor, $\bar{R}$. Substituting (8) and (9) we get

$$\frac{d}{dt} \left\{ \left(\bar{R}^2 - R_o^2 \right) \dot{R}_o \right\} = - \frac{c^2 \left(\int \mathcal{E}_o \, dt \right)^2}{4 \pi \rho_o R_o \ln^2 \frac{\bar{R}}{R_o}} \tag{12}$$

Thus we have an ordinary differential equation for the behavior of the pinch. It may easily be reduced to dimensionless form. If, for example, $\mathcal{E}_o$ is constant in time, then we may substitute

$$\eta = \frac{R_o}{\bar{R}} \; ; \; \tau = \frac{t \left\{ \sqrt[4]{\frac{c^2 \mathcal{E}_o^2}{4 \pi \rho_o}} \right\}}{\bar{R}} \tag{13}$$

$$\frac{d^2}{d\tau^2}\left(\eta - \frac{\eta^3}{3}\right) = -\frac{\tau^2}{\eta[b - \ln \eta]^2} \tag{14}$$

Equation (14) has been modified to allow for the possibility that the initial radius of the gas is less than that of the copper conductor. This makes the initial inductance non-zero. The parameter b is the logarithm of the ratio of copper radius to initial gas radius. The scaling laws apply for a given value of b. In practice, because of the glass containing tube, b is about .1.

Equation (14) has been numerically integrated by C. Kazek. The results are shown in Fig. 2 for various values of b.

Equation (13) gives the scaling laws for the system. It should be noted that the same scaling applies for any hydrodynamic model (though not if mixed models are needed--i.e. hydrodynamic for high density, free-particle for low density). The experimental agreement with (14) is only fair. Quantities seem correctly given within about a factor 2. In particular, the fourth-root dependence of velocity on density does not agree well with the experiment which indicates more of a square-root law. However, it must be noted that the quantity ρ_o is the density of <u>ionized material</u>, not the overall density, and the degree of ionization is not well-known. In fact there are good reasons to expect that it is small and variable. The presence of neutral material complicates the problem, for it may make some collisions with the ionized material, thus receiving some energy and momentum, but eventually leaking through the magnetic walls.

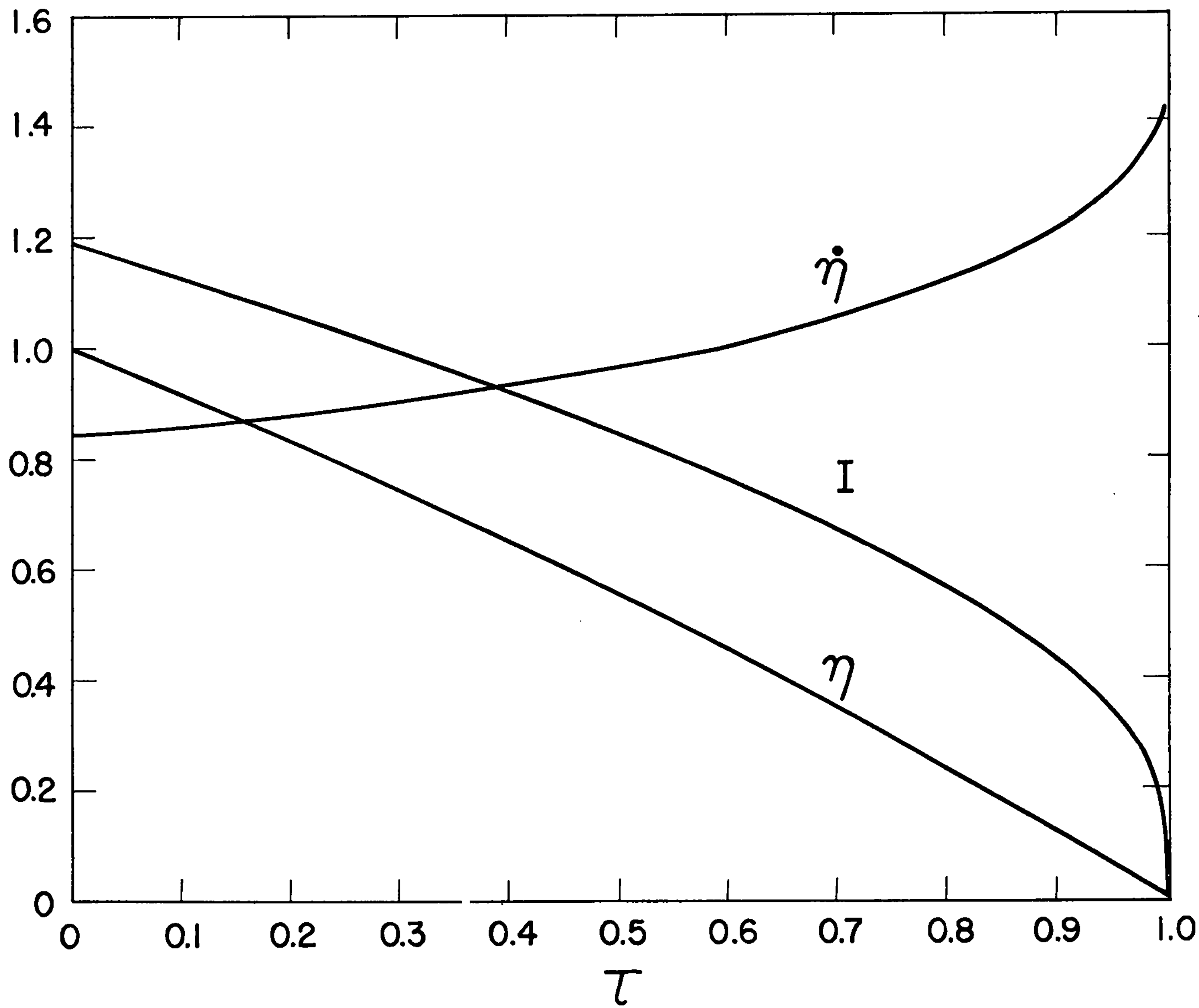

Fig. 2a Snow-plow solution of pinch equations for b = 0.

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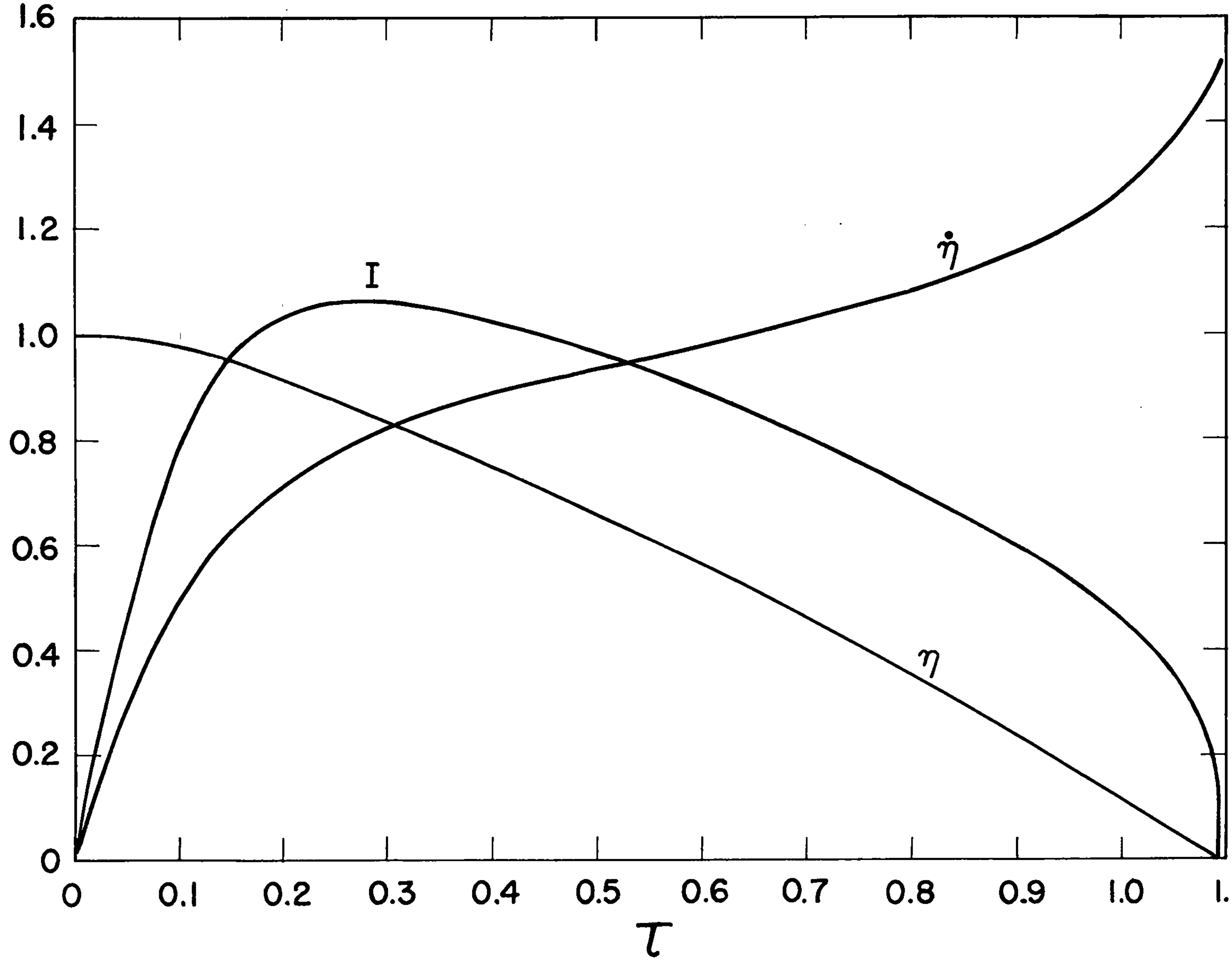

Fig. 2b Snow-plow solution of pinch equations for b = .1.

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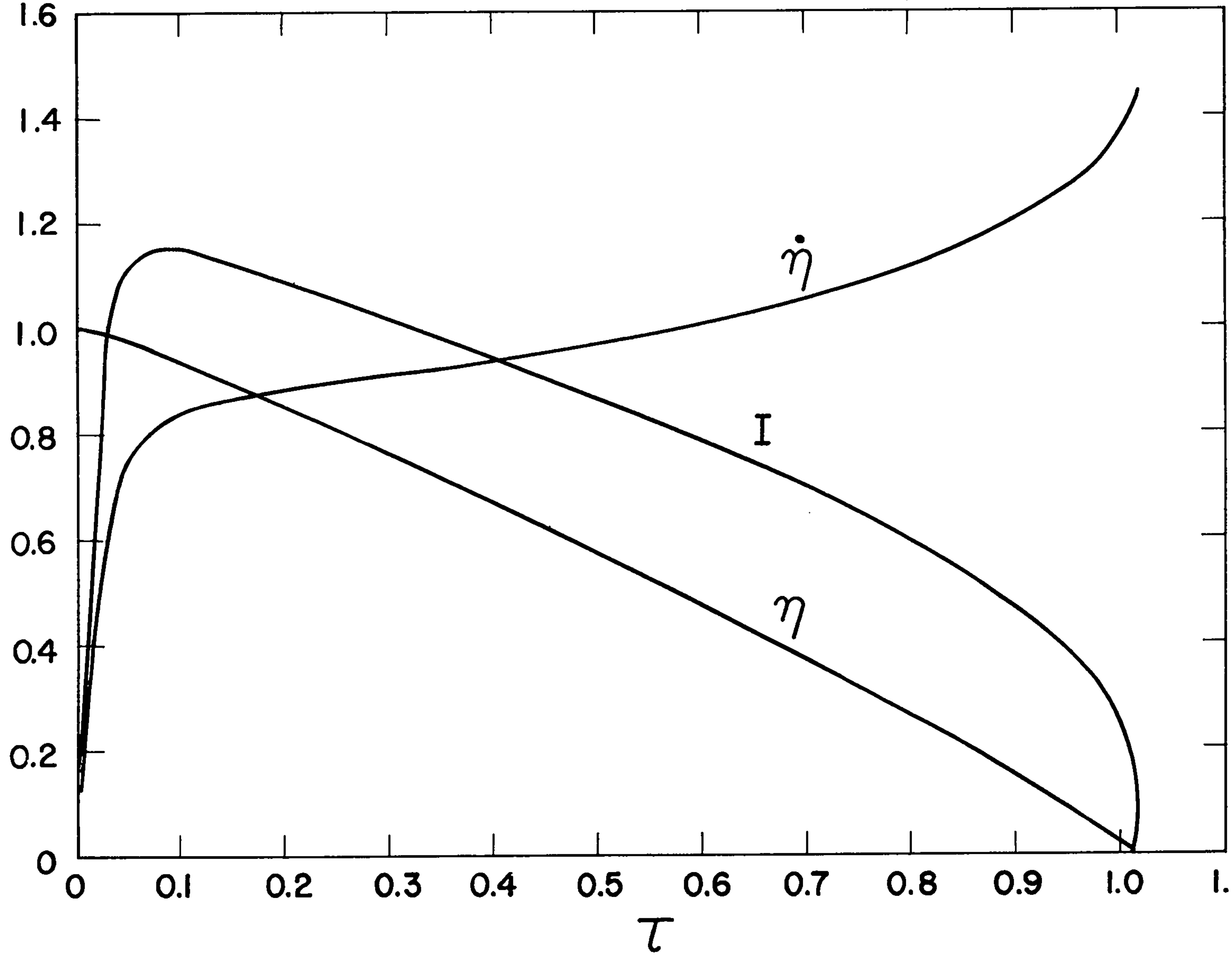

Fig. 2c Snow-plow solution of pinch equations for b = .2.

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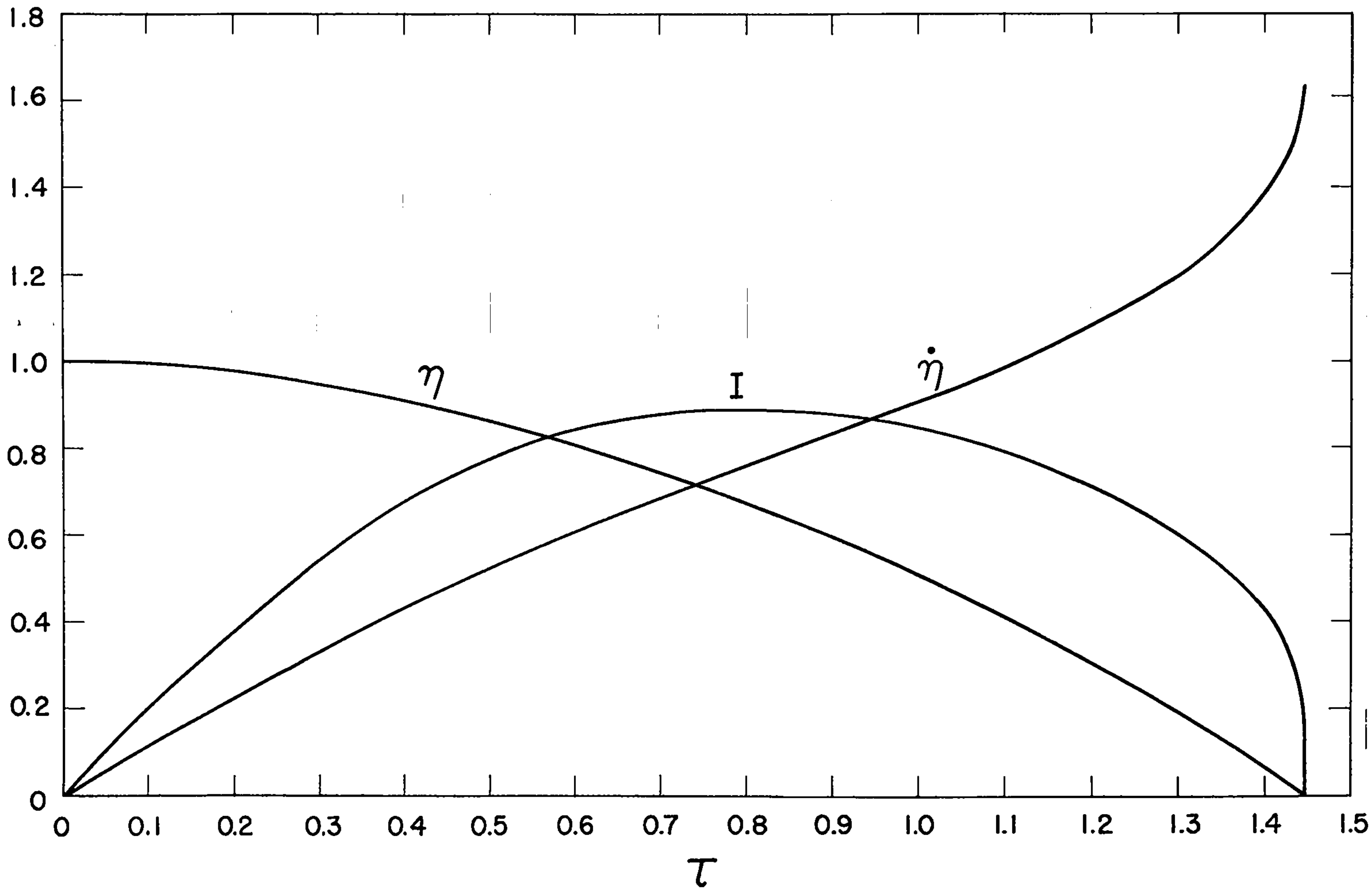

Fig. 2d Snow-plow solution of pinch equations for b = .5.

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Comparing the orders of magnitude of (13) with the current allowed by the $T^{3/2}$ conductivity, one can find for completely ionized material with only Rutherford Scattering a rough criterion for the validity of the infinite conductivity approximation during the implosive phase:

$$10^{-6} \gg \frac{\mathcal{E}_o^{-2} R_o^{-1} \rho_o}{A^{3/2}} \qquad (14')$$

where A is the ion atomic mass number.

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IV. MORE COMPLEX HYDRODYNAMIC MODELS

As has been mentioned earlier, if there are many mean-free paths of material, the particles reflected from the magnetic walls will make scatterings with the other particles ahead of the wall and share their energy and momentum. In this case we have a standard implosion problem with the magnetic wall acting like a piston with the boundary condition given by (8) and (9). It will be shown later that it is reasonable to assume that most of the energy is in thermal motion so that a γ of 5/3 is used.

Several such implosions have been run by A. Rosenbluth as shown in Figs. 3, 4, and 5. Qualitatively the sequence of events is that a first shock develops and in the early stages we have a behavior much like the snow-plow model. Then, however, a back-shock turns the surface around and it bounces outwards. After a time it turns around again and another shock develops. As can be seen from Fig. 3 these wiggles are dying down fairly rapidly and the radius asymptotically goes to zero. For the constant $\mathcal{E}_o$ case the density at the first maximum is about 30, at the fourth maximum about 100.

Unfortunately the happy achievement of infinite density can not be attained in practice since instabilities seem to disrupt the pinch after about the first shock. It should be mentioned that the scaling laws (13) can be used to relate this problem to other cases with different $\mathcal{E}_o$ and ρ_o. Those particular laws, of course, apply only

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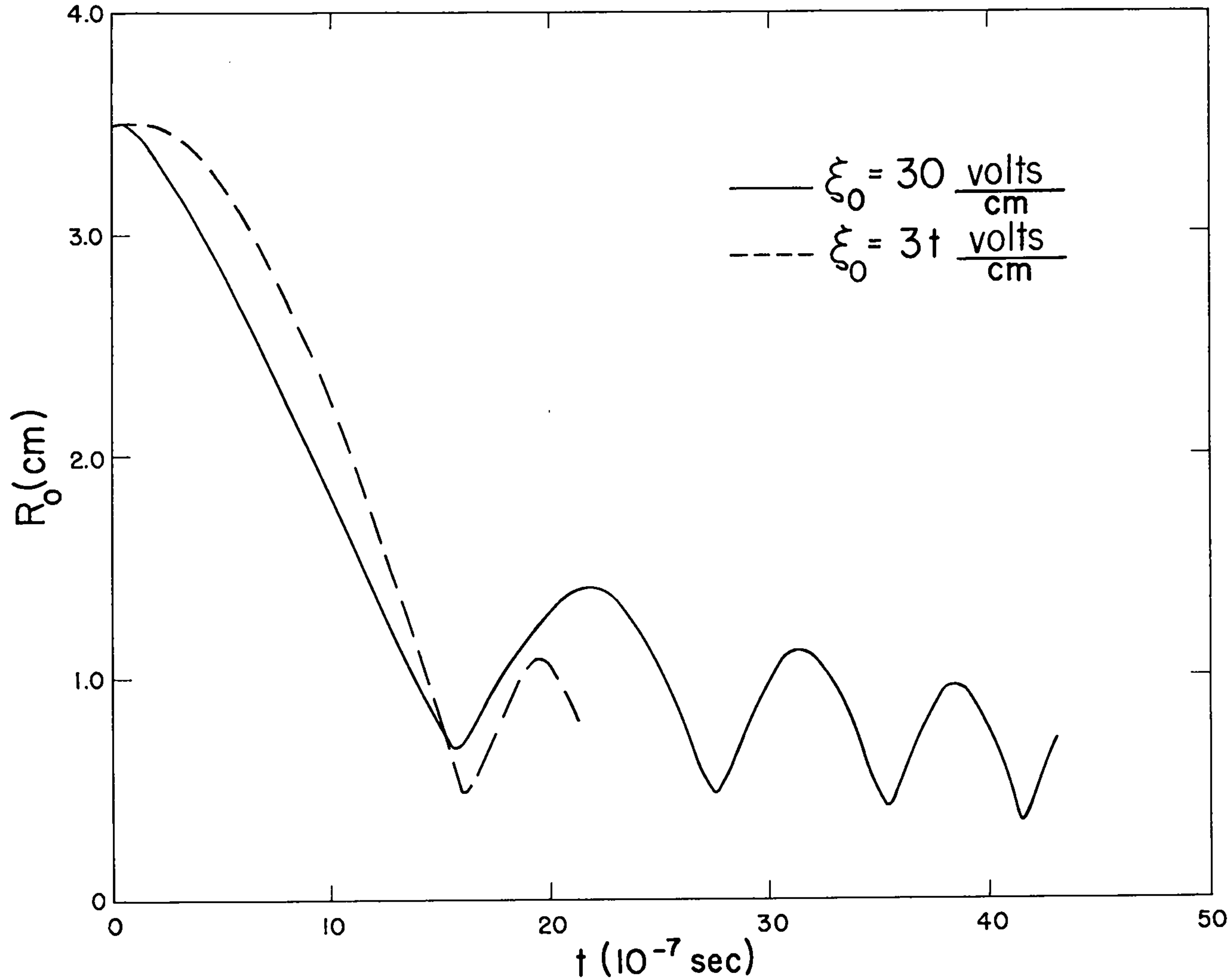

Fig. 3 Hydrodynamic solution of pinch equations. Radius as a function of time for the two values of ξ_o indicated. In both problems $\rho_o = 2.041 \times 10^{-8}$ gms/cc; radius of conductor = 3.86 cm.

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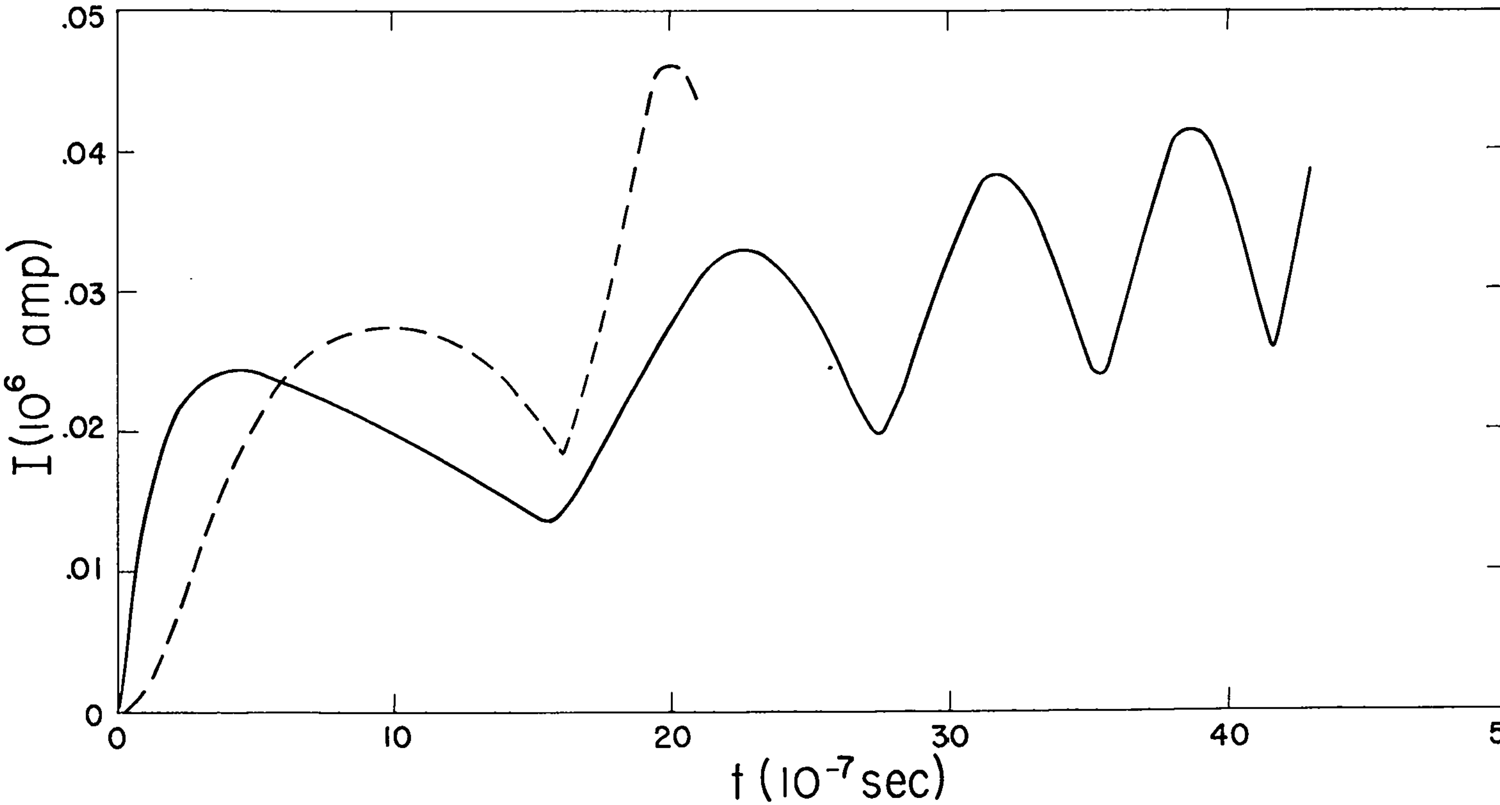

Fig. 4 Current as a function of time for the two problems described in Fig. 3.

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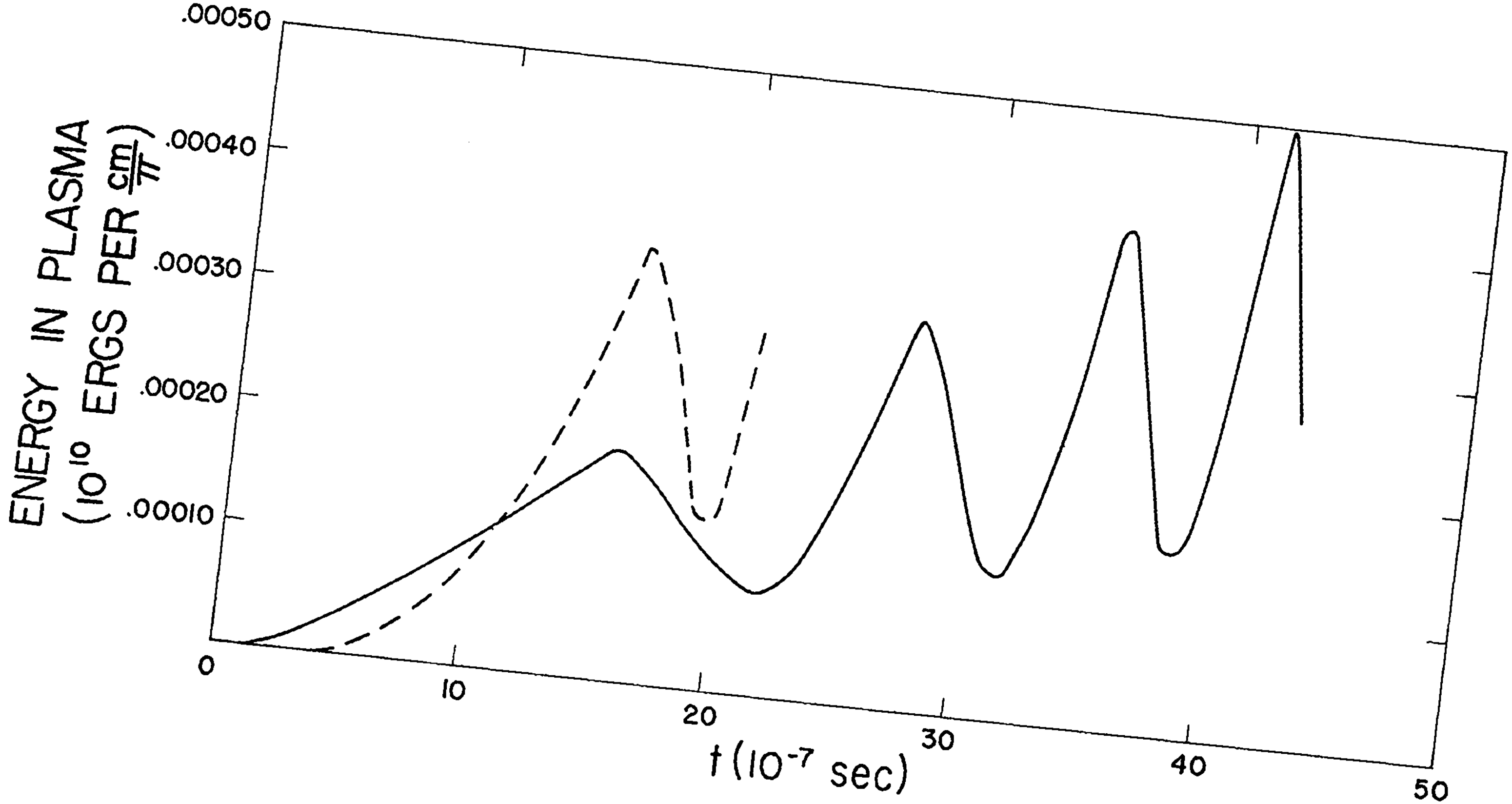

Fig. 5. Energy in plasma as a function of time for the two problems described in Fig. 3.

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to $\mathcal{E}_o$ constant in time and a given ratio of initial gas radius to conductor radius.

It is also planned to run a problem with an alternating r.f. $\mathcal{E}_o$.

The opposite extreme to the hydrodynamic model, where thermal equilibrium is assumed ahead of the piston, is the free-particle model. In this model the particles are assumed to make no collisions and hence move freely except when being reflected from the magnetic wall. The condition of reflection is that, in the frame of reference where the wall is stationary, the component of particle velocity normal to the wall is reversed in sign upon reflection. In this model all particles move purely radially. For a while the wall reflects only particles at rest, but eventually a wave of particles which have been once reflected will hit the opposite wall. This is analagous to a hydrodynamic back shock. Petschek has integrated the momentum equation for times prior to the back shock and has shown in the constant $\mathcal{E}_o$ case that the back shock occurs at density about 16 times initial density. A plot of a complete free-particle implosion would look very similar to Figs. 3, 4, and 5. One can show that after many waves of particles are involved the free-particle gas acts like a material of γ equals 3.

One interesting feature of the free-particle model is that all particles must pass through the axis, creating a high local density there. One can also show that if the applied $\mathcal{E}_o$ is varied so as to make the current constant in time, then all particles make their first traversal of the axis at the same time, thus creating a super-pinch. This might be achieved by putting a large inductance in series with the system. Pure hydrogen or deuterium at high temperatures should behave like a free-particle gas.

V. SURFACE LAYER MODEL IN THE LIMIT OF NO COLLISIONS

In order to understand the mechanism by which the current is carried and the particles are reflected at the surface, it is instructive to look at the microscopic conditions in the surface layer. We will assume the layer is thin so we can use the plane approximation. We will also assume, to make the algebra easier, that all particles ahead of the layer are at rest, so that in the frame in which the layer is stationary all particles ahead of the layer move into it with a constant velocity, which we will call V. Finally we must assume that in this stationary frame of reference the longitudinal electric field is zero. As we have already remarked, this is the condition for an infinitely conducting medium, and it is easy to show that no static solution can exist in the presence of a longitudinal electric field.

The conditions in the layer are shown in Fig. 6 in a picture of particle orbits. Here x is the direction of a minor radius of the torus, y is along the axis of the torus.

At large negative values of x all particles travel with a velocity V. As they come into the surface layer they are deflected by

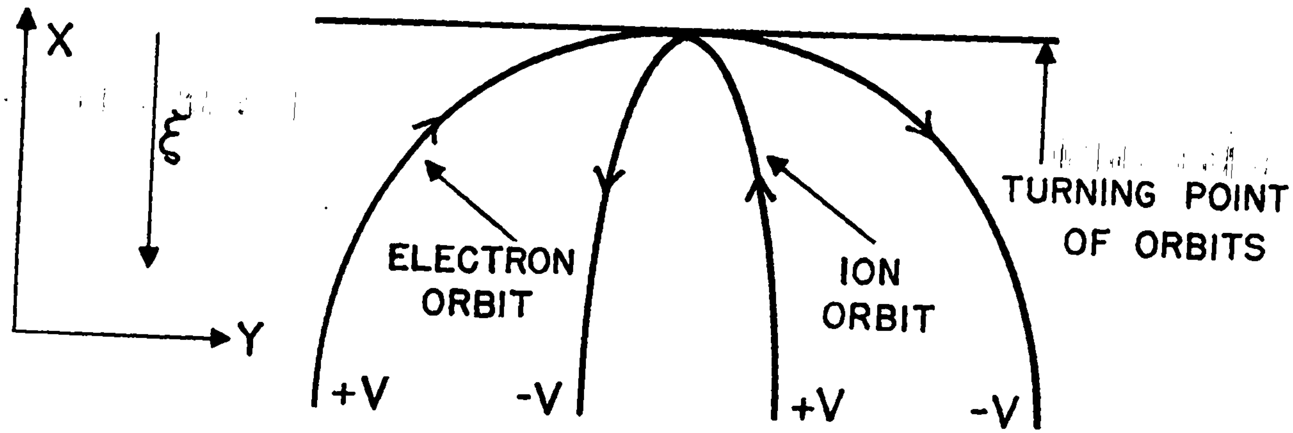

Fig. 6 Qualitative picture of orbits and fields in surface layer.

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the fields in the layer and perform an orbit finally being reflected with a velocity V. In the layer the electric field is in the negative x direction. The electric field arises from charge separation. The magnetic field is in the z direction. It arises from the y component of velocity in the electron orbits. All particles of a given kind perform the same orbit. We assume two kinds of particles, electrons and singly charged heavy ions.

There are no forces acting in the z direction so there will be no motion in the z direction, and the system is assumed infinite in the y direction so all quantities depend only on x.

Our procedure will be to solve for the particle orbits with arbitrary $\mathcal{E}$ and H. Having found the orbits, we know the current and charge densities, hence can solve Maxwell's Equations for the fields. In this way a completely self-consistent solution is obtained so that we have calculated the mechanism of conductivity and reflection.

First we unite the equations of motion for the particles

$$\frac{d^2x}{dt^2} = \frac{e}{m}\left[\mathcal{E} + \frac{H}{c}\frac{dy}{dt}\right]$$

$$\frac{d^2y}{dt^2} = -\frac{eH}{mc}\frac{dx}{dt} \tag{15}$$

We now eliminate the time and use x as the independent variable. We let

$$p = \frac{dx}{dt} \quad ; \quad q = \frac{dy}{dt}$$

Then we can write

$$p \frac{dp}{dx} = \frac{e}{m} \left[\mathcal{E} + q \frac{H}{c} \right] \qquad \text{(a)}$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(16)}$$
$$p \frac{dq}{dx} = - \frac{eH}{mc} p \qquad \text{(b)}$$

(16b) can be immediately integrated to give

$$q = - \frac{e}{mc} \int H \, dx. \qquad \text{(17a)}$$

Substituting into (16a) and integrating we obtain

$$p = \pm \sqrt{2 \frac{e}{m} \int \mathcal{E} \, dx - \left(\frac{e}{mc} \right)^2 \left[\int H \, dx \right]^2 + V^2} \qquad \text{(17b)}$$

The solution (17) obeys the restriction that at large negative x, where the field strengths are small, the y velocity is zero and the x velocity is V. The ± sign on (17b) refers to the fact that after the turning point of the orbit the x velocities are negative. The orbits are, of course, symmetric about the turning point.

To calculate the number density we use the continuity equation. The continuity equation will hold even if we only apply it to the density of particles which have not yet turned around. To see this suppose that there were a sink of particles at the turning point. We would still have a static situation since a constant flux of particles is coming into the surface layer.

Thus

$$\operatorname{div} (N_i \vec{v}) = 0$$

where N_i is the number density of particles that have not yet been reflected and $\vec{v}$ is their vector velocity. Since there is no variation with y or z we have

$$\frac{\partial}{\partial x} (N_i \ p) = 0$$

or

$$N_i = \frac{N_o \ V}{|p|}$$

where N_o is the initial number density ahead of the layer. Finally since the number of reflected particles equals the number of incoming particles we have:

$$N \ (x) = \frac{2 \ N_o \ V}{|p \ (x)|} \tag{18}$$

Equation (18) of course applies to either electrons or ions but with the appropriate value of e and m inserted into the expression (17b) for p.

Next we need an expression for the current density. We see from (17a) that the heavy ions receive almost no y component of velocity so we need consider only electron current. We may write therefore:

$$\vec{j} = N_e \ e \ q = - 2 \frac{N_o \ Ve^2 \int H \ dx}{mc \ p} \tag{19}$$

Now from Maxwell's Equations we may write our final equations which must be solved for self-consistent $\mathcal{E}$ and H. Here we will use m for electron mass, M for ion mass, e for ion charge, - e for electron charge.

From

$$\nabla \times \vec{H} = \frac{4\pi \vec{j}}{c} \quad \text{we have}$$

$$\frac{dH}{dx} = \frac{8\pi N_o e^2 v \int H\,dx}{mc^2 \sqrt{-2\frac{e}{m}\int \mathcal{E} dx - \left(\frac{e}{mc}\right)^2 \left[\int H\,dx\right]^2 + v^2}} \tag{20}$$

and from

$$\nabla \cdot \vec{\mathcal{E}} = 4\pi\rho = 4\pi e \,(N_i - N_e) \tag{21a}$$

$$\frac{d\mathcal{E}}{dx} = 8\pi N_o v_e \left\{ \frac{1}{\sqrt{2\frac{e}{M}\int \mathcal{E} dx - \left(\frac{e}{Mc}\right)^2 \left(\int H\,dx\right)^2 + v^2}} - \frac{1}{\sqrt{-2\frac{e}{m}\int \mathcal{E} dx - \left(\frac{e}{mc}\right)^2 \left(\int H\,dx\right)^2 + v^2}} \right\} \tag{21b}$$

A complete program would now be to solve these coupled equations for $\mathcal{E}$ and H. This would represent a self-consistent solution.

A complete solution would be a horrible thing to try. We can simplify matters by making the following assumption: That the charge separation is small, i.e., that at all points the density of electrons and ions is nearly equal. After making this assumption we can carry through the integration, obtain $\mathcal{E}$ and use (21) to see whether the assumption is self-consistent. Thus we must equate N_e and N_i. Doing so we obtain

$$- 2 \frac{e}{m} \int \mathcal{E}\, dx - \left(\frac{e}{mc}\right)^2 \left[\int H\, dx\right]^2 = 2 \frac{e}{M} \int \mathcal{E}\, dx - \left(\frac{e}{Mc}\right)^2 \left(\int H\, dx\right)^2 \tag{22}$$

Solving we obtain

$$\int \mathcal{E}\, dx = - \frac{\left[\left(\frac{e}{mc}\right)^2 - \left(\frac{e}{Mc}\right)^2\right]\left[\int H\, dx\right]^2}{2\left(\frac{e}{m} + \frac{e}{M}\right)} \tag{23}$$

Neglecting terms of order $\frac{m}{M}$ we get

$$\int \mathcal{E}\, dx = - \frac{e}{2\ mc^2}\left[\int H\, dx\right]^2 \tag{24}$$

Substituting (24) into (22) and neglecting terms of order $\frac{m}{M}$

$$- 2 \frac{e}{m} \int \mathcal{E}\, dx - \left(\frac{e}{mc}\right)^2 \left[\int H\, dx\right]^2 = - \frac{e^2}{M\ mc^2} \left(\int H\, dx\right)^2$$

Finally we substitute into (20) and obtain

$$\frac{\partial H}{\partial x} = \frac{8 \pi N_o e^2 V}{mc^2} \frac{\int H\, dx}{\sqrt{V^2 - \frac{e^2}{M\ mc^2}\left[\int H\, dx\right]^2}} \tag{25}$$

We put (25) in dimensionless form by substituting

$$H = H' \sqrt{8\pi N_o M V^2}$$

$$x = x' \sqrt{\frac{mc^2}{8\pi N_o e^2}} \qquad (26)$$

$$\chi = \int H' \, dx'$$

This yields

$$\frac{d^2 \chi}{d x'^2} = \frac{\chi}{\sqrt{1 - \chi^2}} \qquad (27)$$

We can integrate by writing $\phi = \frac{d\chi}{d x'}$ so

$$\phi \frac{d\phi}{d\chi} = \frac{\chi}{\sqrt{1 - \chi^2}}$$

$$\phi^2 = -2\sqrt{1 - \chi^2} + \text{Const}$$

At $x' = -\infty$ we want both $\int H\,dx$ and H to vanish so the constant must be chosen so that $\phi = 0$ when χ vanishes.

Thus we have

$$\frac{d\chi}{d x'} = \sqrt{2} \sqrt{1 - \sqrt{1 - \chi^2}} \qquad (28)$$

(28) is an elementary quadrature so after half an hour with Pierce we find

$$x' = \ln \tan \left(\frac{1}{4} \sin^{-1} \chi\right) + 2 \cos \left(\frac{1}{2} \sin^{-1} \chi\right) - .532 \qquad (29)$$

The maximum value of χ is 1. This corresponds to the turning point of the orbits. The constant of integration in (29) has been chosen so that $\chi = 1$ at $x = 0$ and the plasma extends into the infinite negative half-plane.

Using (29), (26), (24), and (18) we can write down the solution for all quantities of interest.

$$H' = \frac{H}{\sqrt{8\pi N_o M V^2}} = 2 \sin \left(\frac{1}{2} \sin^{-1} \chi\right) \qquad (30)$$

$$\mathcal{E}' = - \frac{\mathcal{E}}{\sqrt{\dfrac{8\pi N_o}{mc^2} M V^2}} = \chi H' \qquad (31)$$

$$N = \frac{2 N_o}{\sqrt{1 - \chi^2}} \qquad (32)$$

These quantities have been evaluated numerically and are shown in Fig. 7.

First let us check our assumption that the densities of electrons and ions are nearly equal at all points. We have seen from (26)

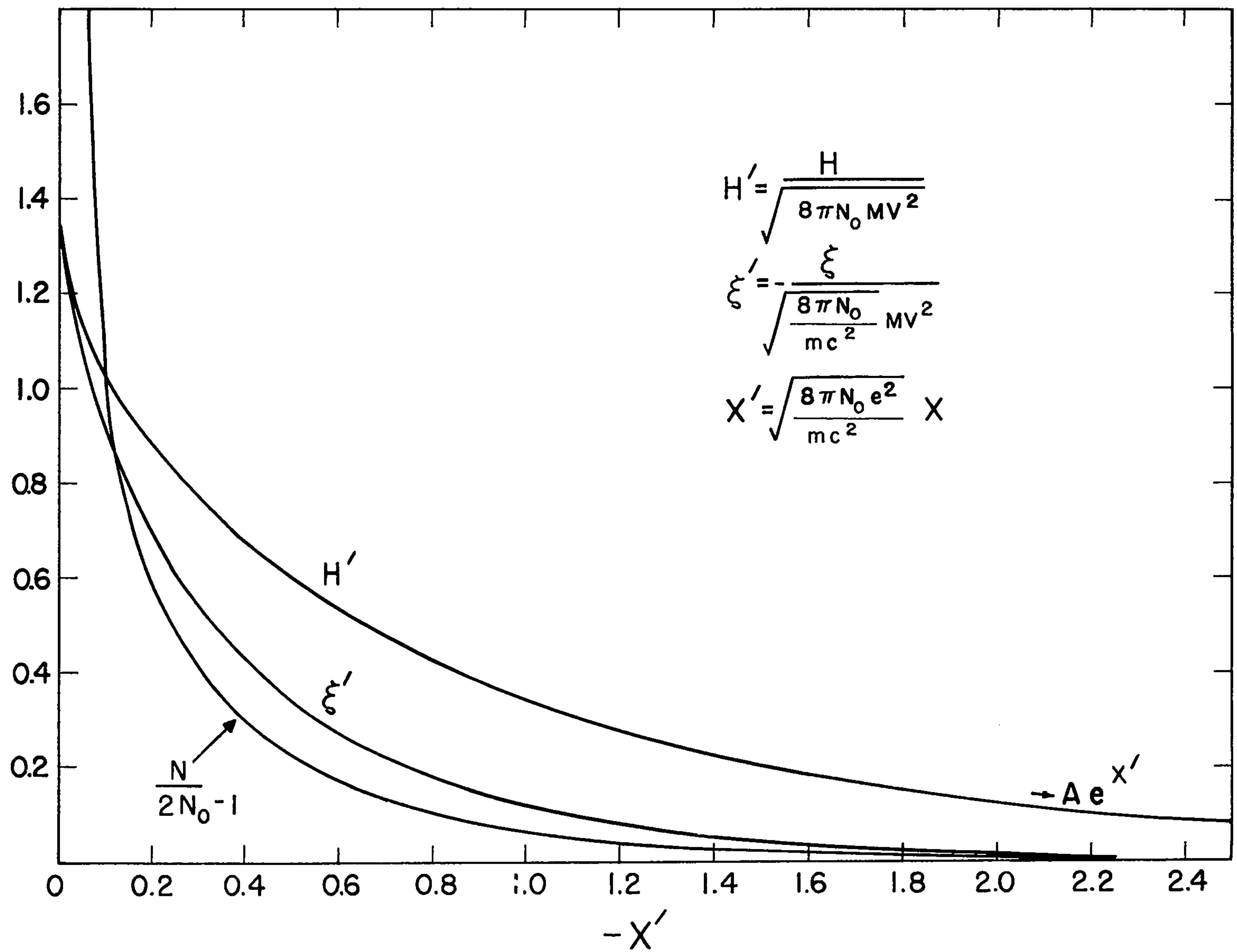

Fig. 7 Fields and density as functions of depth in surface layer.

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that the surface layer extends roughly over a distance

$$d = \sqrt{\frac{mc^2}{8\pi N_o e^2}} \tag{33}$$

We can write an approximate solution of (21a) in the form:

$$\xi = 4\pi N_o e d f \tag{34}$$

Here f represents the average fractional excess of one sign of charge. Equating (34) and (31)

$$f = -\frac{\sqrt{\frac{8\pi N_o}{mc^2}}\, M V^2}{4\pi N_o e \sqrt{\frac{mc^2}{8\pi N_o e^2}}} \approx -2\frac{M V^2}{mc^2} \tag{35}$$

Thus the charge separation is very small, being of order of magnitude the ratio of the "temperature" to the electron rest energy. Hence the solution is consistent with the assumption.

Another point to be noted here is that the electric field should vanish at the edge of the layer, instead of reaching its maximum value there. What happens of course is that the turning point of the ions is actually a distance f d beyond the electron turning point. Thus on the graph ξ' should actually drop to zero between $x' = -.001$ and zero.

Equation (33) gives us a measure of the thickness of the surface layer. In cases of interest it is order of magnitude 1 mm thick. Thus the approximation of treating it as infinitely thin should be good. One exception should be noted. If indeed the degree of ionization is small or the density very low, then the surface layer may become quite thick.

Next we may note from (30) that at X = 0 we have

$$\frac{H^2}{8\pi} = 2 N_o M V^2 \qquad (36)$$

The left-hand side is the magnetic pressure. The right-hand side is precisely the momentum which is imparted to the ions in reflecting them. Thus our solution satisfies momentum conservation.

Finally it should be noted that since the layer reflects both electrons and ions with the same velocity, almost all energy is in the ions. Thus the effective ion temperature is very large compared to the electron temperature. In fact, it is only the current carrying electrons near their turning points which have energy comparable to the ions. It is also difficult for the electrons to pick up energy from the ions due to the large mass ratio. Probably the easiest mechanism is for a high energy electron near its turning point to be scattered back into the plasma. An important corollary of low electron energy is that it is difficult for ionization to occur despite the high kinetic temperature of the ions, since the degree of ionization depends on electron temperature. This prediction of the theory is well borne out by experimental evidence.

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