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MISSION ANALYSIS FOR NUCLEAR PROPULSION

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MISSION ANALYSIS FOR NUCLEAR PROPULSION

by

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ABSTRACT

This report, originally prepared for a UCLA Short Course on Advances in Space Propulsion, presents elementary methods for determining the performance of high thrust rockets with emphasis on nuclear propulsion. Simple models are used to describe the vehicles and to evaluate the mission requirements. Performance comparisons, both in general and for specific missions, are made of various propulsion systems using typical values for the system parameters.

CONTENTS

Introduction

Although the over-all optimization of a space vehicle requires detailed knowledge and integration of all the subsystems and flight objectives, a simple model can serve to give a good estimate of the performance as well as an understanding of the significance of the vehicle parameters. The simple models are particularly suited to preliminary design and advanced systems where the parameters (e.g., engine weight and specific impulse) are not well known and to comparisons among different classes of propulsion systems. We shall make simplifying assumptions concerning the vehicle components and concerning the flight kinetics and will point out their limitations and more detailed methods.

Vehicle Description and Performance

In field free space, we can apply the law of conservation of momentum to a rocket of mass m, which ejects a small mass dm at an exhaust velocity v_e with respect to the vehicle. Using V for the vehicle velocity, we have

$$v_e dm = -mdV \tag{1}$$

Integrating

$$\Delta V(t) = V(T) - V_o = -v_e \int_{m_o}^{m(t)} \frac{dm}{m}$$

$$= v_e \, \ell n \frac{m_o}{m(t)} \, . \qquad (2)$$

Note that this is independent of the rate of acceleration (or thrust program). By specifying the thrust program, one can integrate further to get the distance (x)t; in particular, for the common case of constant $dm/dt = \alpha$

$$V(t) = V_o + v_e \, \ell n \frac{m_o}{m_o - \alpha t} \qquad (3)$$

which leads to

$$X(t) = X_o + V_o t + v_e t \left[1 - \left(\frac{m(t)}{m_o - m(t)} \right) \ell n \frac{m_o}{m(t)} \right] . \qquad (4)$$

If we consider the burnout time when the thrust ceases, Eq. (2) gives the familiar rocket equation

$$\Delta V = v_e \, \ell n \frac{M_o}{M_b}$$

$$= v_e \, \ell n \, R . \qquad (5)$$

The ratio M_o/M_b is termed the "mass ratio," R. The burnout mass includes engine, tankage, etc., in addition to the payload. For simplicity, we shall divide this "dead" weight into two categories labeled M_e and M_t. M_t will represent the tank mass which is considered separately for several reasons. We shall be interested in the effects of dropping useless tankage and in carrying fuel to orbit for refuel purposes. M_t will be assumed to be a fixed fraction of the propellant mass M_p, which is true for sufficiently large tanks and accurate enough for our purpose. Thus

$$f = \frac{M_t}{M_p} . \tag{6}$$

Since $M_o = M_b + M_p$

$$M_t = f(M_o - M_b) = f(1 - \frac{1}{R})M_o . \tag{7}$$

The remainder of the dead weight, including engine, structure, guidance and control, holdover propellant, etc., will be lumped into M_e and assumed to be proportional to the gross vehicle weight (including upper stages and payload):

$$\epsilon = \frac{M_e}{M_o} . \tag{8}$$

The payload M_L is thus given by

$$M_L = M_b - M_t - M_e \tag{9}$$

$$= M_o \left[\frac{1 + f}{R} - \epsilon - f \right] . \tag{10}$$

Letting $y = M_L/M_o$, the payload fraction for a stage, and using Eq. (5) to replace R, we have

$$y = \frac{M_L}{M_o} = (1 + f)e^{-\Delta V/v_e} - f - \epsilon . \tag{11}$$

For an n stage vehicle, the over-all payload fraction is the product of the y_i's for each stage:

$$\left(\frac{M_L}{M_o}\right)_n = \prod_{i=1}^{n} \left[(1 + f_i)e^{-\Delta V_i/v_e^i} - f_i - \epsilon_i \right] , \tag{12}$$

where the sum of the ΔV_i is the total mission velocity requirement ΔV_T. For brevity, we will occasionally drop the Δ. Eqs. (11) and (12) will form the basis for most of our analyses and computations. The vehicle parameters f_i, ϵ_i, and v_e^i can be determined or guessed from more detailed studies. The ΔV_i will be determined from the total velocity increment (including losses) required to perform a given mission, which will be divided among the stages to optimize the payload or have staging at appropriate points (e.g., termination of a phase of the mission).

One can optimize the multistage payload fraction only by trial and error, but an elegant method devised by Dr. R. H. Fox does this by finding a single root of a polynomial equation. This method is presented in Appendix A. If we assume the same kind of propulsion (v_e^i) for all stages but allow for different values of f_i (to account for tank size or insulation) and ϵ_i (for different size engines or lower thrust to weight ratio for upper stages), then we can optimize the payload fraction by choosing

$$\Delta V_i(\text{optimum}) = \frac{\Delta V_T}{n} + v_e \, \ln\left(\frac{1 + f_i}{E_i + f_i}\right) - \frac{v_e}{n} \sum_{i=1}^{n} \ln\left(\frac{1 + f_i}{\epsilon_i + f_i}\right) \quad (13)$$

With equal parameters for all stages,

$$\Delta V_i(\text{optimum}) = \frac{\Delta V_T}{n} \quad (14)$$

and

$$\frac{M_L}{M_o} = \left[(1 + f)e^{-V_T/nv_e} - \epsilon - f\right]^n . \quad (15)$$

We can perform an approximate optimization of the number of stages (which is generally $n \approx V_T/v_e$) to obtain

$$\left(\frac{M_L}{M_o}\right)_{\text{opt.}} \approx e^{-(1+1.4f+2.9\epsilon)V_T/v_e} . \quad (16)$$

This equation, the derivation of which is given in Appendix A , is useful for making gross comparison between different propulsion systems and estimating the payload fraction for "difficult" (high V_T) missions. Often one wishes to compare "dry" or manufactured weights (M_d) as an estimate of vehicle costs. In view of the uncertainties of ϵ and f, a crude but sufficiently precise estimate is

$$M_d = (\epsilon + f)M_o \tag{17}$$

or

$$\frac{M_d}{M_L} = (\epsilon + f)e^{kV_T/v_e} \tag{18}$$

where

$$k = (1 + 1.4f + 2.9\epsilon). \tag{18a}$$

To the same approximation, we can estimate the propellant volume per unit payload to be

$$\frac{Vol}{M_L} = \frac{1}{\rho_p}(1 - \epsilon - f)\left[e^{kV_T/v_e} - 1\right] \tag{19}$$

where ρ is the propellant density. For large V_T, we can drop the (-1).

We can determine the effects of the parameters upon the payload and each other (exchange ratios) by differentiation of Eq. (11). Of special interest is the value of specific impulse (I_{sp}) which is the exhaust velocity divided by g, the gravitational constant. Thus

$$\frac{\partial(M_L/M_o)}{\partial I_{sp}} = \frac{\partial}{\partial I_{sp}} \left[(1 + f)e^{-V/gI_{sp}} - \epsilon - f\right]$$

$$= (1 + f)e^{-V/gI_{sp}} \frac{V}{gI_{sp}^2} . \tag{20}$$

Writing this in terms of the payload and letting

$$\beta = V/gI_{sp},$$

we have

$$\frac{\partial M_L}{\partial I_{sp}} = \frac{(1 + f)M_o}{I_{sp}} \beta e^{-\beta}. \tag{21}$$

Since $\beta e^{-\beta}$ is a very slowly varying function in the range of interest in practice ($.6 < \beta < 1.5$ or for nuclear propulsion, $15{,}000 < V < 40{,}000$ ft/sec), we can replace it by its average over the interval which is 0.352. This value multiplied by the factor $(1 + f)$ is equal to e^{-1} for $f \sim 0.05$, and thus a convenient approximation is

$$\frac{\partial M_L}{\partial I_{sp}} \approx \frac{M_o}{eI_{sp}} . \tag{22}$$

For nuclear H_2 propulsion, neglecting dissociation, I_{sp} is proportional to the square root of the absolute temperature (T) of the exit gas, and this leads to

$$\frac{\partial M_L}{\partial T} \approx \frac{M_o}{2eT} . \tag{23}$$

The effect of changes in the mission velocity requirement V, such as to account for the earth's rotation, for example, is

$$\frac{\partial M_L}{\partial V} = - \frac{(1 + f)M_o e^{-V/v_e}}{v_e} = \frac{(1 + f)M_o}{v_e R} . \tag{24}$$

Similarly for the component weights:

$$\frac{\partial M_L}{\partial f} = -M_o(1 - \frac{1}{R}) \tag{25}$$

$$\frac{\partial M_L}{\partial \epsilon} = -M_o . \tag{26}$$

One can make the analysis more detailed by separately considering the various components. For example, what we have lumped into M_e is frequently broken down into reactor, pressure shell, pump, nozzle, and thrust structure; and relations for the component weights written in terms of flow rates, pressures, reactor size and void fraction, etc. We shall not go into such detail here (for examples, see references 1, 2) but will make some comments concerning them. The reactor itself usually constitutes the bulk of the engine weight. At temperatures of interest and H_2 as propellant, about 50 pounds of thrust are developed per megawatt (Mw) of reactor power. Nuclear engines described in the open literature average about 2 pounds/Mw, making their contribution to ϵ equal to .04 for a thrust equal to the initial gross weight. For ground launching, the thrust to weight ratio (T/W_o) must be at least 1.2 in practice, while lower values are optimum for upper stages, and 0.2 is quite satisfactory for orbital vehicles. For the latter, the low thrust requirement greatly offsets the large reactor weight per unit thrust.

One can also compute tank weights in terms of tank pressure and geometry and material properties. This is complicated by the possible need for insulation, structure, or additional pressure for supporting the payload, interstage material, etc. Hydrogen has a very low density which leads to relatively high values for f, the tank fraction. These range from .03 to .10 in the open literature which illustrates the uncertainty associated with advanced systems.

Mission Requirements

Circumplanetary Operations

We shall first present the results of simple dynamics for impulsive velocity changes and then refine these with suitable additions and corrections which may be included by altering the mission velocity requirement V_T.

Since nuclear propulsion is more appropriate for difficult missions, we will skip sounding and ground-to-ground missions (which are discussed in references 1 and 3) and begin with ground-to-orbit missions. The velocity required for a circular orbit about the earth is given by

$$v_c = \sqrt{\frac{g_e r_o^2}{r}} = \sqrt{\frac{K_e}{r}} \qquad (27)$$

where

g_e = earth's gravitational constant (surface acceleration)

r_o = earth's radius

r = orbit radius

$K_e = g_e r_o^2$, another form of the gravitational constant.

To go into an orbit at the earth's surface would require a velocity of 25,900 ft/sec. In practice, one must go to altitudes of 100 miles (v_c = 25,600 ft/sec, but the mission velocity requirement is higher to account for the potential energy increase) or more to reduce atmospheric friction so the orbit can be maintained. In addition, the rocket must fire over an appreciable period of time (i.e., non-impulsively), start vertically, and follow some trajectory through the atmosphere and into orbit. Solving this problem with the effects of an air drag, the earth's rotation, gravity, etc. is formidable and is discussed in references 1, 3. For example, one might integrate the equation of motion with an added term for gravity, which represents a major portion of the correction. The result would be similar to Eq. (3), i.e.,

$$V(t) = V_o + v_e \ln \frac{m_o}{m_o - \alpha t} - gt\,\overline{\sin\theta} \qquad (28)$$

where $\overline{\sin\theta}$ is the time averaged value of the sine of the angle between the velocity and the horizontal.

The gravitational term would depend upon the trajectory and acceleration but is usually of the order of several thousand ft/sec. The rotation of the earth can result in as much as ±1500 ft/sec, while atmospheric drag is much less important, especially for large boosters. The atmospheric pressure reduces the thrust (and thus the effective exhaust velocity) for liquid propellant engines to about 90% of their vacuum value at sea level. This loss could be included in the mission velocity or accounted for by using an average atmospheric value for v_e for ground launched stages. The over-all velocity requirement (including losses) for a low earth orbit typically ranges from 30,000 to 33,000 ft/sec, and we shall use 32,000 ft/sec as a representative value for our computations.

Velocity requirements for transfer to other circular orbits will depend upon the path, but the method which usually* requires the minimum ΔV is the Hohmann transfer ellipse (Figure 1) which is tangential to each of

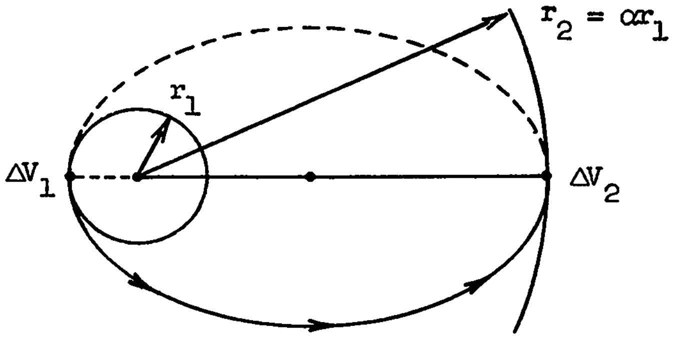


Figure 1. Hohmann Transfer

*E.g., see "Two impulse maneuvers" in reference 3, Chapter 8.

the circular orbits. Two impulses are required, at the ellipse's extremums and parallel to the velocity. The first velocity change (ΔV_1) for transfer from a circular orbit r_1 to an elliptic orbit with apogee $r_2 = \alpha r_1$ is

$$\Delta V_1 = \left(\sqrt{\frac{2\alpha}{1 + \alpha}} - 1\right) v_{c1} \tag{29}$$

where v_{c1} is the circular velocity at r_1. To transfer to the new circular orbit requires

$$\Delta V_2 = \left(1 - \sqrt{\frac{2}{1 + \alpha}}\right) v_{c2} = \frac{1}{\sqrt{\alpha}} \left(1 - \sqrt{\frac{2}{1 + \alpha}}\right) v_{c1}. \tag{30}$$

Thus the total ΔV required for transfer between circular orbits is

$$\Delta V_1 + \Delta V_2 = \left\{\frac{1}{\sqrt{\alpha}} \left(1 - \frac{\sqrt{2}\,(1 - \alpha)}{\sqrt{1 + \alpha}}\right) - 1\right\} v_{c1} \tag{31}$$

which holds for all $\alpha > 0$.

In particular, the total ideal velocity necessary to go from the ground to a circular orbit at x earth radii ($x = r/r_o$) is

$$\Delta V_I \text{ (cir. orb.)} = \frac{1}{\sqrt{x}} \left[1 + \frac{\sqrt{2}\,(x - 1)}{\sqrt{1 + x}}\right] \sqrt{g_o r_o}. \tag{32}$$

For escape from the ground, $x \rightarrow \infty$ and

$$\Delta V_I \text{ escape} = \sqrt{2g_o r_o} = \sqrt{2} v_{co} = 36{,}600 \text{ ft/sec.}$$

In general, escape from any circular orbit requires $(\sqrt{2} - 1)$ times the orbital velocity. Surprisingly, in Eq. (32), ΔV_I is not monotonic with x but has a shallow maximum at x = 15.6 (altitude = 58,400 miles). Achieving a "stationary" orbit at 22,000 miles requires 2300 ft/sec more than escape.

In Figure 2, we show results for ascent to a 100 mile circular orbit and Hohmann transfer to higher circular orbits. The perigee (v_p) and apogee (v_a) velocities are given for the elliptic transfer orbits as well as the circular velocities. Conservation of momentum gives $v_p = \alpha v_a$. Also presented in Figure 2 is the total velocity requirement v_I^{C+E} for circular orbit at altitude (h) and subsequent escape, showing that the most economical assembly orbit for interplanetary flight is a low one. Not much penalty is exacted for orbits up to about 500 mile altitude. These results, derived for impulsive motion in a conservative field, can be used for inward or outward transfers. The velocity requirement for other types of maneuvers, e.g., rotation of the plane of the orbit, can be found from the vector difference of the initial and final vehicle velocities. These ΔV's can be large, in the above example

$$\Delta V \approx 2v_c \sin \frac{\theta}{2} \qquad (33)$$

to rotate the plane of the orbit through an angle θ. We shall consider low thrust to weight ratios for orbital stages in an appendix where we

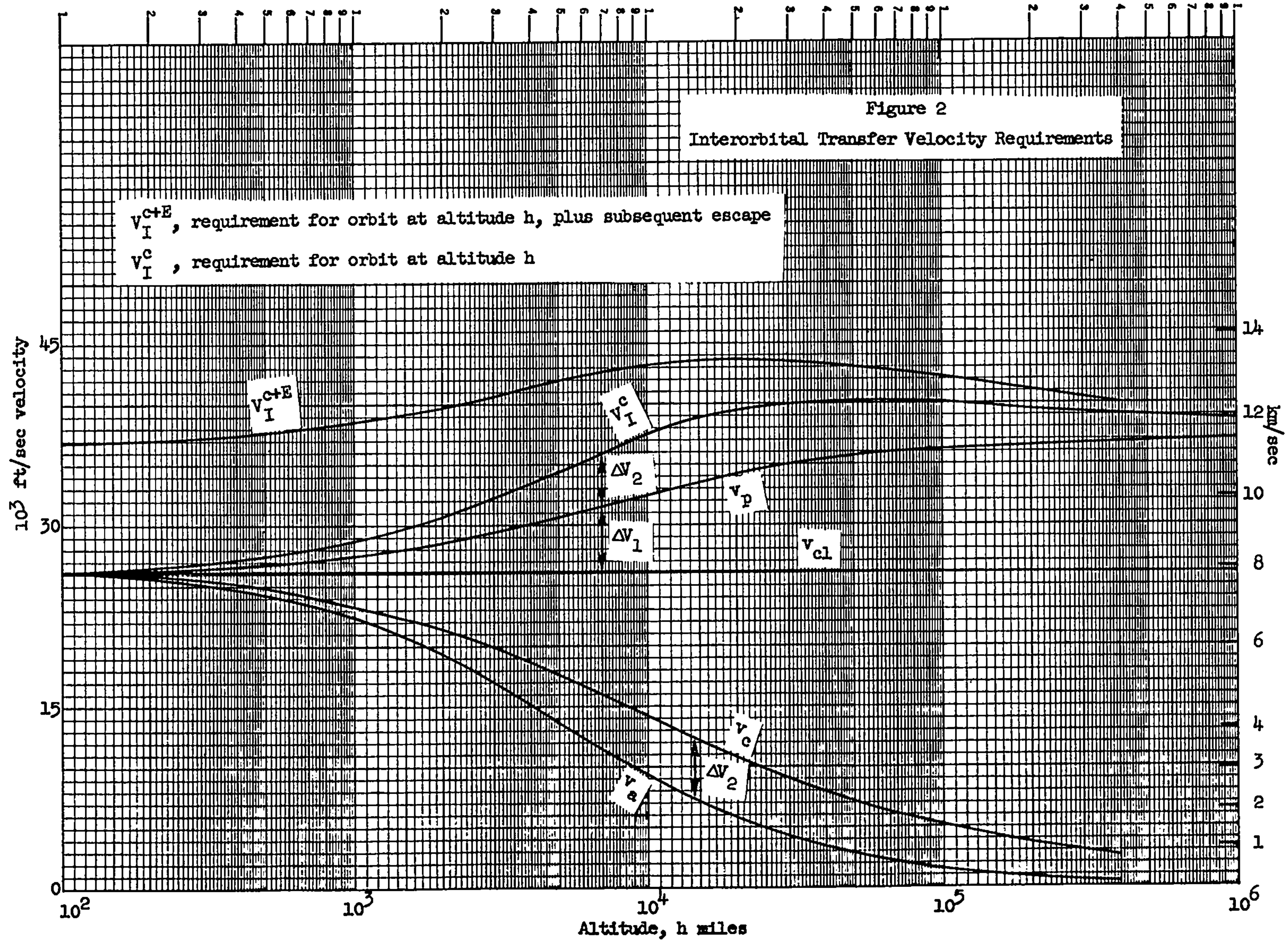


Figure 2

Interorbital Transfer Velocity Requirements show values of T/W_o of the order of .1 to .2 are acceptable.

Lunar Operations

The moon has an average orbit radius of 238,000 miles, which places it almost at infinity with respect to the velocity requirement for reaching it, as can be seen from Figure 2. This value is 36,000 ft/sec (not including losses) compared to 36,600 for escape. From its gravitational constant (g_m = 5.30 ft/sec^2) and radius (1080 miles), we find its escape and low orbit velocities to be 7783 ft/sec and 5500 ft/sec, respectively. Circumlunar flights require very little more than the velocity necessary for impact. Orbital maneuvers can be computed as for the earth. The ΔV for going from escape to a low lunar orbit is ~2300 ft/sec, which makes this mission only as difficult as a stationary (22,000 mile) earth orbit.

Interplanetary Operations

While the same simple dynamics applies, a few more simplifying assumptions are necessary, and some additional effects should be noted. We shall assume the planets to have circular concentric coplanar orbits and consider the gravitational effects only of the single major body affecting the vehicle's motion during each phase of the trip. For interplanetary voyages, we are interested in the gravitational constant of the sun, and the earth's orbital radius (1 A.U.) is a convenient unit as is the earth's velocity (v_E = 97,800 ft/sec). Eqs. (29) through (31) hold for transfer between orbits, where the vehicle remains free of planetary influence at either end of the transfer. However, just as the total velocity increment for escape from the earth is less for departure from a low orbit, it is more economical to make interplanetary departure from a low orbit. We can evaluate the velocity requirement and demonstrate the above statement.

Let $v_{\infty 1}$ be the actual velocity required for the vehicle when it is essentially free of the earth's attraction but still at the same distance from the sun, i.e., having "escaped" from the earth (Figure 3). We can

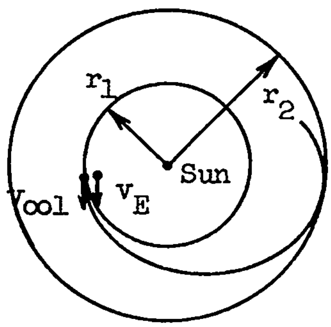


Figure 3

then use Eq. (29) to compute the excess velocity (v_{ex}) above the earth's orbital velocity required to place the vehicle in an elliptical orbit with apogee

$$r_2 = \alpha r_E;$$

$$v_{ex} = v_{\infty 1} - v_E = \left(\sqrt{\frac{2\alpha}{1 + \alpha}} - 1\right) v_E. \qquad (34)$$

Using a low earth orbit as a starting and reference point, we can calculate the total ΔV required if escape is accomplished separately, i.e.,

$$\Delta V_1 = \Delta V \text{ (escape)} + v_{ex}$$

$$= (\sqrt{2} - 1)v_{co} + v_{ex}. \qquad (35)$$

On the other hand, if we give the vehicle a single impulse in a low orbit to a velocity v_h, we can obtain the velocity of the vehicle after escape (v_{ex}) from conservation of energy;

$$\frac{v_h^2}{2} - \frac{K_e}{r_o} = \frac{v_{ex}^2}{2} . \qquad (36)$$

Using Eq. (27) and rearranging

$$v_h = \sqrt{v_{ex}^2 + 2v_{co}^2}. \qquad (37)$$

Thus the one impulse velocity requirement is

$$\Delta V_1 = v_h - v_{co} = \sqrt{v_{ex}^2 + 2v_{co}^2} - v_{co} , \qquad (38)$$

which can easily be shown to be less than that given in Eq. (35). The analysis can be easily extended to orbits at intermediate altitudes which are of interest in capture operations. At the apogee of the transfer orbit, the vehicle can be injected into the target planet's orbit with a ΔV_2 of the form given in Eq. (30), and very little more is required to go into a very high orbit about the planet. Results for ΔV_1 and ΔV_2 are shown in Figure 4. Should lower orbits be desired,

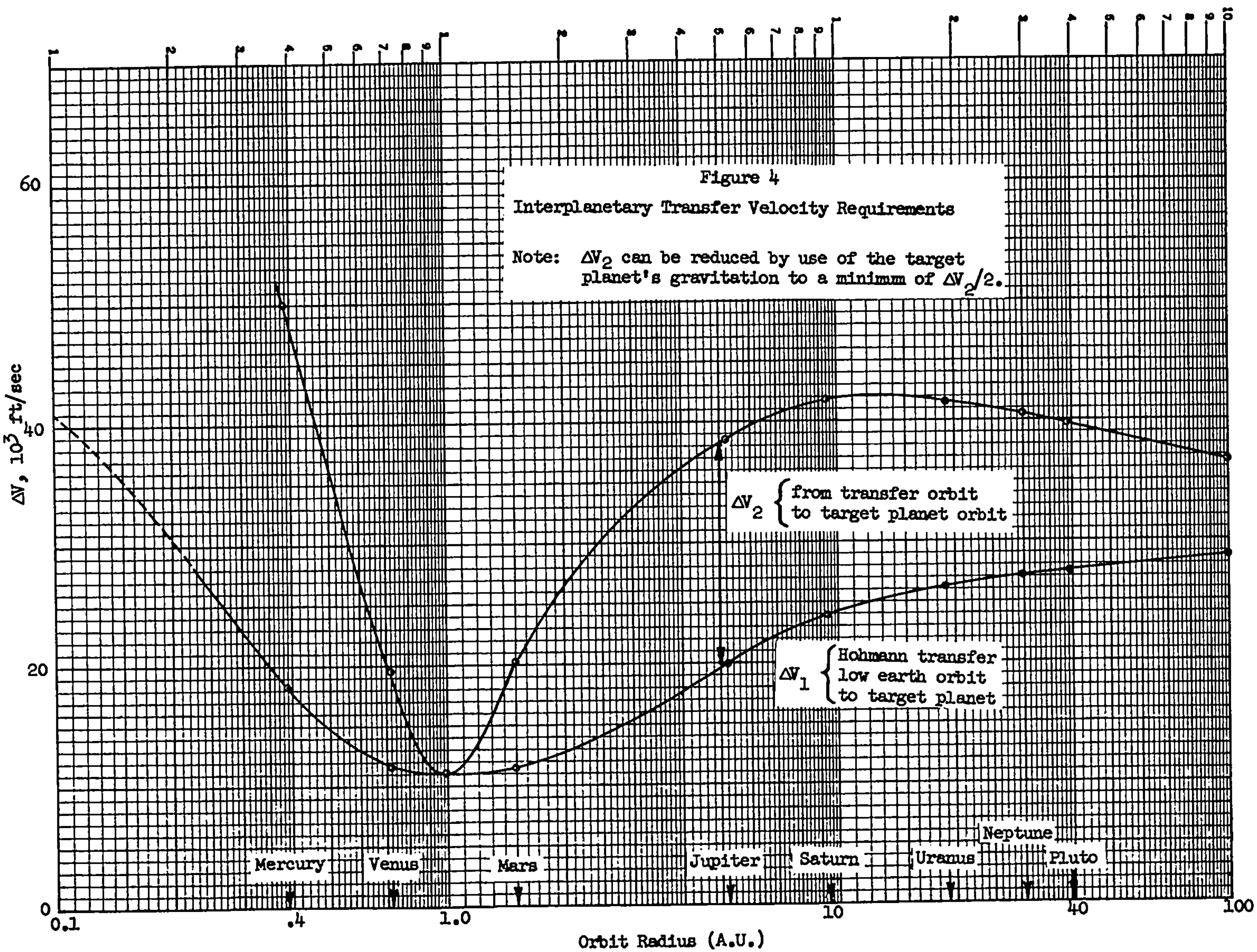


Figure 4
Interplanetary Transfer Velocity Requirements
Note: ΔV_2 can be reduced by use of the target planet's gravitation to a minimum of $\Delta V_2/2$.
ΔV_2 from transfer orbit to target planet orbit
ΔV_1 Hohmann transfer low earth orbit to target planet
ΔV, 10^3 ft/sec
Orbit Radius (A.U.)
0
20
40
60
0.1
.4
1.0
10
40
100
Mercury
Venus
Mars
Jupiter
Saturn
Uranus
Neptune
Pluto

a combined single impulse capture would be more economical, just as in the escape case. One can use Eq. (38), written in a more general way,

$$\Delta V \text{ (capture)} = v_h - v \text{ (desired orbit)}$$

$$= \sqrt{v_{ex}^2 + \frac{2K_p}{r}} - v \text{ (desired orbit)}. \qquad (39)$$

As a special case of a circular capture orbit,

$$\Delta V \text{ (capture)} = \sqrt{v_{ex}^2 + \frac{2K_p}{r}} - \sqrt{\frac{K_p}{r}} \qquad (40)$$

from which one can find the circular orbit radius which minimizes ΔV capture and which is

$$r = \frac{2K_p}{v_{ex}^2}. \qquad (41)$$

If r is not smaller than the planet radius, then ΔV (capture) can be 0.707 times the value computed without the planet's gravitational effect.

This is also of interest when returning to earth, as the returning passengers and cargo can be picked up and returned to earth by a vehicle from earth, and thus be relieved of carrying propellant for this maneuver on the entire round trip.

The minimum energy (Hohmann) transfer has several drawbacks; it can be started only at predetermined widely separated times and if used in both directions, forces a particular waiting time at the target planet. Furthermore, the transit times are quite long, especially to the outer planets, and the miss distances at the target planets are a sensitive function of the injection velocity. Thus there is much interest in non-minimum energy missions or as they are more frequently termed "fast transfer" missions. While these also can be treated analytically under the assumptions we have made, there exists a two parameter family of transfer orbits from which one must select on the basis of desired transit and stay times, available ΔV vs. payload etc. These missions are treated in varying amounts of detail in several of the references, and only a few examples will be presented here to give a feeling for the quantities involved.

Hohmann transfer to Mars from a low earth orbit requires ~12,000 ft/sec for the earth impulse phase and ~6000 ft/sec for capture in a 1000 mile altitude Martian orbit. The transit time is 250 days, but can be reduced to half that by increasing the earth escape ΔV by only 2000 ft/sec. However, the intersection of the transfer orbit with Mars' orbit becomes steep, and the capture velocity requirement increases by 16,000 ft/sec. One can minimize the total ΔV (escape + capture) for a fixed transit time by varying both the magnitude and direction of the escape impulse such that the vehicle's path makes an angle with the earth's orbit.

Another example is of probes to the outer planets. Minimum energy transfers to Jupiter, Saturn, and Uranus require 2.9, 6, and 16.1 years, respectively (and ΔV's of 21,500 to 27,000 ft/sec). Increasing the ΔV to ~29,000 ft/sec would give the vehicle solar system escape velocity and decrease the transit times to 1.2, 2.7, and 6.9 years, respectively.

To summarize, we present a table (I) listing the approximate velocity requirements for a variety of missions.

Illustrations and Examples

Engines and Vehicles

We shall give some examples from the unclassified literature of component weights for nuclear propulsion systems, including some equations for estimating them.

The reactor weight, which usually is the bulk of the engine weight, depends upon the reactor type, operating pressure, power level, void fraction, materials, heat transfer, etc. and cannot be evaluated from a simple equation. Instead we will give some examples for graphite reactors, including core and reflect, excluding pressure shell, external structure, controls, pump, and nozzle. These give 16,000 pounds for a 16,600 Mw reactor (reference 4) and 4,000 pounds for 3000 Mw (reference 2) with both reactors operating at ~1000 psi inlet pressure. Both are ~1 pound/Mw or about 1 pound of engine per 40 pounds of thrust. Detailed formulae are given in reference 2 for pump, pressure shell, and nozzle weights, but here we shall present only an example to give a feeling for their relative importance.

Table I

Approximate Mission Velocity Requirements
Earth Satellite and Cislunar Operations

Mission	Partial ΔV ft/sec	ΔV_{Total} ft/sec (with losses)
Low Earth Orbit		32,000
Ideal velocity	26,000	
Gravitational losses	5,000	
Drag and pressure losses	1,000	
Escape		43,000
Low orbit to escape	11,000	
24 Hour (21,000 mile) orbit		45,000
Low to high orbit	13,000	
Lunar Orbit		45,000
Earth escape to lunar orbit	2,000	
Lunar Landing		52,000
Ideal lunar escape velocity	8,000	
Lunar Round Trip		
(Parabolic earth re-entry)		60,000
(Circular earth re-entry)		70,000

Interplanetary Operations

Planet	Min. E. Probes ΔV	Min. E. Probes Time Years	High Orbit Capture ΔV, ft/sec	Planetary Escape Velocity ft/sec	Manned Trip ΔV From Earth Orbit, ft/sec
Mercury	18,200	.27	31,700	11,600	
Venus	11,500	.38	8,300	33,000	60-90x10^3 ft/sec
Mars	11,600	.7	8,500	16,400	60-90x10^3 ft/sec
Jupiter	20,000	2.9	18,500	195,000	
Saturn	23,900	6.0	17,900	116,000	
Uranus	26,200	16.1	15,300	68,000	
Solar Escape	29,000	--	--	--	

Typical Engine

		Weight, pounds
Power, 4500 Mw	Engine	9000
Thrust, 210,000 pounds	Reactor	7000
Flow Area, 3.3 ft^2	Pump	700
Flow Rate, 340 pounds/sec	Pressure shell	900
Exit Mach No., 0.4	Nozzle	400
Exit Pressure, 420 psi		

The pump, pressure shell, and nozzle weights are roughly proportional to the operating pressure, while the reactor is insensitive to it.

The propellant tankage weight is significant and can be computed from the tank pressure (p_t), material density (ρ_t), and tensile strength (σ). For a sphere, this gives

$$f = \frac{M_t}{M_p} = \frac{2p_t\rho_t}{3\sigma_t\rho_p}$$

which is independent of tank size. Depending upon the choice of materials, this equation gives values of f between .02 and .08 for H_2. However, in typical cases in practice, the interstage structures, fittings, insulation, etc. can add up to 50% of this value to give the entire fuselage weight fraction and thus at least a preliminary vehicle design is required to pin this value down. For our purposes, we shall select a set of vehicle parameters for each of several propulsion systems (Table II).

Table II

Vehicle Parameters

Propulsion	I_{sp}, sec	v_e, ft/sec	f	ϵ	ρ_p, lbs/ft^3
LOX-RP	300	9,660	.02	.02	63
LOX-H_2	420	13,500	.04	.02	17
Nuclear	860	27,700	.10	.04 .06*	4.3

*Ground launched stages only; where an average value is required, use 0.05.

General Comparisons

Using the assumed parameters and Eqs. (16), (18), and (19), we can make rough estimates of the payload fraction, the ratio of hardware weight to payload, and the propellant volume per unit payload. Significant quantities are listed in Table III and results in Figures 5, 6, and 7.

Table III

Quantities for General Propulsion System Comparisons

	v_e/k	$(\epsilon + f)$	$1/\rho\ (1 - \epsilon - f)$
LOX-RP	8,900 ft/sec	.04	.015 ft^3/lb
LOX-H_2	12,100	.06	.055
Nuclear	21,600	.15	.20

These equations allow direct comparisons among the systems. For example, compare the gross weight of LOX-H_2 and nuclear propulsion for the same payload:

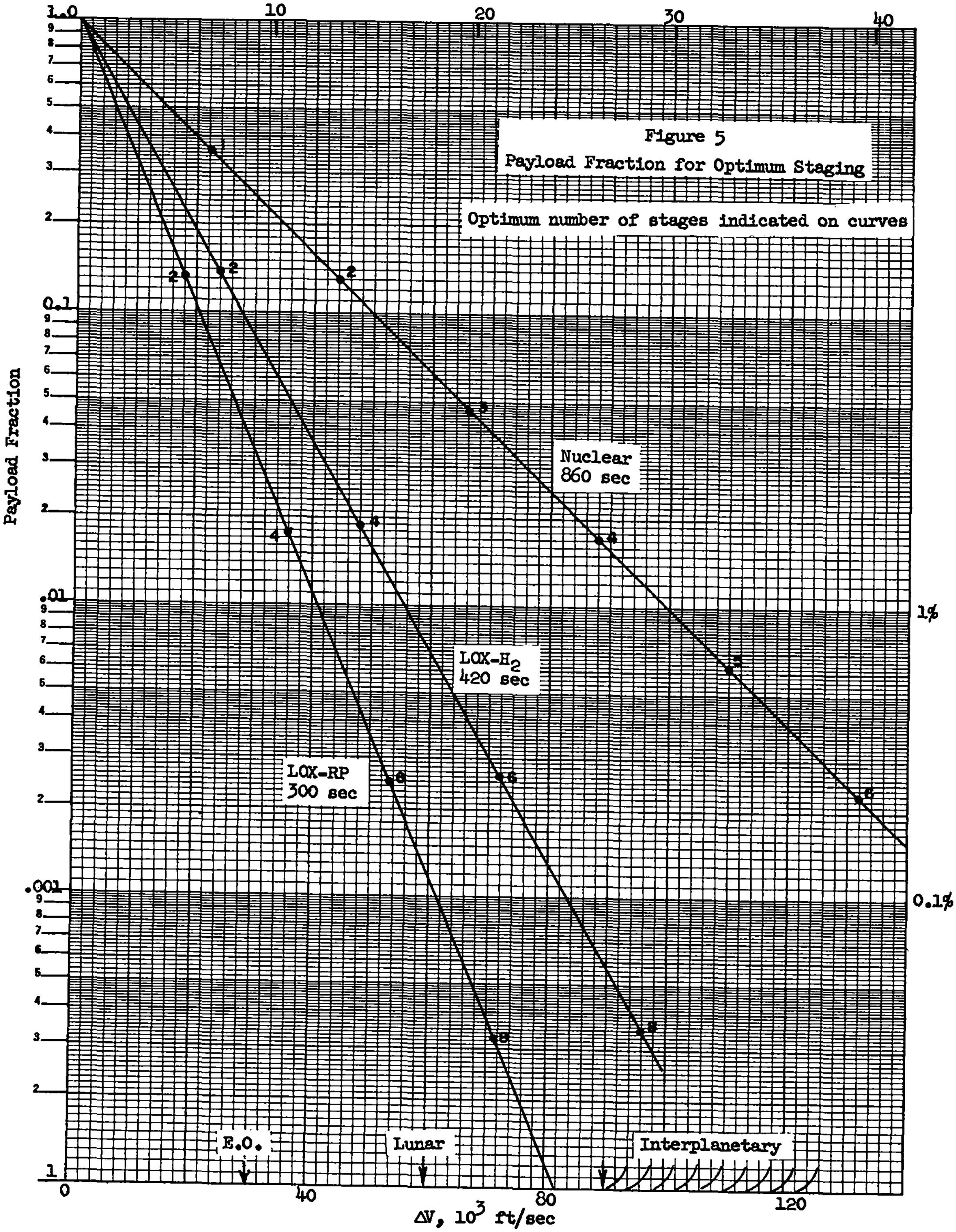
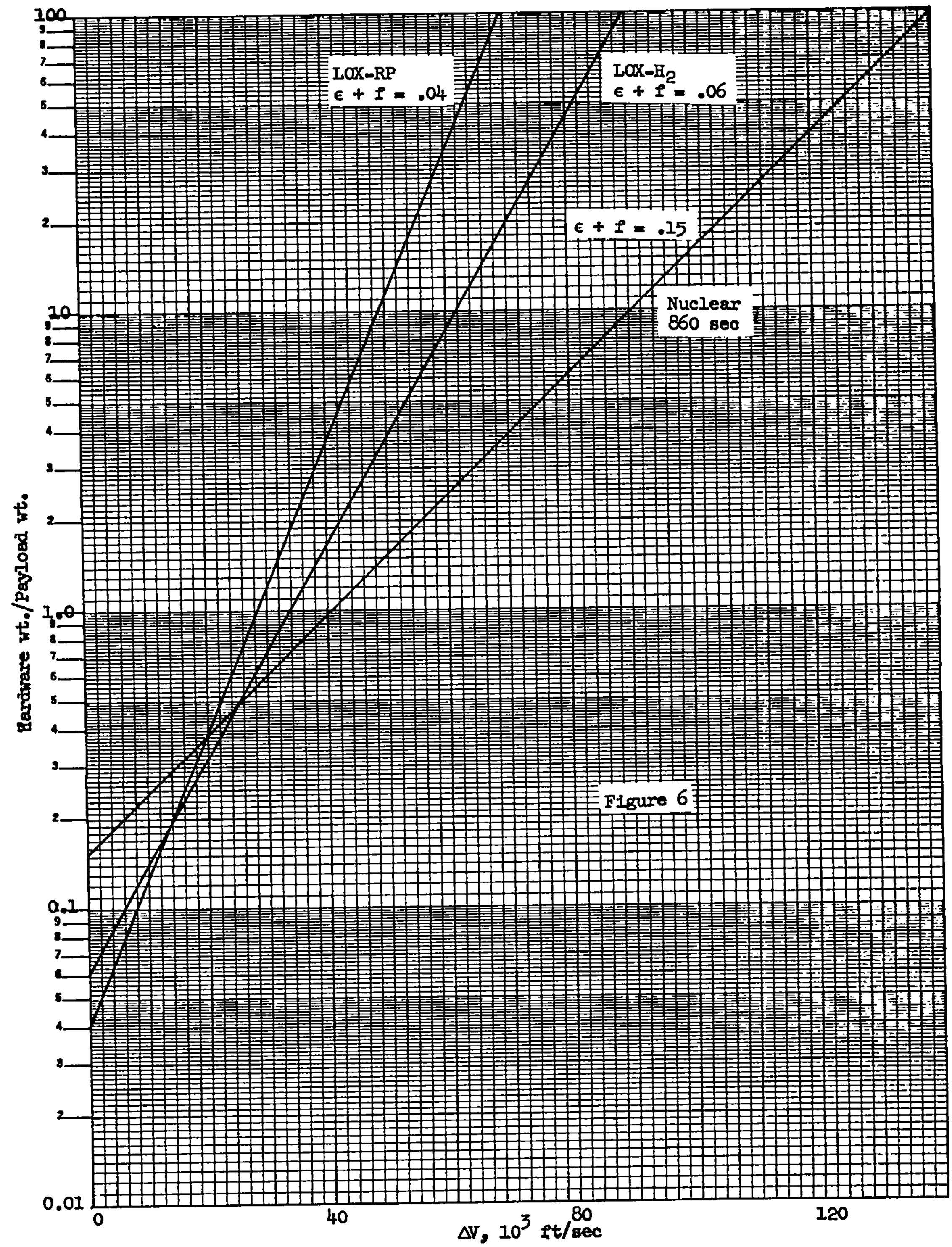
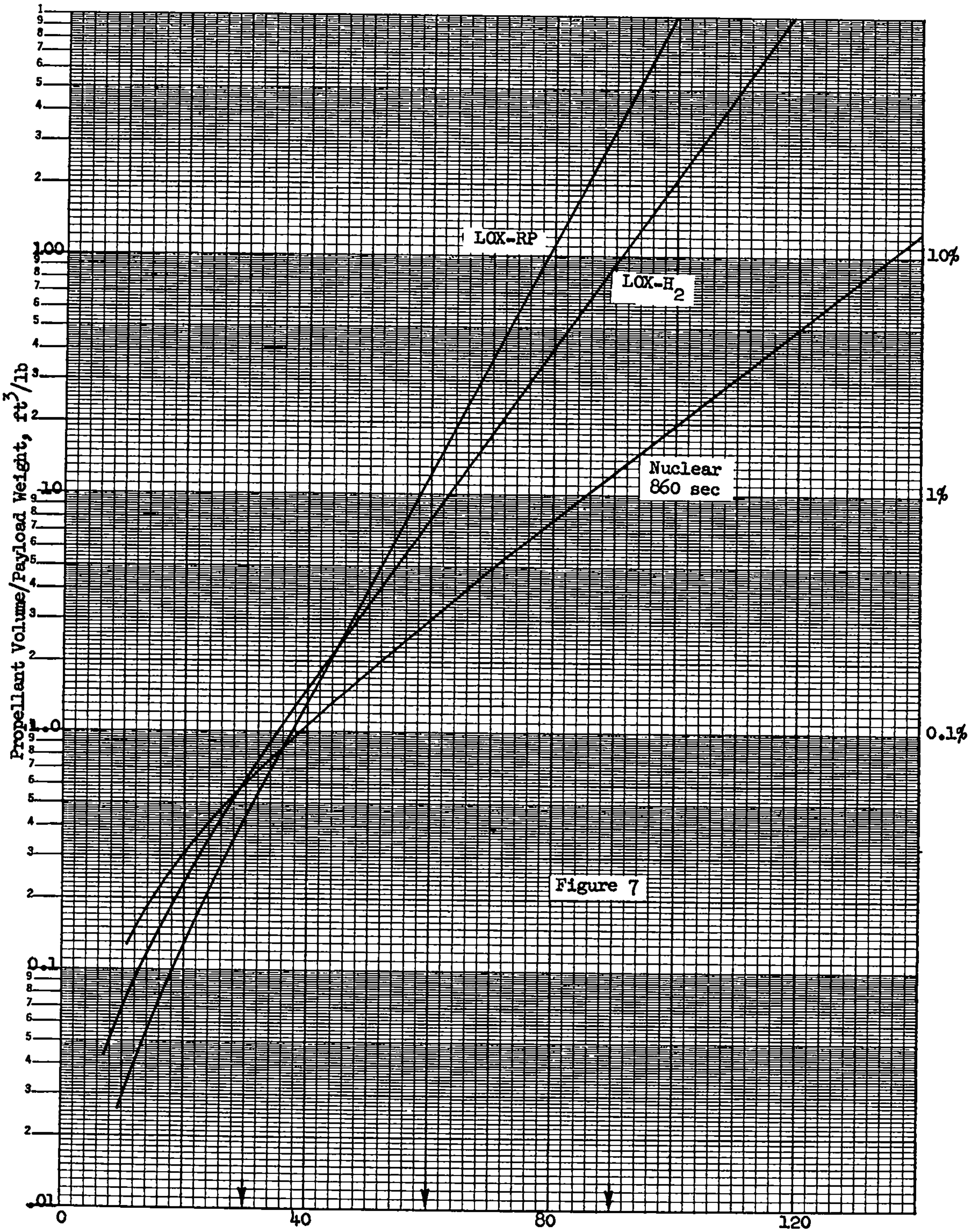


Figure 5
Payload Fraction for Optimum Staging
Optimum number of stages indicated on curves
Payload Fraction
ΔV, 10³ ft/sec
Nuclear
860 sec
LOX-H2
420 sec
LOX-RP
300 sec
1%
0.1%
E.O.
Lunar
Interplanetary



LOX-RP
ε + f = .04
LOX-H2
ε + f = .06
ε + f = .15
Nuclear
860 sec
Figure 6
Hardware wt./Payload wt.
100
10
1.0
0.1
0.01
0
40
80
120
ΔV, 10^3 ft/sec



LOX-RP
LOX-H$_2$
Nuclear
860 sec
Propellant Volume/Payload Weight, ft^3/lb
100
10
1.0
0.1
.01
10%
1%
0.1%
0
40
80
120

Figure 7

$$\frac{M_o\ (\text{chem.})}{M_o\ (\text{nuc.})} = \frac{e^{kV_T/v_e}\ (\text{chem.})}{e^{kV_T/v_e}\ (\text{nuc.})} = e^{V_T(k_c/v_e^c)-(k_n/v_e^n)}$$

$$= e^{V_T/27,500} .$$

Thus the advantage of nuclear propulsion increases with mission velocity, which is the result of higher specific impulse. There is the same dependence upon V_T for the dry weight and volume, but the chemical systems have initial advantages due to their lower dry weight fraction and higher propellant density:

$$\frac{M_d\ (\text{chem.})}{M_d\ (\text{nuc.})} = .4e^{V_T/27,500}$$

and

$$\frac{\text{Vol. (chem.)}}{\text{Vol. (nuc.)}} = .27e^{V_T/27,500} .$$

These have crossover points at 25,000 ft/sec and 36,000 ft/sec respectively, above which the nuclear system is superior. No single parameter will determine the choice of a propulsion system; all factors, including reliability, safety, availability, ground support, over-all cost, future potential, etc. will be involved. Our aim here is to make those comparisons which follow from the vehicle and mission parameters and are partial inputs to the decision-making process.

The above comparisons are for vehicles using only a single type of propulsion for all stages. Composite vehicles using different propulsion schemes in different stages can have smaller values of dry weight or volume. Most of these two quantities for a given vehicle are in the first stage, and the use of chemical propulsion for it might materially reduce these for the vehicle as a whole. We do not have a simple technique for minimizing structure or volume of multistage composite vehicles though they would be useful. Results of such optimizations should be used with caution, as we have indicated, because of the many other factors involved. However, the degree of sensitivity of the optimized quantity to changes from the optimum is valuable and can usually be obtained through analytical or numerical techniques. Choosing the correct subsidiary conditions or restrictions is also important in performing optimization analyses. For example, in optimizing the thrust/initial weight, one gets very different results if the vehicle weight is fixed for one calculation and the engine size for another. Either method might be the correct one under particular circumstances. Finally, the results, though analytically correct, might be ignored for a reason not contained in the analysis; for example, a stage or propulsion system with poorer performance might be chosen because of its availability, simplicity, etc. A case in point is combined nuclear-chemical propulsion where the propellant fuel is first heated in a nuclear reactor and after partial expansion is burned with an oxidizer. For the H_2-O_2 system, this gives an increased performance under certain conditions (reactor size fixed and of a size too small for the stage considered). The H_2 issuing from the reactor might have a specific impulse of 800 seconds and a temperature of ~2500°C. The addition and combustion of the oxygen does not raise the temperature much but does supply energy to raise the combustion products to about the same temperature. However, they have much higher molecular weight than the H_2 resulting in higher thrust and lower specific impulse. The mixture ratio and duration of oxidizer flow can be varied to optimize the performance of a given stage. While this scheme might give better performance (based on some criterion) than a simple nuclear engine plus a separate LOX-H_2 stage, it is the author's personal opinion at the present time that the complexities of such an engine are not worth the possible performance gain.

The missions, as well as the vehicle parameters, can be a source of confusion if results for one are carelessly generalized. One can easily choose the mission such that small differences in the vehicle parameters make the difference between no payload and substantial payload or such that one or another of various systems is superior. One should keep in mind the old admonition, "Figures don't lie, but liars do figure." Remembering this, let us continue with (figuring) further illustrations.

Earth to Orbit

We shall present results for a variety of vehicles intended for the low earth orbit mission. We will examine solid LOX-RP and LOX-H_2 boosters with LOX-H_2 or nuclear propelled upper stages, and a single stage nuclear vehicle. (For the solid rocket, I_{sp} = 250 seconds, ϵ = .02, f = .05.) For ease of visualization, we will compare vehicles designed to place

200,000 pounds in orbit. Since optimization on a gross weight/payload basis would lead to an all nuclear stage, the booster velocity increments are arbitrarily taken to be equal to the boost exhaust velocity (i.e., mass ratio = e) where the second stage is nuclear.

Table IV

Chemically Boosted Nuclear Stages
200,000 Pounds in Low Earth Orbit

Weights in 10^3 Pounds
Volumes in 10^3 ft^3

	Solid	LOX-RP	$LOX-H_2$
M_o	2080	1810	1520
M_d	185	130	121
Volume	100	96	113
First Stage			
M_d	108	60	68
Volume	12	18	57
Second Stage			
M_o	655	606	490
M_d	77	70	53
Volume	88	78	56

The results show there is relatively little difference in gross weight, dry weight, and volume among these cases. Notice that much of the dry weight and volume lie in the nuclear stage. There are clearer differences if we compare the previous cases with a single stage nuclear vehicle and a two stage chemical vehicle.

Table V

Chemical and Nuclear Vehicles for Low Earth Orbit Missions

200,000 Pounds in Orbit

$\Delta V = 32,000$ ft/sec

	Nuclear	$LOX-H_2$	LOX-RP, $LOX-H_2$
No. of Stages	1	2	2
M_o, 10^3 lbs	1070	3000	4300
M_d, 10^3 lbs	126	181	217
Vol., 10^3 ft^3	170	154	112

The single nuclear stage has the smallest gross weight and largest propellant volume, while the all-chemical vehicles have significantly largergross weights and dry weights. The engine for nuclear second stages requires power levels of 10,000 to 12,000 Mw, while the ground launched single stage requires at least 30,000 Mw. Chemical engines in the million pound thrust class are being developed, while the power level of the first nuclear engine to be flight tested is reported* to be of the order of 1000 Mw (50,000 pounds thrust). Such a difference in the relative states of the art could influence the choice of vehicles. Other factors involved in the all-nuclear vehicle are the air-scattered radiation to the payload and the launch site and the possible greater safety and reliability of a single stage. It might be possible to return the nuclear engine or the entire stage for reuse, though many

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*Robert E. L. Adamson, Nucleonics 19, 56 (April, 1961).

technical problems would have to be solved with high reliability for this to be practical.

Lunar Exploration

A round trip to the lunar surface with parabolic re-entry of the earth's atmosphere requires 60,000 ft/sec. The optimum number of stages are 5 and 3 for LOX-H_2 and nuclear propulsion, respectively, and in practice fewer can be used with small weight penalty. Here there is a great difference on all counts between the nuclear and chemical vehicles with intermediate results for composite vehicles (Table VI).

Table VI

Lunar Exploration Vehicles

Payload = 20,000 Pounds
ΔV = 60,000 ft/sec

	LOX-H_2	Nuclear	LOX-H_2, Nuc., LOX-H_2
No. of Stages	4	2	3
M_o, 10^3 lbs	3440	406	1130
M_d, 10^3 lbs	194	60	82
Vol. Prop., 10^3 ft^3	190	74	97

The use of a single nuclear stage reduces the gross weight by a factor of 3 and the dry weight and volume by a factor of 2, and these factors are increased to 8 and 3, respectively, for an all-nuclear vehicle.

Interplanetary Missions

The same ΔV (60,000 ft/sec) could apply for an interplanetary reconnaissance mission, starting from a low earth orbit and returning to a high orbit. Here the gross weight is more significant than dry weight as the cost of placing weight in orbit is high, and low thrust/weight ratios ($\sim$.1) can be used with small performance penalty. Extensive exploration, even of the inner solar system, will require higher ΔV's, to 100,000 ft/sec. Comparison of conventional systems can be made from Figures 5 to 7, but the competition may be among more advanced systems, e.g., 1000-1600 second I_{sp} nuclear heat exchangers, nuclear bomb propulsion, and nuclear electric (ion, arc, plasma, etc.) schemes. The latter have high I_{sp}'s (2000 to 10,000 seconds) but very low thrusts. Our method of analysis is not completely applicable to low thrust systems ($T/W \leq 10^{-3}$) where the critical parameter is the specific weight of the power generating equipment. The low thrust systems require long times ($\sim$ months) for escape from a low earth orbit which leads to higher gravitational losses and much time in the radiation belts. Thus it appears that even when electric propulsion is operational, the high thrust nuclear heat exchanger will have important areas of application.

APPENDIX A

Stage Optimization

First we can show that the optimum number of stages is approximately V_T/v_e, i.e., that the mass ratio for each stage be about e. Defining $y = M_L/M_o$ and $\beta = V_T/v_e$, Eq. (15) of the text becomes

$$y = \left[(1 + f)e^{-\beta/n} - \epsilon - f\right]^n . \tag{A.1}$$

Assuming f, $\epsilon \ll 1$, we can drop the f in the first term (equivalent to assuming all the dry weight is proportional to gross vehicle weight), rearrange to

$$y = e^{-\beta}\left[1 - (\epsilon + f)e^{\beta/n}\right]^n \tag{A.2}$$

and expand approximately to obtain

$$y \approx e^{-\beta}\left(1 - n(\epsilon + f)e^{\beta/n}\right) . \tag{A.3}$$

Now $dy/dn = 0$ yields $n = \beta = V_T/v_e$.

Differentiation of (A.1) exactly plus a numerical solution for n gives $n \approx \beta$. We can use this fact to derive Eq. (16) of the text. Let u be the terms in the square brackets in Eq. (A.1) and maximize y with respect to n:

$$\frac{dy}{dn} = \frac{du^n}{dn} = 0.$$

This leads to

$$u \ln u + \frac{\beta}{n}(u + f + \epsilon) = 0 \tag{A.4}$$

or

$$u \ln u = (1 + f)a \ln a \tag{A.5}$$

where

$$a = \frac{u + f + \epsilon}{1 + f} = e^{-\beta/n} = \frac{1}{R} . \tag{A.6}$$

We solve (A.5) by letting $u = a - \lambda$ and expanding the left side, giving

$$\lambda \approx \frac{af \ln a}{1 + \ln a} \tag{A.7}$$

whence

$$u \approx a + \frac{af \ln a}{1 + \ln a} \tag{A.8}$$

and, using the definition of u,

$$a \approx \frac{(f + \epsilon)(1 + \ln a)}{f} . \tag{A.9}$$

(The analysis to this point is due to Dr. K. Brueckner.) Since $a = e^{-\beta/n}$, which we know to be approximately e^{-1} for optimized n, let us write $a = e^{-1} + \delta$, leading to

$$\delta \approx \frac{f}{e[e(f + \epsilon) - f]} \qquad \text{(A.10)}$$

and

$$a \simeq \frac{1}{e(1 - b)} \qquad \text{(A.11)}$$

where

$$b = \frac{f}{e(f + \epsilon)} \leq \frac{1}{e}. \qquad \text{(A.12)}$$

From (A.6) and (A.11),

$$n\ (\text{optimum}) = \frac{\beta}{\ln a^{-1}} = \frac{\beta}{\ln e(1 - b)} \simeq \beta(1 + b + \frac{1}{2} b^2) \qquad \text{(A.13)}$$

which gives n (optimum) more exactly in terms of β, f, and ϵ. Thus

$$y\ (\text{optimum}) = \left[(1 + f)a - f - \epsilon\right]^n$$

$$= \left[\frac{(1 + f)}{e(1 - b)} - f - \epsilon\right]^{\beta/\ln e(1-b)} . \qquad \text{(A.14)}$$

This is of the form

$$y = A^{-k_1\beta}$$

or

$$\ln y = -\beta k(f,\epsilon). \tag{A.15}$$

We can evaluate k under various assumptions concerning f and ϵ.

$$\left\{\begin{array}{l} f = \epsilon \ll 1 \\ k \simeq (1 + 4.32f) = (1 + 4.32\epsilon) \end{array}\right\} \tag{A.16.1}$$

$$\left\{\begin{array}{l} \epsilon \ll f \ll 1 \\ k \simeq (1.33f + 3.18\epsilon) \end{array}\right\} \tag{A.16.2}$$

$$\left\{\begin{array}{l} f \ll \epsilon \ll 1 \\ k \simeq (1 + 1.72f + 2.72\epsilon). \end{array}\right\} \tag{A.16.3}$$

Since this is only an approximate limiting case, that all forms give similar results and that in practice, $\epsilon \approx f$ for both chemical and nuclear vehicles, we choose a compromise which is correct for $\epsilon = f$ and reflects the relative importance of the two parameters, i.e.,

$$\ln y = -(1 + 1.42f + 2.9\epsilon)\beta. \tag{A.17}$$

If the stages have different propulsion systems and thus different values for ϵ, f, and v_e for each stage, the stage mass ratios can be adjusted to obtain a maximum ΔV_T for a fixed payload/gross weight ratio by a method developed by Dr. Robert Fox of UCRL (reference 5 in Bibliography).

We shall present the method without proof. Choose the smallest of the exhaust velocities (v_i) to be v_o. For each stage, calculate

$$c_i = \frac{v_i - v_o}{v_i} \quad . \tag{A.18}$$

Compute

$$Q = \left[\prod_{i=1}^{n} (\epsilon_i + f_i)\right] \left[\prod_{i=1}^{n} \frac{v_o}{v_i}\right] y^{-1}, \tag{A.19}$$

where y is the chosen value of the payload fraction. Solve for q the polynomial equation

$$\prod_{i=1}^{n} (c_i + q) - Q = 0. \tag{A.20}$$

A good trial value is $q = Q^{1/n}$, and Dr. Fox has shown that there is only one real positive root of Eq. (20), which we shall label q_o. Then

$$y_i = \frac{(\epsilon_i + f_i)}{(c_i + q_o)} \frac{v_o}{v_i} \tag{A.21}$$

and

$$V_T = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} v_i \, \ln\left(\frac{1 + f_i}{\epsilon_i + f_i + y_i}\right) . \tag{A.22}$$

An analysis with V_T fixed and y optimized leads to an irrational equation similar to (A.20). Equation (A.20) can be solved analytically for two or three stages, which is the most that are of interest in practice, since difficult missions requiring many stages will have several natural staging steps (e.g., earth orbit or escape, lunar landing or planetary approach, etc.). Furthermore, where there is a large difference in the parameters, particularly v_e, the high performance stage will tend to "swallow" the others. In the interesting case of a chemically boosted nuclear stage optimized for a low earth orbit mission, the chemical stage will entirely disappear.

APPENDIX B

Low Thrust Operations*

Here we evaluate the effects of using low thrust to weight ratios for orbital operations. The advantage of lower engine weight is offset to some extent by increased gravitational losses, and usually an optimum value can be found. In practice, thrust/weight ratios or reactor power levels considerably lower than the optimum can be used with small performance losses.

The equations for motion in a gravitational field:

$$\frac{dv_r}{dt} - \frac{v_\theta^2}{r} = - \frac{(dm/dt)}{m} (v_e)_r - g_o \left(\frac{r_o}{r}\right)^2 \qquad (B.1)$$

$$\frac{d(rv_\theta)}{dt} = - \frac{(dm/dt)}{m} (v_e)_\theta r, \qquad (B.2)$$

(where subscripts r and θ refer to radial and azimuthal components, and g_o is the field constant at the initial orbit radius r_o) can be solved analytically only in special limiting cases. The high thrust limit gives the familiar results obtainable from the direct vector addition of velocities. Let us introduce a dimensionless energy parameter λ, which is

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*The analysis and results are based on the work of Brueckner et al. in "Topics of Thrust Orbit Optimization (Los Alamos Scientific Laboratory).

the final total energy (with respect to a body at rest and infinitely far from the attracting body) in terms of the kinetic energy of the final mass in the initial orbit. Thus

$$\lambda = \frac{\text{Kinetic Energy + Potential Energy}}{\frac{1}{2} m v_o^2} . \tag{B.3}$$

For a body in the initial circular orbit, $\lambda = -1$ and for escape with zero final velocity, $\lambda = 0$.

In the high thrust (impulsive) limit, the velocity increment ΔV is simply related to λ;

$$v_h = v_o + \Delta V \text{ (impulsive)} \tag{B.4}$$

$$\lambda = \frac{v_h^2}{v_o^2} - 2 \tag{B.5}$$

or

$$\Delta V \text{ (impulsive)} = (\sqrt{2 + \lambda} - 1)v_o . \tag{B.6}$$

The mass ratio R is given by

$$R = e^{\Delta V(\text{impulsive})/v_e} . \tag{B.7}$$

The following table illustrates the significance of λ for cases of interest.

Table B.1

High Thrust Operations and Significance of λ

	In General		Low Earth Orbit $v_o = 25{,}200$ ft/sec	
λ	v_h/v_o	$\Delta V/v_o$	ΔV(impulsive), ft/sec	Typical Mission
0	$\sqrt{2}$	$(\sqrt{2} - 1)$	10,400	Escape
1	$\sqrt{3}$	$(\sqrt{3} - 1)$	18,400	Mercury Probe
2	2	1	25,200	Saturn Probe

The low thrust limit is also soluble for $\lambda \leq 0$ (missions up to and including escape with zero final velocity). Once out of the earth's field, the rest of the operations would be the same as the impulsive limit (no gravitational losses) except that now the sun's field must also be considered. For $\lambda \leq 0$, the equations can be solved to give the mass ratio:

$$R(\text{low thrust limit}) = e^{\frac{v_o}{v_e}(1-\sqrt{-\lambda})} \tag{B.8}$$

In particular, $\lambda = 0$

$$R(\text{low thrust escape}) = e^{v_o/v_e}.$$

This is equivalent to a velocity requirement of v_o compared to $(\sqrt{2} - 1)v_o = .414\ v_o$ for the imulsive limit. Similarly, for $\lambda = 1$, the values are $2v_o$ and $(\sqrt{3} - 1)v_o$, respectively, for low and high thrust limits.

To gain an understanding of the intermediate cases, let us examine the mass ratio as a function of thrust/initial weight for $\lambda = 0$ and 1, starting in a low earth orbit (v_o = 25,200 ft/sec) and assuming a propellant exhaust velocity equal to v_o (I_{sp} = 783 sec). We shall give results (Figure B.1) based on the thrust being kept parallel to the velocity, which has been shown by Baker (LAMS-2403) to be close to optimum. The results show that the mass ratio increases slowly with decreasing T/W until about T/W = 0.2, whereupon it rises faster. These curves, together with the weight-thrust relationship for the engine, would give the optimum power level and off-optimum losses.

Another technique for utilizing low thrust/weight ratios with less gravitational loss is to operate the engine several times only near the perigee of the orbit; thus increasing only the apogee until escape is reached. This has the disadvantage of requiring several engine cycles, passes through the radiation belts, and more difficult timing problems.

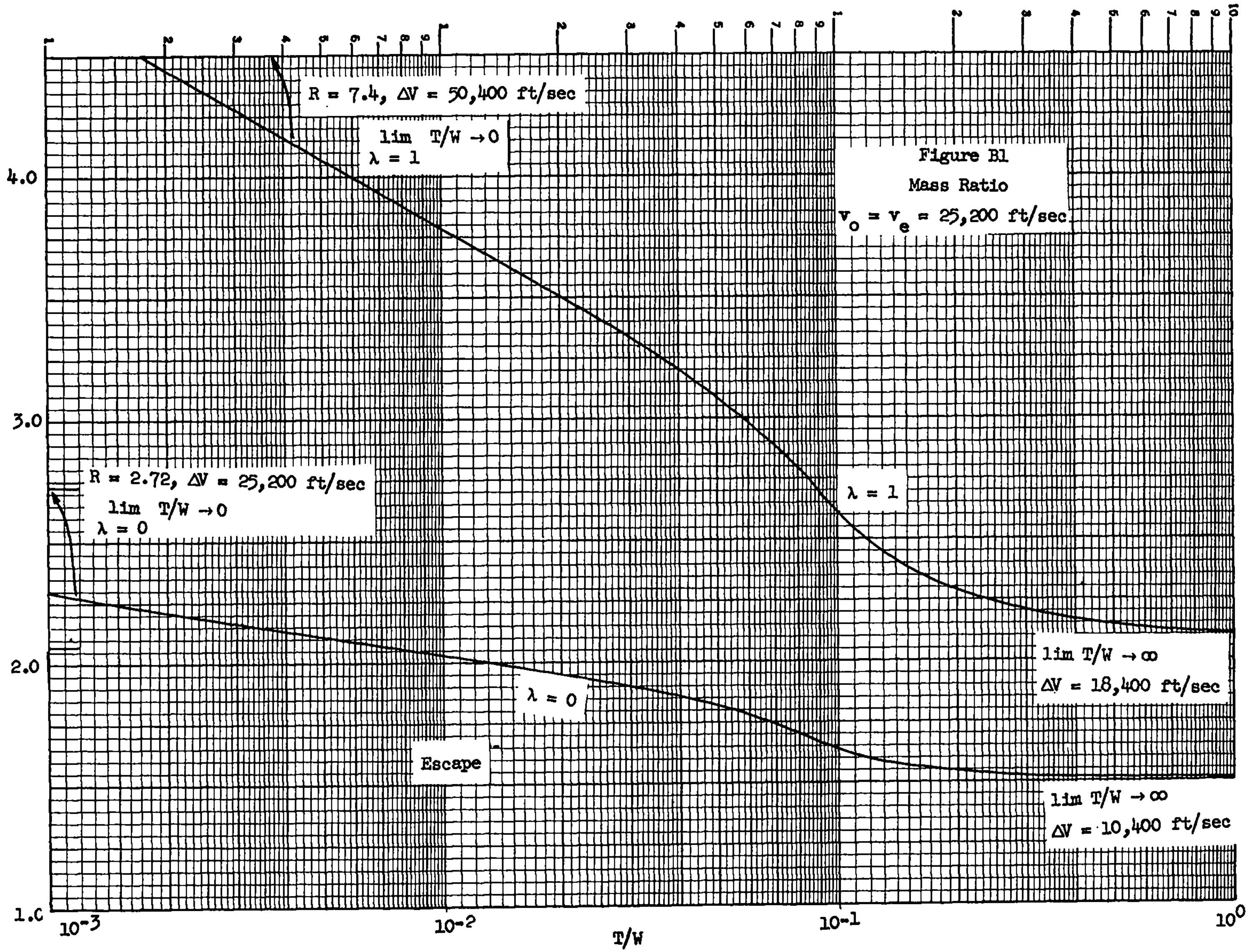

Figure B1
Mass Ratio
$v_o = v_e = 25,200$ ft/sec

SYMBOLS AND UNITS

Equations in the text are generally true with any consistent set of units. We have used engineering units (pounds, feet, seconds) in our numerical examples.

SYMBOLS

e	2.718 ...
f	tankage fraction
g	gravitational constant, ft/sec^2
I_{sp}	specific impulse (sec)
k	a structural parameter depending upon f and ϵ
K	gravitational constant (ft^3/sec^2)
M	mass, usually of vehicles and components (lbs)
m	mass, general (lbs)
n	number of stages
p	pressure (lbs/in^2)
R	mass ratio
r	orbit radius (ft)
r_o	planet radius (usually earth, ft)
T	absolute temperature (^oR or ^oK)
t	time (sec)
v	velocity (ft/sec)
v_c	circular orbit velocity

v_E	velocity of earth in its orbit
v_e	propellant exhaust velocity, ft/sec
V	vehicle velocity, also short for ΔV (ft/sec)
ΔV	change in vehicle velocity; also mission velocity requirement
Vol.	propellant volume, ft^3
x	distance (ft); also orbit radius/planet radius
y	payload fraction
α	propellant flow rate (lbs/sec); also ratio of orbit radii
β	a parameter, V/v_e
ϵ	engine (plus structure) fraction
θ	angle
ρ	density, lbs/ft^3
σ	tensile strength, lbs/in^2

SUBSCRIPTS

b	burnout
c	circular orbit, also chemical
d	dry (manufactured weight)
E	Earth
e	engine (and structure)
i	stage index
L	payload
m	moon
n	nuclear
o	initial
p	propellant
t	tankage

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*See annotation at end.

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Annotation

References 1, 3, and 6 are the most valuable and extensive and contain numerous further references. Reference 1 is currently the only book devoted to its topic. Reference 3 contains several chapters of interest on mission studies. Krafft Ehricke's trilogy will probably become the standard work in this field when it is available. Until then, his chapter in reference 3 should suffice for a brief review of interplanetary operations.