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of the University of California
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ABSTRACT

The AYERM code is a computer program which has been developed for the high-temperature gas-cooled reactor (HTGR) safety research program. It is a conjunction of the heat conduction code, AYER, and a set of special subroutines. This modified AYER code can predict the time-dependent release of volatile fission products from a reactor core during a hypothetical loss-of-forced-circulation (LOFC) accident. The computation scheme is based on the finite element method. The function of the AYER code is to compute the temperature distribution and the temperature history of a reactor during an LOFC accident. The subroutines perform two functions. One group of the subroutines provides the essential input data, such as the properties, configuration, initial and boundary conditions, etc., of the reactor core. The other group combines the computed instant local temperature with the fuel model parameters (i.e., the decay and release constants, and the irradiation history of the fuel) to perform the fission product release calculations.

I. INTRODUCTION

The design of the graphite-moderated high temperature gas-cooled reactor (HTGR) is discussed in Ref. 1. One problem of concern in safety studies of this reactor is the elevating temperature of the core during a hypothetical loss-of-forced-circulation (LOFC) accident. The rising temperature causes the coatings on the fuel particles to fail. Since the function of the coatings is to retain the fission products, the probability for release of the fission products will increase. Because of their importance with respect to health effects,

it is essential in safety research to predict the time-dependent release of the fission products during the LOFC accident. The AYERM code has been designed for this purpose. The foundation of AYERM is the AYER code,[2] with the addition of eight subroutines:

- GEOMTY provides a model configuration for the reactor core that is needed in the finite element method (FEM) calculation.
- INITIAL supplies all the initial condition data and the fuel failure and fission product release characteristics.
- BNDRY specifies all the boundary conditions which are essential in heat transfer computations.
- PROP supplies all the thermal and material property data for the reactor core and its surroundings.
- POWER produces the local heat source data caused by the decay power in the active core.
- TRANS computes the time-dependent release of a fission product of interest.
- SINT provides a linear interpolation process between data points.
- LEAST performs a least-square fitting process.

The AYER code has been documented in Ref. 2. The essential features of the subroutines are presented in this report. A listing of the subroutines is given in Appendix A.

II. THE MODEL REACTOR CORE CONFIGURATION

To simplify the calculations, the complex configuration of a reactor core is idealized as an axisymmetric cylinder. Figure 1 illustrates the model configuration which consists of the active core, the core support, and the surrounding reflectors. To carry out the heat transfer analysis by the finite element method, the cylindrical model is partitioned into a system of ring elements. The rectangular cross sections of these ring elements are also shown as an example in Fig. 1. The sizes of these elements can be adjusted as one desires. This is accomplished by changing a few parameters in the subroutine GEOMTY. In principle, the degree of accuracy of the computed temperature distribution can be improved by employing more finite elements of smaller size. However, it has been shown[3] that beyond a certain size limit, the improvement in accuracy becomes very small, and the computation time increases considerably.

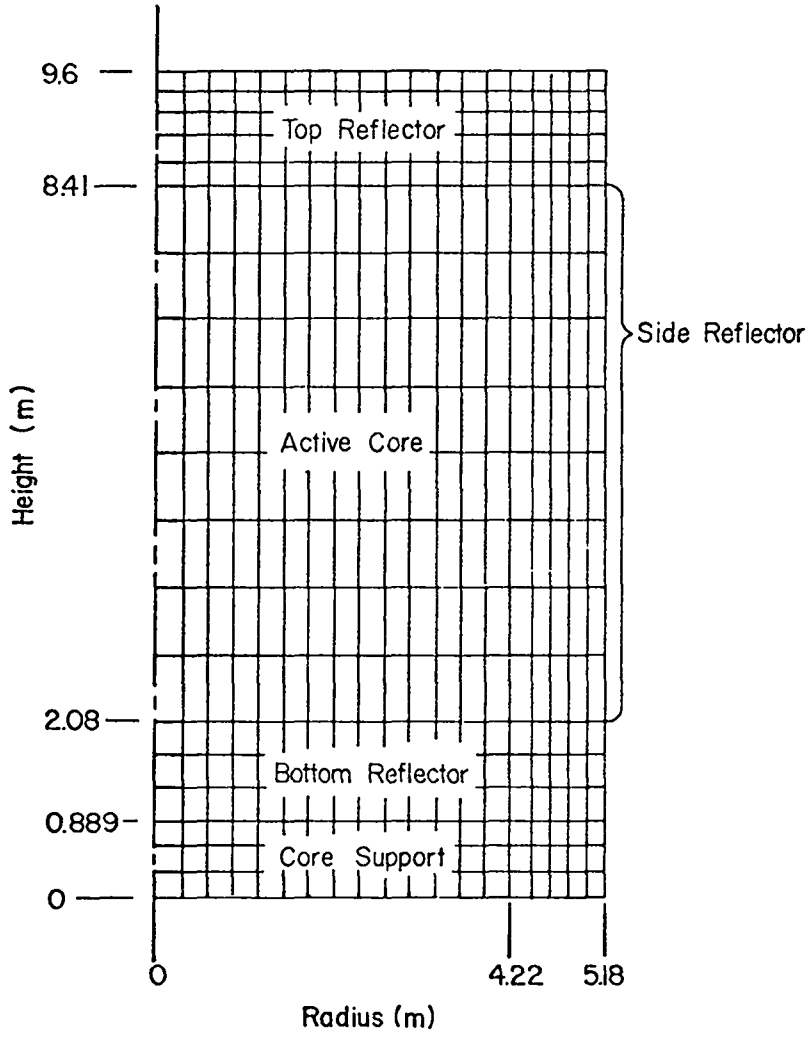

Fig. 1.
Finite element mesh.

III. HEAT TRANSFER MODEL

To formulate the heat transfer analysis for the partitioned reactor core configuration, the following assumptions were made.

A. Properties

The thermal and material properties within each ring element in the model reactor core are taken to be anisotropic and temperature-dependent, but constant within the element during a time step.

B. Heat Transfer Mechanisms

In the interior of the reactor core, the thermal energy is transferred by conduction. On the external boundaries of the configuration, the energy is transferred to the surroundings by conduction and by radiation. We have neglected

the heat loss from the reactor core due to natural convection of the helium coolant. A justification for this is given in Appendix B.

C. Power Density

The decay power density within the active core is a function of position and time. Therefore, the heat source in the core after the shutdown is not uniform. To model this volumetric heat distribution, a set of radial and axial power factors is specified, based on the nominal power density at full power.

D. Integration Procedure

In performing transient calculations, a step-by-step procedure is introduced. In this computing procedure, one uses the results of the previous time, $(t-\Delta t)$, as the initial values to compute the results at time t. Furthermore, the changes in material properties and in decay power are assumed to be negligible during each time step.

IV. INITIAL AND BOUNDARY CONDITIONS

In the model calculations, the initial temperatures at all the node points of the cross section of every ring element are specified at the beginning of the program. This process is accomplished in the subroutine INITIAL. These temperatures are based on the steady-state operating conditions of the reactor just before the LOFC accident.

The external boundary surfaces of the reactor core are enclosed by a layer of helium gas. Beyond this layer is the thermal barrier which is attached to the inside of the prestressed concrete reactor vessel (PCRV). The outer boundary of the barrier is surrounded by cooling water passages embedded in the PCRV. To compute the heat transfer from the reactor to the cooling water, the effective thermal conductance has been employed. The effective thermal conductance is defined as the reciprocal of the sum of the reciprocals of heat transfer coefficient in each layer. That is

$$\frac{1}{h_e} = \frac{1}{h_g} + \frac{1}{h_i} + \frac{1}{h_p} , \qquad (1)$$

where

h_e is the effective conductance used in the AYERM code ($W/m^2 \cdot K$);

h_g is the combined conductance for the helium gas layer which includes the radiation and conduction transfer in the helium ($W/m^2 \cdot K$);

h_i is the conductance for the PCRV thermal barrier ($W/m^2 \cdot K$); and

h_p is the effective heat transfer coefficient between the thermal barrier and the cooling water ($W/m^2 \cdot K$).

A set of effective heat conductances which model the heat transfer from the reactor core configuration to the surrounding cooling water is built in the subroutine BNDRY. A detailed description and pertinent parameters for these effective conductances are given in Appendix C.

V. FUEL MODEL PARAMETERS

The pyrocarbon (PyC) and silicon carbide (SiC) coatings on the fuel kernel of HTGR fuel particles are fission product retainers. The performance of these coatings depends strongly on their temperatures and on their irradiation exposures. The correlation among these quantities is not unique, and there exists no general functional relationship among them. The new General Atomic standard safety analysis report (GASSAR) fuel release model[1,4] has been adapted in the subroutines through numerical interpolation.

Each chemical element has an empirically derived release "constant" used to describe its observed fractional release per unit time from the coated fuel particles. In the subroutine TRANS, we have adapted the Arrhenius formula for the functional relationship between the temperature and a release constant. That is:

$$\text{release constant} = A \exp(-AK/\text{temperature}), \tag{2}$$

where the dimensional constants $A(h^{-1})$ and $AK(K)$ were determined from the data which were furnished by the GASSAR fuel release model.[1,4]

It has been recognized in the previous sections that the decay power density in the active core is inhomogeneous, and that the boundaries of the reactor core are nonadiabatic. Therefore, the temperature distribution in the core elevates nonuniformly during an LOFC accident. Furthermore, it has been established that the release constants and the failed fuel fractions depend on temperature; consequently, both of them are position- and time-dependent. To compute the release of a fission product from the core, the following method was used.

(1) The releases from every ring element were computed to obtain the release during each short time interval; then the release from the core is the sum of these releases weighted by the local radial and axial power factors.

(2) The total amount of the fission product released, from the onset of the LOFC up to time t, is the summation of the releases in all the preceding intervals with corrections due to radioactive decays.

All of these calculations were performed with the subroutine TRANS.

VI. NUMERICAL DATA

The AYERM code was developed to predict the release of fission products from an HTGR during a hypothetical LOFC accident. In order to compare our results with those reported in Refs. 5 and 6, we have adapted the numerical data given in these references. Some of the important ones which have been modified are given in this section.

A. Power Density Factors

The radial and axial power density factors which were used in computing the decay power distribution in the active core are listed in Table I. These factors were deduced from the data given in Ref. 5.

B. Decay Power

Formulas for the decay power following the shutdown of a reactor are empirically determined. The one used in Ref. 5 is

$$\frac{P(t)}{P_o} = 10^{\alpha(t)}, \tag{3}$$

where P_o and $P(t)$ are the power at the time of shutdown and at t hours thereafter, respectively; and the function $\alpha(t)$ is given by

$$\alpha(t) = -1.75 - 0.248\, T(t) + 0.0059\, T^2(t) - 0.00465\, T^3(t) - 0.0021\, T^4(t), \tag{4}$$

where $T(t) = \log_{10} t$,

and t is time in hours.

TABLE I

POWER FACTORS

Radial Zone (center to outer)	Radial Power Factors	Axial Zone (bottom to top)	Axial Power Factors
1	0.83	1	0.67
2	0.83	2	0.90
3	1.16	3	0.97
4	1.16	4	1.01
5	1.16	5	1.17
6	0.96	6	1.14
7	0.96	7	1.08
8	0.96	8	1.03
9	0.96		
10	1.07		
11	1.07		
12	1.07		
13	0.90		
14	0.90		

Equation (3) has a singularity at t = 0. The decay power formula which has been used in the AYERM code is Eq. (3) with a 10% increment.*

Recently some study on the afterheat for the HTGR was reported.[7] The data presented in Ref. 7 can be correlated by the following equation:

$$\frac{P(t)}{P_o} = 0.128\ (t+0.0003796)^{-0.261}, \qquad (6)$$

where the time t is in units of seconds. A comparison of this formula with the one given by Eq. (3) is shown in Fig. 2. The data points from Ref. 7 are also plotted in the figure.

*This 10% modification is based on the advice of F. Silady of GAC in a personal communication.

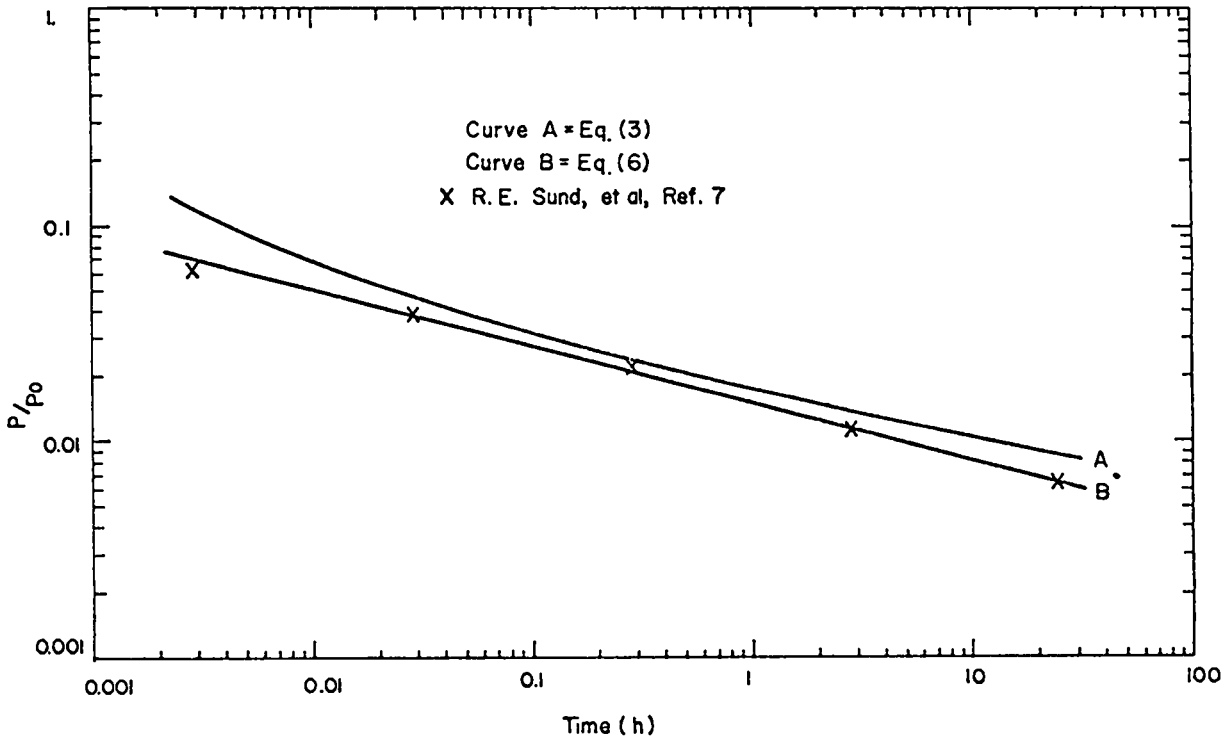

Fig. 2.
Decay power for HTGR.

VII. NOMENCLATURE

A. Model Core Configuration

The nomenclature connected with partitioning the core into finite ring elements is listed below. Some of the key ones are depicted in Fig. 3.

XCORE Radius of the active core.

XSR Radius of the side reflector.

YBS y-coordinate of the upper surface of the core support.

YBR y-coordinate of the upper surface of the bottom reflector.

YCORE y-coordinate of the upper surface of the active core.

YTR y-coordinate of the upper surface of the top reflector.

NXCOD Number of radial dividing node points for the active core used in the finite element method calculation (FEMC).

NXSRD Number of radial dividing node points for the side reflector used in FEMC.

NYBSD Number of axial dividing node points for the core support used in FEMC.

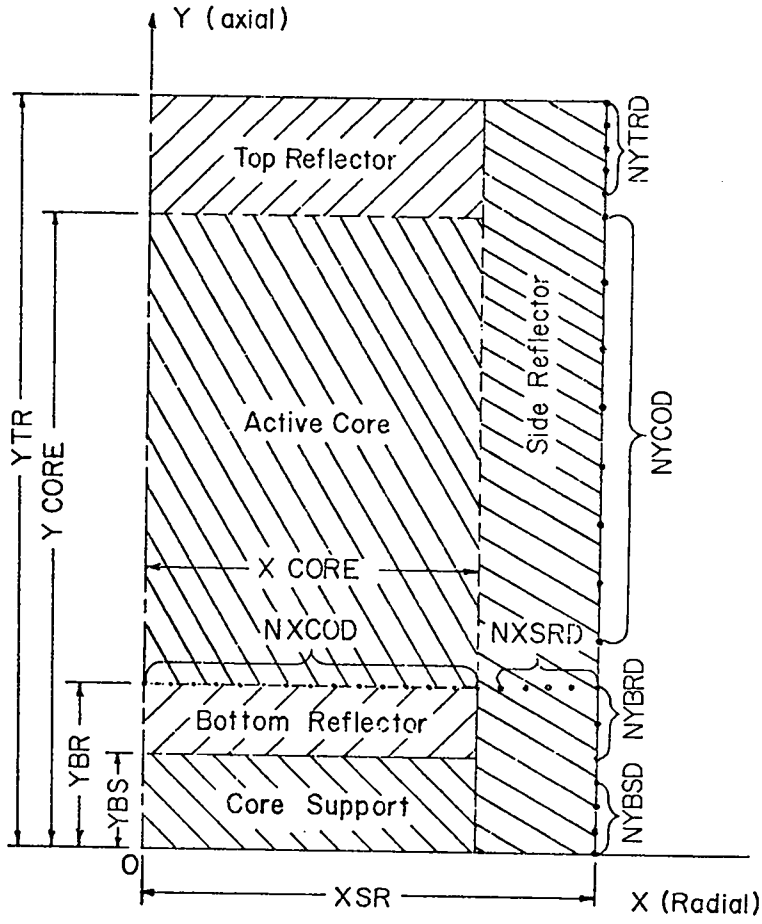

Fig. 3.
Some key nomenclatures for the model core configuration.

NYBRD Number of axial dividing node points for the bottom reflector used in FEMC.

NYCOD Number of axial dividing node points for the active core used in FEMC.

NYTRD Number of axial dividing node points for the top reflector used in FEMC.

NX The total number of radial dividing node points of the radius of the, model core configuration. That is NX = NXCOD + NXSRD.

NY The total number of axial dividing node points for the model core configuration. That is NY = NYBSD + NYBRD + NYCOD + NYTRD.

NNX The total number of radial divisions on the radius of the model core. That is NNX = NX - 1.

NNY The total number of axial divisions for the model core. That is NNY = NY - 1.

NUMNP The total number of node points for the model core used in the FEMC. That is NUMNP = NX x NY.

NUMEL The total number of ring elements used in the FEMC. NUMEL = NNX x NNY.

Y(N) y-coordinate of the Nth node point.

X(N) x-coordinate of the Nth node point.

MATL(N) The material number for the Nth ring element.

IB(N,I) The boundary number of the Ith side of the Nth element.

B. Fuel Model Parameters

As indicated earlier, the release of a fission product from the failed fuel particles depends on the local instant temperature, the fuel failure model, the release and decay constants of the isotope, etc. The nomenclature listed below applies to this aspect.

RF The radial power density factor of a ring element.

AF The axial power density factor of a ring element.

QX The power density of a ring element at a given time after shutdown, in units of (W/m^3).

QF The fractional decay power of the active core at a given time after shutdown.

QSUB The heat of sublimation per unit volume for the graphite (J/m^3).

QLAT The integrated latent heat of sublimation for a ring element at a given time, for comparison with QSUB (J/m^3).

TINC The time increment (s).

VBAR The volume of a ring element (m^3).

TSUB The sublimation temperature of the graphite (K).

TN The instant temperature of a node point at time t (K).

TBAR The volumetric average temperature of a ring element at time t (K).

TAVE The instant average temperature at time t (K).

TMX The instant maximum temperature at time t (K).

XLA The decay constant of the isotope considered (h^{-1}).

R The release constant of the isotope from the failed fuel in a ring element at time t (h^{-1}).

FR The instant local failed fuel fraction.

FFI The instant local inventory-fraction of the isotope due to failed fuel.

B The instant local fractional release of the isotope.

RFI The fractional amount of the isotope released from the onset of the LOFC accident to time t.

FSUM The failed fuel fraction at t-hour after the onset of the LOFC accident.

LBFY The age of the BISO fuel (yr).

LTFY The age of the TRISO fuel (yr).

The following set of nomenclature pertains to the fuel particle coating failure diagrams: RSR, RSR1, TER, TUM, and TUR. The meanings of these parameters are indicated in Figs. 4 and 5. These figures are fuel particle coating failure diagrams for BISO and TRISO fuels, respectively.

C. Heat Transfer

The following list of nomenclature is connected with the material and thermal properties and boundary conditions in heat transfer calculations.

TEMI,TEMJ The bulk fluid temperature adjacent to the Ith and Jth node points, respectively (K).

HI,HJ The heat transfer coefficient at the Ith and Jth node points, respectively ($W/m^2 \cdot K$).

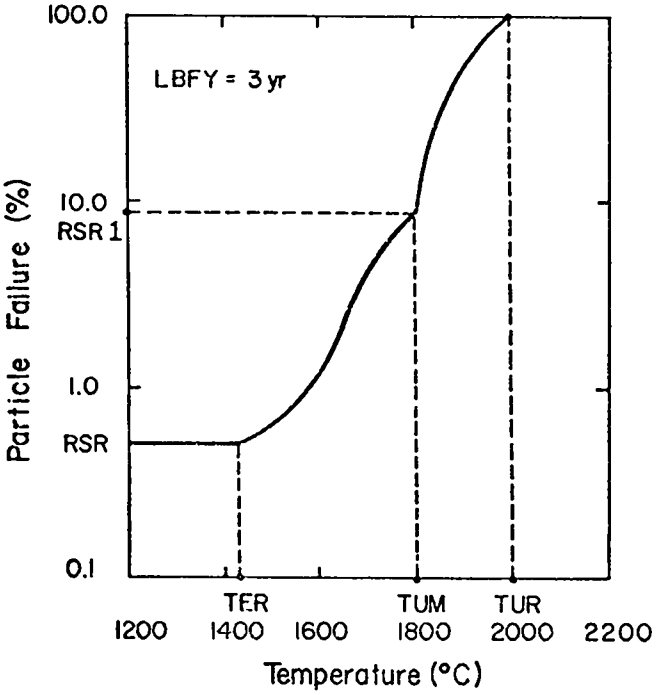

Fig. 4.
BISO fuel particle coating failure diagram and the nomenclature used in fuel model calculations.

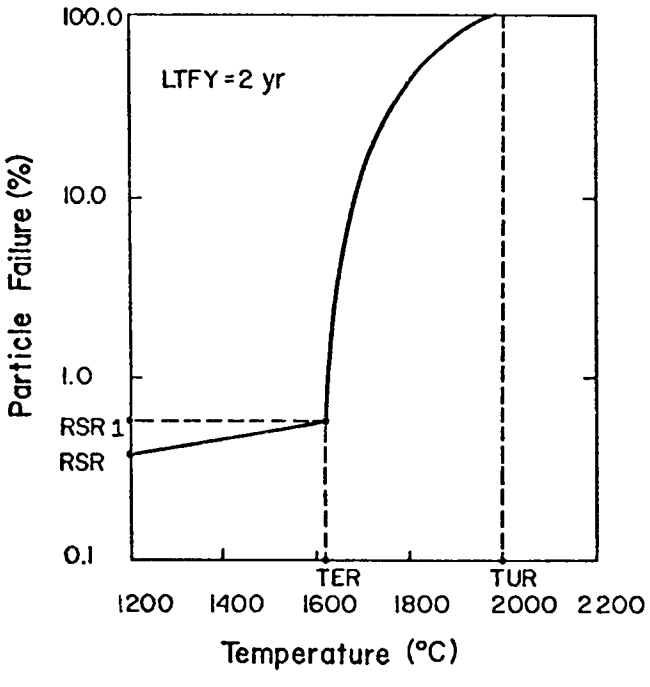

Fig. 5.
TRISO fuel particle coating failure diagram and the nomenclature used in fuel model calculations.

TBA The average temperature of the Ith and Jth node point.

FACT The radiation conductance from the outer surface of the reflector to the PCRV thermal barrier ($W/m^2 \cdot K$).

XLAH The thermal conductivity of the helium ($W/m \cdot K$).

XINS The thermal conductivity of the PCRV thermal barrier ($W/m \cdot K$).

DENS The material density of a ring element (kg/m^3).

HCAP The specific heat capacity of a ring element ($J/kg \cdot K$).

CCAP Table of specific heat capacity for the composite fuel in 1000°F increments, starting at 0°F (Btu/lb-°F).

RCAP Table of specific heat capacity for the reflector in 1000°F increments, starting at 0°F (Btu/lb-°F).

C1 The thermal conductivity of a ring element in radial direction ($W/m \cdot K$).

C2 The thermal conductivity of a ring element in axial direction ($W/m \cdot K$).

D. LEAST

The detailed information about the least-squares polynomial fitting subroutine is given in Ref. 8. The key parameters are:

M The number of input data points.

X A set of independent variables.

F A set of dependent variables.

W A set of weights.

EPS The desired weighted RMS error.

MAXDEG The maximum degree of the fit.

NDEG The degree of the fit.

E. The AYER Code

The AYERM code is based on the AYER code.[2] Some of the important variables used in the AYER code are listed below for convenience.

X x-coordinate of the Nth node.

Y y-coordinate of the Nth node.

QØRT(N) Specified temperature or external heat flow for the Nth node.

KØDE(N) If zero, external heat flow is specified for the Nth node via QØRT(N). If nonzero, temperature is specified via QØRT(N).

TN(N) Temperature of Nth node.

TI(N) Temperature of Nth node for last iteration.

TNI(N)	Temperature of Nth node at the start of a time increment.
IX(N,I)	Node number of the Ith vertex of the Nth element.
IB(N,I)	Boundary number of the Ith side of the Nth element.
MATL(N)	Material number of the Nth element.
NN(N)	Number of nodes describing the Nth element.
NUMNP	Total number of node points.
NUMEL	Total number of elements.
MBAND	Matrix band width.
NPRØB	An integer number to be defined by the user if needed.
NIT	Index of the transient DØ-loop.
NTINC	Total number of time increments.
TINC	Increment of time.
TIME	Actual time (summation of TINCs).
NPRNT	Time increments for which information is printed. (If NPRNT = 4, information for every fourth increment will be printed.)
NI	Index of iteration DØ-loop.
ITER	Total number of iterations requested.
CØNV	Convergence required.
CØNVRG	Actual convergence, defined as the maximum value |TN-TI|.
NCØV	Node number where CØNVRG was obtained.
NPNCH	If node temperatures are to be punched on cards, NPNCH must be nonzero. (Format 10F8.3)
NEPRNT	If nonzero, node and element information will not be printed out.
KAT	If zero, region has uniform thickness of 1. If nonzero, region is axisymmetric about y-axis.
VØL(N)	Volume of material N.
QVG(N)	Total heat generated in material N.
QVT(N)	Transient "source" value for material N.
QVV(N)	Velocity "source" value for material N.
QVTØT(N)	Summation of QVG, QVT, AND QVV.
TAVE(N)	Average temperature of material N.
VØLUME	Total volume.
TBAR	Average temperature of entire body.
QVGTØT	Total heat generated.

QVTTØT	Total transient "source" value.
QVVTØT	Total velocity "source" value.
QVØL	Summation of QVGTØT, QVTTØT, and QVVTØT.
AB(N)	Area of boundary N.
TBAV(N)	Average temperature of boundary N.
QBC(N)	Heat flow by conduction in boundary N.
QBV(N)	Enthalpy transfer across boundary N due to velocity.
QBT(N)	Summation of QBC and QBV.
QBCTØT	Total heat flow by conduction across boundaries.
QBVTØT	Total enthalpy transfer across boundaries due to velocity.
QBTØT	Summation of QBCTØT and QBVTØT.
TBND	Average temperature of all boundaries.
QBAL	Overall heat balance (QVØL + QBCTØT).
HED(8)	Problem title.
KEYPLT	Approximate number of isotherms to be plotted on isoplot.
SCALE(4)	Minimum x, maximum x, minimum y, maximum y for explanded plot.
TISØ(N)	Temperature for the Nth isotherm.
NISØ	Actual number of isotherms.
NMAT	Number of different materials.

VIII. TYPICAL RESULTS

Results of the AYERM code have been published in Ref. 3 and subsequent Los Alamos Scientific Laboratory reports on the Quarterly Progress of Research on Reactor Safety and Technology. This section presents some of these results in order to document them for the reader and indicate what can be expected from the code.

The input data for the calculations are all included by means of FORTRAN DATA and REPLACEMENT statements in the subroutines just described. These are given in Appendix A. Figure 6 shows the maximum and average temperatures in the active core vs time after the onset of the LOFC accident. The hottest part of the core begins to sublimate at about 17 h. A companion plot, more useful to indicate the extent of fuel failure and fission product release is Fig. 7, showing the fraction of the active core above a certain temperature at various times. The core is initially at a very uniform temperature of 1110 K because of the

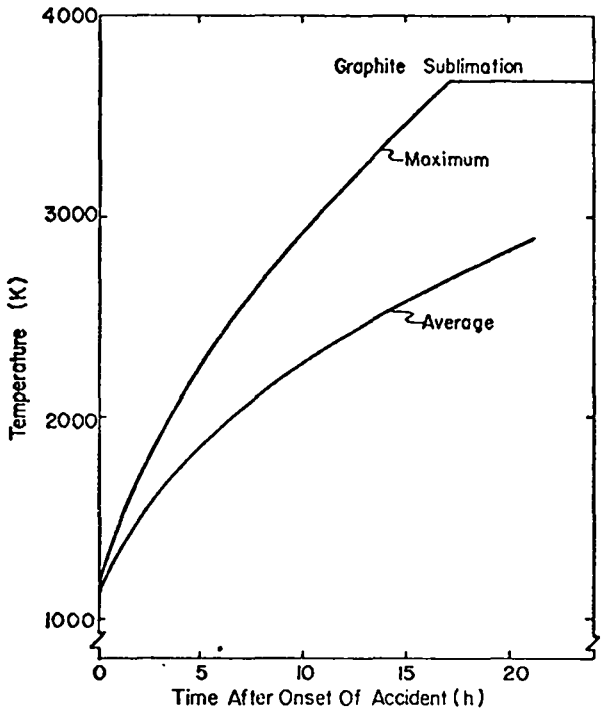

Fig. 6.
Temperatures in the active core vs time.

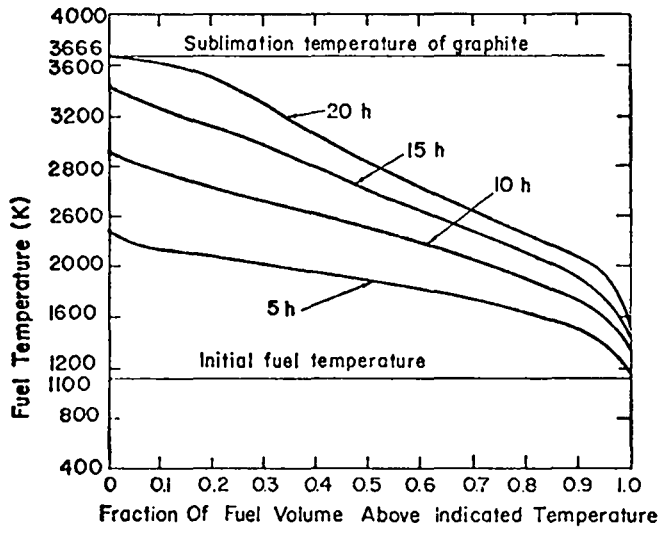

Fig. 7.
Fraction fuel volume above the indicated temperatures.

helium flow which is orificed to achieve optimum temperature flattening at full power. At later times, with no helium cooling, regions with highest heat generation increase most rapidly in temperature, leading to the almost 2000 K temperature difference between various parts of the core. Figures 8 and 9 show the resulting failure of BISO and TRISO fuels, respectively, as a consequence of the rising core temperatures. Figures 10 and 11 give the corresponding fractional release of ^{131}I from the failed fuel. A core consisting of fuel of uniform age would be represented by a single curve on these two figures and the total release would be the sum of the appropriate fraction from BISO and TRISO fuels, multiplied by their respective initial inventory. A core of mixed fuel ages would be further subdivided accordingly. For a particular reactor, where the local distributions of fuel ages, fuel type, fission product inventory, and power peaking factors are known, these variables can be built into the AYERM code model, and resulting total fission product release will be the appropriate sum from all sources.

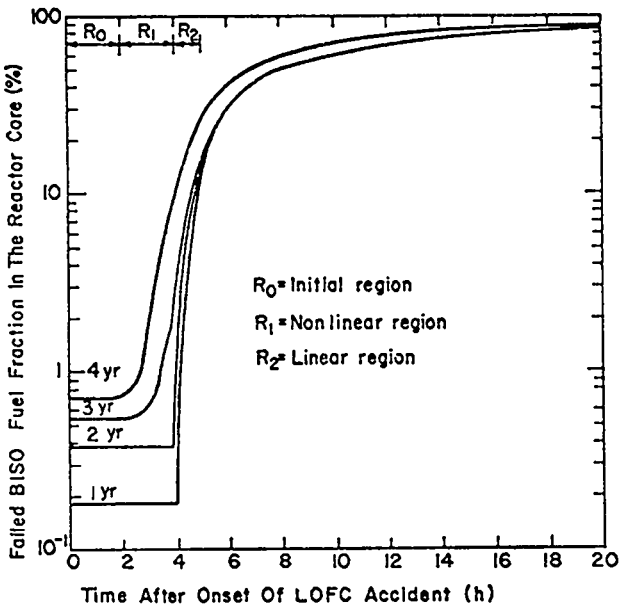

Fig. 8.
Failure fraction for BISO fuels.

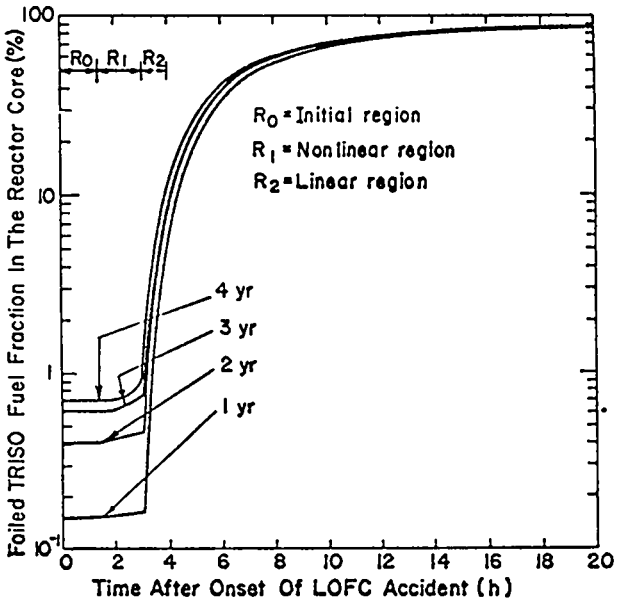

Fig. 9.
Failure fraction for TRISO fuels.

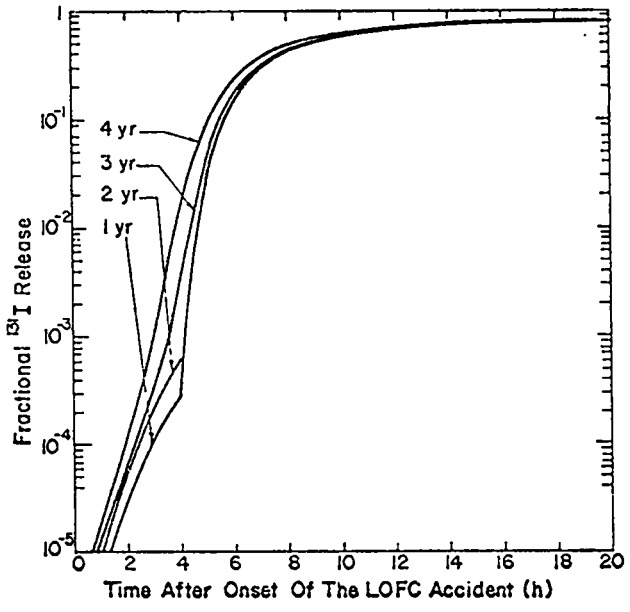

Fig. 10.
Time-dependent fractional releases from BISO fuels.

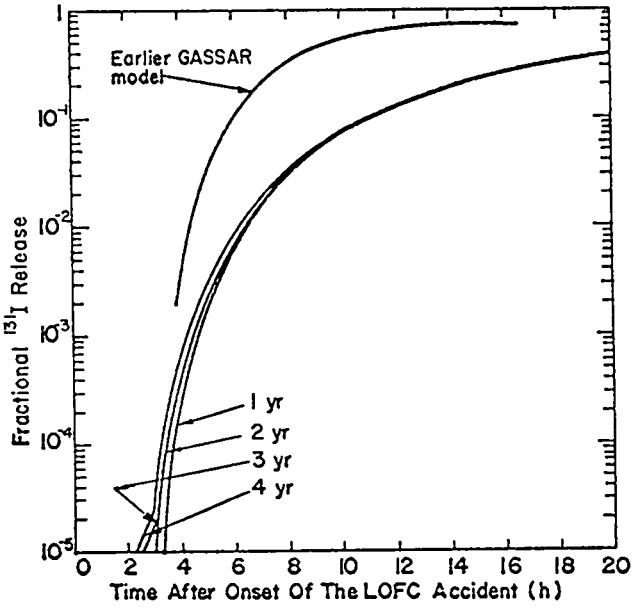

Fig. 11.
Time-dependent fractional releases from TRISO fuels.

IX. SUMMARY AND DISCUSSION

A computing model has been developed that can predict the time-dependent release of fission products from an HTGR core during an LOFC accident. The thermal analysis of the model is based on the finite element method.[2] The time-dependent computations were carried out through the step-by-step approximation scheme. For the relations between the temperature and the fuel elements in the core, the new fuel release parameters[1,4] have been adapted. All the other numerical data constructed in the model were taken from Refs. 5 and 6. The model can easily be adapted to conform with different geometries, properties, boundary conditions, and assumptions.

No genetic relationships among fission products were considered in the present model. In computing the release of a fission product with long-lived precursors, the model produces a more conservative result. To improve this aspect, we have studied the genetic relationships of all the fission products which are of importance to safety research,[9] and have constructed a model decay scheme. This model decay scheme can delegate all the genetic relationships of those fission products whose transformation does not involve neutron absorption processes. When it is incorporated into the AYERM code, this new model will be able to compute the time-dependent release of fission products in any one of the many different decay chains under consideration. The mathematical formation of the model decay scheme is represented in Appendix D.

APPENDIX A

SUBROUTINE LISTINGS

```
      SUBROUTINE SINT(T,PRO,DATA)
      DIMENSION DATA(10)
      COMMON/CAUSE/NM,MN,TEL
C
C SINT
C
      TK     =0.          $ ND    =6
      DO 50 I=1,ND
      IF (T.LT.TK) GO TO 60
   50 TK     =TK+1.
      PRO    =DATA(ND)                                        $ RETURN
   60 IF (TK.EQ.0.)GO TO 70
      PRO    =DATA(I-1)+(DATA(I)-DATA(I-1))*(T-TK+1)          $ RETURN
   70 CONTINUE
      PRO    =DATA(1)
      RETURN
```

```
      END


      SUBROUTINE LEAST(M,X,F,W,EPS,MAXDEG,NDEG,ARRAY,R)
C
      DIMENSION X(1),F(1),W(1),ARRAY(1)
      DOUBLE PRECISION R(1),SUM,CK,TEMP
C
      IB=MAXDEG+1
      IBL2=MAXDEG+1
      IC=IB+IBL2
      I0L1=IC+MAXDEG
      I1L1=I0L1+M
      RM=M
      TOL=RM*EPS**2
C
      NDEG=0
      S=0.0
      SUM=0.0D0
      DO 1 I=1,M
      S=S+W(I)
1     SUM=SUM+DBLE(W(I))*DBLE(F(I))
      RN0=S
C
      CK=SUM/RN0
      ARRAY(IC)=CK
      ERROR=0.0
      DO 2 I=1,M
      R(I)=CK
2     ERROR=ERROR+W(I)*SNGL(CK-DBLE(F(I)))**2
      IF (NDEG.EQ.MAXDEG) GO TO 14
      IF (EPS.LT.0.0) GO TO 3
      IF (ERROR.LE.TOL) GO TO 14
C
3     NDEG=1
      ES=ERROR
      SUM=0.0D0
      DO 4 I=1,M
4     SUM=SUM+DBLE(W(I))*DBLE(X(I))
C
      ARRAY(1)=SUM/RN0
C
      S=0.0
      SUM=0.0D0
      DO 5 I=1,M
      ARRAY(I1L1+I)=X(I)-ARRAY(1)
      S=S+W(I)*ARRAY(I1L1+I)**2
      TEMP=DBLE(F(I))-R(I)
5     SUM=SUM+DBLE(W(I))*DBLE(ARRAY(I1L1+I))*TEMP
      RN1=S
C
      CK=SUM/RN1
      ARRAY(IC+1)=CK
C
      ERROR=0.
      DO 6 I=1,M
      R(I)=R(I)+CK*DBLE(ARRAY(I1L1+I))
6     ERROR=ERROR+W(I)*SNGL(R(I)-DBLE(F(I)))**2
      IF (ERROR.GT.ES.AND.EPS.GE.0.0) GO TO 12
      IF (NDEG.EQ.MAXDEG) GO TO 14
      IF (ERROR.LE.TOL.AND.EPS.GE.0.0) GO TO 14
      DO 7 I=1,M
```

```
7       ARRAY(I0L1+I)=1.0
        NDEG=2
        K=2
C
8       ES=ERROR
C
        ARRAY(IBL2+K)=RN1/RN0
C
        SUM=0.0D0
        DO 9 I=1,M
9       SUM=SUM+DBLE(W(I))*DBLE(X(I))*DBLE(ARRAY(I1L1+I))**2
        ARRAY(K)=SUM/RN1
C
        S=0.0
        SUM=0.0D0
        DO 10 I=1,M
        ARRAY(I0L1+I)=(X(I)-ARRAY(K))*ARRAY(I1L1+I)
       1                -ARRAY(IBL2+K)*ARRAY(I0L1+I)
        S=S+W(I)*ARRAY(I0L1+I)**2
        TEMP=DBLE(F(I))-R(I)
10      SUM=SUM+DBLE(W(I))*DBLE(ARRAY(I0L1+I))*TEMP
        RN0=RN1
        RN1=S
C
        IT=I0L1
        I0L1=I1L1
        I1L1=IT
C
        CK=SUM/RN1
        ARRAY(IC+K)=CK
C
        ERROR=0.0
        DO 11 I=1,M
        R(I)=R(I)+CK*DBLE(ARRAY(I1L1+I))
11      ERROR=ERROR+W(I)*SNGL(R(I)-DBLE(F(I)))**2
        IF (ERROR.GT.FS.AND.EPS.GE.0.0) GO TO 12
        IF (NDEG.EQ.MAXDEG) GO TO 14
        IF (ERROR.LE.TOL.AND.EPS.GE.0.0) GO TO 14
        NDEG=NDEG+1
        K=K+1
        GO TO 8
C
12      NDEG=NDEG-1
        ERROR=ES
        DO 13 I=1,M
13      R(I)=R(I)-CK*DBLE(ARRAY(I1L1+I))
C
14      EPS=SQRT(ERROR/RM)
        RETURN
        END
        FUNCTION EVAL(Y,N,ARRAY,MAXDEG)
C
        DIMENSION ARRAY(1)
        IB=MAXDEG+1
        IC=MAXDEG+IB-1
C
        IF (N.GT.0) GO TO 1
        EVAL=ARRAY(IC)
        RETURN
1       IF (N.GT.1) GO TO 2
        EVAL=ARRAY(IC)+ARRAY(IC+1)*(Y-ARRAY(1))
        RETURN
```

```
C
2       DKP2=ARRAY(IC+N)
        DKP1=ARRAY(IC+N-1)+(Y-ARRAY(N))*DKP2
        NL2=N-2
        IF (NL2.LT.1) GO TO 4
        DO 3 L=1,NL2
        K=1+NL2-L
        DK=ARRAY(IC+K)+(Y-ARRAY(K+1))*DKP1-ARRAY(IB+K)*DKP2
        DKP2=DKP1
3       DKP1=DK
4       EVAL=ARRAY(IC)+(Y-ARRAY(1))*DKP1-ARRAY(IB)*DKP2
        RETURN
        END


        SUBROUTINE GEOMTY
        COMMON X(1000),Y(1000),QORT(1000),KODE(1000),TN(1000),T1(1000),     32
       1TNI(1000),IX(1000,7),IB(1000,6),MATL(1000),NN(1000),NUMNP,NUMEL,    33
       2MBAND,NPROB,NIT,NTINC,TINC,TIME,NPRNT,NI,ITER,CONV,CONVRG,NCOV,    00763
       3NPNCH,NEPRNT,KAT,VOL(50),QVG(50),QVT(50),QVV(50),QVTOT(50),        00764
       4TAVE(50),VOLUME,TBAR,QVGTOT,QVTTOT,QVVTOT,QVOL,AB(50),TBAV(50),    00765
       5QBC(50),QBV(50),QBT(50),QBCTOT,QBVTOT,QBTOT,TBND,QBAL,HED(8),      00766
       6KEYPLT,SCALE(4),TISO(20),NISO,NMAT                                 00767
        COMMON/GEOX/NX,NNX,NXCOD,NNXCO,NXSRD,JB
        COMMON/GEOY/NY,NNY,NYCOD,NNYBS,NYBRD,NYTRD,NEBS,NYBSD
        COMMON/ELEM/NEBR,NECO,NETR,NXX,NXX1,NE1
C
C * BETTER MESHER
C
        DIMENSION NR(12),MAT(20),YN(60),XN(60),XL(10,10),XR(10,10),YB(10,100771
       10),YT(10,10),II(20),KY(20,20)                                       00772
C   GEOMTY
        XCORE=139.          $ NXCOD= 15
        NNXCO=NXCOD-1       $ XX   = FLOAT(NNXCO)
        DX    =XCORE**2/XX
        YBS   =35.          $ NYBSD=  4
        XN(1)=  0.0         $ YN(1)=  0.0
        NNYBS=NYBSD-1       $ YY   =FLOAT(NNYBS)
C
C Y-COORDINATE OF DIVIDING POINTS IN BOTTOM SUPPORT MATERIAL
C
        DY    =(YBS  -YN(1))/YY
        DO 310  I=2,NYBSD
  310 YN(I)=YN(I-1)+DY
C
C Y-COORDINATE OF DIVIDING POINTS IN BOTTOM REFLECTOR
C
        YBR   = 82.         $ YBRD =  3.
        DY    =(YBR  -YBS  )/YBRD
        NYBRD=IFIX(YBRD)
        I1    =NYBSD+1      $ I2   =NYBSD+NYBRD
        DO 320  I=I1,I2
  320 YN(I)=YN(I-1)+DY
C
C Y-COORDINATE OF DIVIDING POINTS IN THE CORE
C
        YCORE=331.          $ YCORD=  8.
        DY    =(YCORE -YBR )/YCORD
        NYCOD=IFIX(YCORD)
        IC1   =I2    +1     $IC2  =I2   +NYCOD
        DO 330  I=IC1,IC2
  330 YN(I)=YN(I-1)+DY
```

```
C
C Y-COORDINATE OF DIVIDING POINTS IN THE TOP REFLECTOR
C
      YTR  =378.          $ YTRD =  5.
      DY   =(YTR  -YCORE)/YTRD
      NYTRD=IFIX(YTRD)
      ITR1 =IC2  +1       $ITR2 =IC2  +NYTRD
      DO 340  I=ITR1,ITR2
  340 YN(I)=YN(I-1)+DY
C
C X-COORDINATE OF DIVIDING POINTS IN THE CORE
C
      DO 350  I=2,NXCOD
  350 XN(I)=XN(I-1)+DX
      DO 351  I=2,NXCOD
  351 XN(I)=SQRT(XN(I))
C
C X-COORDINATE OF DIVIDING POINTS IN THE SIDE REFLECTOR
C
      XSR  =178.          $ XSRD =  5.
      DX   =(XSR  -XCORE)/XSRD
      NXSRD=IFIX(XSRD)
      IXSR1=NXCOD+1       $ IXSR2=NXCOD+NXSRD
      DO 360  I=IXSR1,IXSR2
  360 XN(I)=XN(I-1)+DX
      NX   =NXCOD+NXSRD
      NY   =NYBSD+NYBRD+NYCOD+NYTRD
      NNY  =NY-1          $NNX  =NX-1
      NUMNP=NX*NY         $NUMEL=NNY*NNX        $ NE=1
C     NXX  =NEBELCO  JB=NEBELTR    NE1= (NNYBS+NYBRD-1)*NNX
      NEBS =  NNX*NNYBS
      NEBR =  NNX*NYBRD
      NECO =  NNX*NYCOD
      NETR =  NNX*NYTRD
      NXX  =  NEBS+ NEBR
      JB   =  NXX+ NECO
      NXX1 =  NXX+1
      NE1  =  NXX-NNX
C
C MATERIAL NUMBER
C
      DO 410  I=1,NUMEL
      DO 102  J=1,6
      IB(I,J)  =0
  102 IX(I,J)  =0         $ IX(I,7)=0
  410 MATL(I)  =1
C
C     MESH TOPOLOGY
C
      DO 200 I=1,NNY
      DO 205 J=1,NNX
      IX(NE,1)=J +(I-1)*NX                    $ IX(NE,2)=IX(NE,1)+1
      IX(NE,4)=IX(NE,1)+NX                    $ IX(NE,3)=IX(NE,4)+1
      IF(I.EQ.1)    IB(NE,1)=1
      IF(J.EQ.NNX)  IB(NE,2)=2
      IF(I.EQ.NNY)  IB(NE,3)=3
      IF(J.EQ.1)    IB(NE,4)=4
  205 NE   =NE+1
  200 CONTINUE
C GRID COORDINATES
      NP   =1
      DO 300 I=1,NY
```

```
      DO 305 J=1,NX
      Y(NP)=YN(I)
      X(NP)=XN(J)
      X(NP)=X(NP)*.0254 $ Y(NP)=Y(NP)*.0254
  305 NP   =NP+1
  300 CONTINUE
      DO 420   I=1,NXX
      MATL(I)   =2
      MATL(I+JB)=2
  420 CONTINUE
      NEBS =NNX*((NY-NYTRD-NYCOD-NYBRD)-1)
      DO 440  I=1,NEBS
  440 MATL(I)  =4
      DO 430  I=NNX,NUMEL,NNX
      MATL(I-1)=MATL(I-2)=MATL(I-3)=MATL(I-4)=3
  430 MATL(I)  =3
      RETURN
      END


      SUBROUTINE PROP(N,I,J,K,C1,C2,THET,DENS,HCAP,VX,VY,VZ,DTDZ)        00886
      COMMON X(1000),Y(1000),QORT(1000),KODE(1000),TN(1000),T1(1000),    00887
     1TNI(1000),IX(1000,7),IB(1000,6),MATL(1000),NN(1000),NUMNP,NUMEL,   00888
     2MBAND,NPROB,NIT,NTINC,TINC,TIME,NPRNT,NI,ITER,CONV,CONVRG,NCOV,    00889
     3NPNCH,NEPRNT,KAT,VOL(50),QVG(50),QVT(50),QVV(50),QVTOT(50),        00890
     4TAVE(50),VOLUME,TBAR,QVGTOT,QVTTOT,QVVTOT,QVOL,AB(50),TBAV(50),    00891
     5QRC(50),QBV(50),QBT(50),QRCTOT,QBVTOT,QBTOT,TBND,QBAL,HED(8),      00892
     6KEYPLT,SCALE(4),TISO(20),NISO                                      00893
C                                                                        00894
      COMMON/CAUSE/NM,MN,TEL
      COMMON/ELEM/NEBR,NECO,NETR,NXX,NXX1,NE1
      COMMON/GEOX/NX,NNX,NXCOD,NNXCO,NXSRD,JB
      COMMON/GEOY/NY,NNY,NYCOD,NNYBS,NYBRD,NYTRD,NEBS,NYBSD
      COMMON/HCA1/HCAF(50)
      COMMON/ROCK/TM,DE,DH,RM,RS                                         00896
      COMMON/TRANS/QTOT,AMP,KG,VP,RE(7),TO                               00830
      DIMENSION CCAP(6),RCAP(6)
      DATA(CCAP(I),I=1,6)/.15,.335,.42,.442,.453,.457/
      DATA(RCAP(I),I=1,6)/.15,.39,.48,.5,.514,.52/
      DIMENSION RK(15), RT(15)                                           00898
      THET=DENS=HCAP=VX=VY=DTDZ=0.                                       00899
      TEL=(TN(I)+TN(J)+TN(K))/3.
      IF(TEL.GT.3666.)TEL=3666.
      IF(TEL.LT.300.) TEL=300.                                           00901
C
C PROP
C
      NM   =N   $ MN   =MATL(N)          $ T   =(TEL*1.8)-460.
      GO TO (10,20,30,40) MN
C
C     COMPOSITE FUEL
C
   10 DENS = 91.*16.018                  $ T   =T/1000.
      CALL SINT(T,PRO,CCAP)              $ HCAP=PRO*4187.
      HCAF(NIT)=PRO*4187.
      T  =  T*1000.
      IF (T.GT.5000.) GO TO 11
      C2   =1.73*(15.+0.00300*(T-1000.)) $ IF (T.GT.4000.) GO TO 12
      C1   =1.73*( 7.+0.00425*(T-1000.)) $ GO TO 13
   11 C2   =1.73*(27.+0.00900*(T-5000.))
   12 C1   =1.73*(17.-0.00467*(T-4000.))
   13 IF (C1.GT.(1.73*15.5)) C1=1.73*15.5
```

```
      RETURN
C
C     TOP AND BOTTOM REFLECTOR
C
   20 DENS = 85.*16.018                      $ IF (T.GT.5000.) GO TO 21
      C2   =1.73*(20.+0.00325*(T-1000.))     $ IF (T.GT.4000.) GO TO 22
      C1   =1.73*( 7.+0.00650*(T-1000.))     $ GO TO 23
   21 C2   =1.73*(33.+0.00600*(T-5000.))
   22 C1   =1.73*(18.-0.00825*(T-5000.))
   23 IF (C1.GT.(1.73*21.5)) C1=1.73*21.5
   24 T    =T/1000.
      CALL SINT(T,PRO,RCAP)                  $ HCAP=PRO*4187.  $ RETURN
C
C     SIDE REFLECTOR
C
   30 DENS =107.*16.018                      $ IF (T.GE.2250.)  GO TO 31
      C2   =1.73*(24.5+.007*(T-1000.))          $ IF (T.GE.2100.) GO TO 31
      C1   =1.73*(16.+0.00700*(T-1000.)) $ GO TO 24
   31 C1   =1.73*(19.-0.00433*(T-3000.))
      C2   =1.73*(30.-0.00500*(T-3000.)) $ GO TO 24
   40 DENS = 93.*16.018
      C1=C2=1.73*13.9                    $ GO TO 24
      END


      SUBROUTINE INITIAL                                                    00821
      COMMON X(1000),Y(1000),QORT(1000),KODE(1000),TN(1000),T1(1000),       00822
     1TNI(1000),IX(1000,7),IB(1000,6),MATL(1000),NN(1000),NUMNP,NUMEL,      00823
     2MBAND,NPROB,NIT,NTINC,TINC,TIME,NPRNT,NI,ITER,CONV,CONVRG,NCOV,       00824
     3NPNCH,NEPRNT,KAT,VOL(50),QVG(50),QVT(50),QVV(50),QVTOT(50),           00825
     4TAVE(50),VOLUME,TBAR,QVGTOT,QVTTOT,QVVTOT,QVOL,AB(50),TBAV(50),       00957
     5QBC(50),QBV(50),QBT(50),QBCTOT,QBVTOT,QBTOT,TBND,QBAL,HED(8),         00827
     6KEYPLT,SCALE(4),TISO(20),NISO                                         00828
C                                                                           00829
      DIMENSION TX(10),FY(10),W(10),ANSF(10)
      DOUBLE PRECISION R(10)
      COMMON/ELEMENT/XLA,A,AKT
      COMMON/FACT/AF(50),RF(50),QF,AXP
      COMMON/ELEM/NEBR,NECO,NETR,NXX,NXX1,NE1
      COMMON /FIT/AR(50)
      COMMON /FTACP/FUEL,FUELT,RSR,RSR1,TER,TUM,TUR,LBFY,LTFY,TER1
      COMMON/GEOX/NX,NNX,NXCOD,NNXCO,NXSRD,JB
      COMMON/GEOY/NY,NNY,NYCOD,NNYBS,NYBRD,NYTRD,NEBS,NYBSD
      COMMON/ROCK/TM,DE,DH,RM,RS                                            00831
      COMMON/TRANS/QTOT,AMP,KG,VP,RE(7),TO                                  00830
C     INITIAL
C      CONSTANTS
C*
      XLA=0.00358                                                           I=131
      XLA=1.04E-1                                                           I=135
      XLA =8.91E-3                                                          TE=132
      XLA=3.34E-2                                                           I=133
C     FUEL=1.0 FOR BISO  $  FUEL=2.0 FOR TRISO
C
      FUEL=1.
      LBFY=1
      LBFY=2
      LBFY=4
      LBFY=3
C
      FUEL=2.
      LTFY=4
```

```
      LTFY=3
      LTFY=2
      LTFY=1
C     FUELT=2, FOR LINEAR MODEL  $  FUELT=1, FOR NONLINEAR MODEL
      FUELT=2,0
      FUELT=',0
      IF (FUEL.EQ,1,)4,5
C       BISO   ************************************************
    4 CONTINUE
      A  =2578,82                                                    BISO
      AKT=-18490,9                                                   BISO
      TUM=2073,
      TUR=2273,                                                      1,2BISO
      TX(2)=1723,
      TX(3)=1773,
      TX(4)=1823,
      TX(5)=1873,
      TX(6)=1923,
      TX(7)=1973,
      TX(8)=2023,
      TX(9)=2073,
      IF (LBFY-3)9,7,8
    9 CONTINUE
      TER=2073,                                                      1,2BISO
      IF (LBFY.EQ,1) 17,18
C     GASSAR 7/18/75  BISO 1 YR FUEL T-FAIL 2073-2273 RSR=0,0018
   17 RSR=0,0018                                                     1YBISO
      GO TO 21
C       GASSAR 7/18/75  BISO 2 YR FUEL T-FAIL 2073-2273 RSR=0,0038
   18 RSR=0,0038                                                     2YBISO
      GO TO 21
    7 CONTINUE
C     GASSAR 7/18/75  BISO 3 YR FUEL T-1693-2073-2273 RSR=0,0054
      RSR=0,0054
      TER=1693,
      TX(1)=TER
      FY(1)=0,0054
      FY(2)=0,0058
      FY(3)=0,007
      FY(4)=0,0088
      FY(5)=0,012
      FY(6)=0,023
      FY(7)=0,048
      FY(8)=0,07
      FY(9)=0,088
      GO TO 6
    8 CONTINUE
C     GASSAR 7/18/75  BISO 4 YR FUEL T-1683-2073-2273 RSR=0,0071
      RSR=0,0071
      TER=1683,
      TX(1)=TER
      FY(1)=0,0071
      FY(2)=0,0104
      FY(3)=0,017
      FY(4)=0,035
      FY(5)=0,086
      FY(6)=0,16
      FY(7)=0,25
      FY(8)=0,34
      FY(9)=0,44
      GO TO 6
C       BISO   ************************************************
```

```
C
C       TRISO          $$$$$      $$$$$     $$$$$
    5 CONTINUE
      A=7,5957                                                          TRISO
      AKT=-13823,2                                                      TRISO
      TER1=1473,                                                        TRISO
      TUM=2073,
      TUR=2273,                                                         TRISO
      TX(1)=1473,
      TX(2)=1550,
      TX(3)=1600,
      TX(4)=1650,
      TX(5)=1700,
      TX(6)=1750,
      TX(7)=1800,
      TX(8)=1850,
      GO TO (11,12,13,14)LTFY
   11 CONTINUE
C     * 1-YR--TRISO
      RSR=0,0015
      TER=1940,                                                         1YTRISO
C       1-YR--TRISO
      FY(1)=0,0015
      FY(2)=0,00156
      FY(3)=0,00159
      FY(4)=0,00162
      FY(5)=0,00164
      FY(6)=0,00168
      FY(7)=0,00170
      FY(8)=0,00173
      FY(9)=0,0018
      GO TO 6
   12 CONTINUE
C       2-YR--TRISO
      RSR=0,004
      TER=1898
      FY(1)=0,0040
      FY(2)=0,00428
      FY(3)=0,00446
      FY(4)=0,00465
      FY(5)=0,00485
      FY(6)=0,00508
      FY(7)=0,0053
      FY(8)=0,00554
      FY(9)=0,0058
      GO TO 6
   13 CONTINUE
C       3-YR--TRISO
      RSR=0,006
      TER=1883
      FY(1)=0,006
      FY(2)=0,00635
      FY(3)=0,00655
      FY(4)=0,00678
      FY(5)=0,0070
      FY(6)=0,0072
      FY(7)=0,00745
      FY(8)=0,0077
      FY(9)=0,0079
      GO TO 6
C     3-YR--TRISO
   14 CONTINUE
```

```
C       4-YR--TRISO
        RSR=0.007
        TER=1873
        FY(1)=0.007
        FY(2)=0.00722
        FY(3)=0.00742
        FY(4)=0.00758
        FY(5)=0.00778
        FY(6)=0.00795
        FY(7)=0.0081
        FY(8)=0.00835
        FY(9)=0.0084
C       TRISO           $$$$$       $$$$$     $$$$$
      6 CONTINUE
        IF (FUEL.EQ.2.) TX(9)=TER
        RSR1=FY(9)
        M=9
        DO 1 I=1,M
        W(I)=1.0
        FY(I)=ALOG(FY(I))
      1 CONTINUE
        MB=5
        ME=5
        DO 2 J=MB,ME
        MAXDEG=J
        EPS=-1.0
        CALL LEAST (M,TX,FY,W,EPS,MAXDEG,NDEG,AR,R)
        MTIME=9
        DO 3 K=1,MTIME
        Z=TX(K)
        ANS=EVAL(Z,J,AR,J)
        ANSF(K)=EXP(ANS)
        R(K)=EXP(R(K))
      3 CONTINUE
        PRINT 1002
        PRINT 3500,(R(I),I=1,M)
        PRINT 1002
        PRINT 4002,TX(1),EPS,NDEG
        PRINT 1002
        PRINT 4001, (I,TX(I),FY(I),ANSF(I),I=1,MTIME)
        PRINT 1002
      2 CONTINUE
     21 CONTINUE
C
C * * * *
C       CONTROL PARAMETERS
C * * * *
        KEYPLT=-10          $ KAT  =1
        NEPRNT= 0           $ ITER =5           $ CONV =10.
        NEPRNT=1
        NTINC = 29          $ NPRNT=29
        TIME  =0.0          $ TINC =720.
        CONV = 10.
        TTI   =(TIME+TINC )/ 3600.
        TL    =ALOG10(TTI)
        AXP    =-1.75+TL*(-.248+TL*(.0059+TL*(-.00465-.0021*TL)))
        QF    =10.**AXP
        DO 100 K=1,NUMEL
        I     =NUMEL+1-K
        MN    =MATL(I)
        IF (MN.LT.1 .OR. MN.GT.9) GO TO 90
        GO TO (10,20,30,40) MN
```

```
   10 TO=1110,                              $ GO TO 90
   20 TO=1086,  $ IF(I,GT,JB) TO=678,  $ GO TO 90
   30 TO= 736,  $ IF(I,GT,JB) GO TO 20 $ N = I
   34 IA=N/NNX+1                            $ IR=N-(IA-1)*NNX
      IF (IR,LT,NXCOD) GO TO 51
      IR   =IR+NNXCO
      GO TO 33
   51 CONTINUE
      IF (IR,EQ,0) 39,90
   33 CONTINUE
      GO TO (35,36,37,38)IR
   35 TO= 736,  $ GO TO 90
   36 TO= 721,  $ GO TO 90
   37 TO= 706,  $ GO TO 90
   38 TO= 692,  $ GO TO 90
   39 TO= 677,  $ GO TO 90
   40 TO=1086,  $ N=I      $ GO TO 34
   90 CONTINUE
      IF (TO ,LE, 500) TO=926,
      DO  95 J=1,6          $ NNU    =IX(I,J)
      TNI(NNU)=TO           $ T1(NNU)=TO
   95 TN (NNU)=TO
  100 CONTINUE
C KG USED TO IDENTIFY GAS USED = +1 FOR HE, -1 FOR N2                          00861
      KG=1                                                                     00862
 1002 FORMAT(//)
 3500 FORMAT (1H, 4(5X,D25,10)/)
 4001 FORMAT(1H ,1(I6),4X,3(3X,E15,6))
 4002 FORMAT (1H , *  TO= *F10,3*   ERR= *E15,6*    DEGREE=  *I2//)
      RETURN
      END


      SUBROUTINE BNDRY(N,NB,I,J,IA,JA,HI,HJ,TEMI,TEMJ,QFI,QFJ)                 00935
      COMMON X(1000),Y(1000),QORT(1000),KODE(1000),TN(1000),T1(1000),          00936
     1TNI(1000),IX(1000,7),IB(1000,6),MATL(1000),NN(1000),NUMNP,NUMEL,         00937
     2MBAND,NPROB,NIT,NTINC,TINC,TIME,NPRNT,NI,ITER,CONV,CONVRG,NCOV,          00938
     3NPNCH,NFPRNT,KAT,VOL(50),QVG(50),QVT(50),QVV(50),QVTOT(50),              00939
     4TAVE(50),VOLUME,TBAR,QVGTOT,QVTTOT,QVVTOT,QVOL,AB(50),TBAV(50),          00940
     5QBC(50),QBV(50),QBT(50),QBCTOT,QBVTOT,QBTOT,TBND,QBAL,HED(8),           00941
     6KEYPLT,SCALE(4),TISO(20),NISO                                            00942
C                                                                              00943
      COMMON/ELEM/NEBR,NECO,NETR,NXX,NXX1,NE1
      COMMON/GEOX/NX,NNX,NXCOD,NNXCO,NXSRD,JB
      COMMON/GEOY/NY,NNY,NYCOD,NNYBS,NYBRD,NYTRD,NEBS,NYBSD
      COMMON/ROCK/TM,DE,DH,RM,RS                                               00945
      COMMON/TRANS/QTOT,AMP,KG,VP,RE(7),TO                                     00830
C
C BNDRY
C
      QFI  =QFJ  =0,    $ TEMI =TEMJ =0,    $ HI=HJ=0, $ IA=JA=0
      IF (NB,EQ,4) RETURN
      TEMI =TEMJ =339,
      TBA  =(TN(I)+TN(J))*0,5
      IF (TBA,GT,3666,) TBA=3666,
      TBF  =(TBA+TEMI )*0,9 -460,
      XLAH =0,00397*TBA**0,646
      XINS =1,73*(0,15+1,5E-4*TBF)
      FACT =5,67E-8*0,279*(TBA+TEMI)*(TBA*TBA+TEMI*TEMI)
      GO TO (10,20,30) NB
C
C     BOTTOM
```

```
C
   10 XINS =1.73*(0.625+3.75E-4*TBF)
      HGAP =(FACT +XLAH/1.96 )
      HINS =XINS/0.254
      RHP  =0.005
      HI=HJ=1/(1/HGAP+1/HINS+RHP)            $ RETURN
C
C     SIDE
   20 HI=HJ=1/(1/(FACT+XLAH/0.381)+0.1016/XINS+0.00367) $ RETURN
C
C     TOP
C
   30 HI=HJ=1/(1/(FACT+XLAH/2.64 )+0.1016/XINS+0.005  )
      RETURN
      END


      SUBROUTINE POWER(N,I,J,K,QX)                                          00865
      COMMON X(1000),Y(1000),QORT(1000),KODE(1000),TN(1000),T1(1000),       00866
     1TNI(1000),IX(1000,7),IB(1000,6),MATL(1000),NN(1000),NUMNP,NUMEL,      00867
     2MBAND,NPROB,NIT,NTINC,TINC,TIME,NPRNT,NI,ITER,CONV,CONVRG,NCOV,       00868
     3NPNCH,NEPRNT,KAT,VOL(50),QVG(50),QVT(50),QVV(50),QVTOT(50),           00869
     4TAVE(50),VOLUME,TBAR,QVGTOT,QVTTOT,QVVTOT,QVOL,AB(50),TBAV(50),       00870
     5QBC(50),QBV(50),QBT(50),QBCTOT,QBVTOT,QBTOT,TBND,QBAL,HED(8),         00871
     6KEYPLT,SCALE(4),TISO(20),NISO                                         00872
C                                                                           00873
      COMMON/ELEM/NEBR,NECO,NETR,NXX,NXX1,NE1
      COMMON/FACT/AF(50),RF(50),QF,AXP
      COMMON/GEOX/NX,NNX,NXCOD,NNXCO,NXSRD,JB
      COMMON/GEOY/NY,NNY,NYCOD,NNYBS,NYBRD,NYTRD,NEBS,NYBSD
      COMMON/TRANS/QTOT,AMP,KG,VP,RE(7),TO                                  00830
      DATA(RF(I),I=1,14)/0.83,0.83,1.16,1.16,1.16,0.96,0.96,0.96,0.96,
     1 1.07,1.07,1.07,0.90,0.90/
      DATA(AF(I),I=1,8)/.67,.9,.97,1.01,1.17,1.14,1.08,1.03/
C
C POWER
C
      QX   =0.0          $ PR   =1.
      IF (MATL(N).NE.1) RETURN
      QX   =8.4E6
      IA   =(N-NXX1)/NNX+1
      IR   =(N-NE1)- IA*NNX
      QX   =QX*RF(IR)*AF(IA)            $ QX   =QX*QF
      RETURN
      END


      SUBROUTINE TRANS                                                      00952
      COMMON X(1000),Y(1000),QORT(1000),KODE(1000),TN(1000),T1(1000),       00953
     1TNI(1000),IX(1000,7),IB(1000,6),MATL(1000),NN(1000),NUMNP,NUMEL,      00954
     2MBAND,NPROB,NIT,NTINC,TINC,TIME,NPRNT,NI,ITER,CONV,CONVRG,NCOV,       00955
     3NPNCH,NEPRNT,KAT,VOL(50),QVG(50),QVT(50),QVV(50),QVTOT(50),           00956
     4TAVE(50),VOLUME,TBAR,QVGTOT,QVTTOT,QVVTOT,QVOL,AB(50),TBAV(50),       00957
     5QBC(50),QBV(50),QBT(50),QBCTOT,QBVTOT,QBTOT,TBND,QBAL,HED(8),         00958
     6KEYPLT,SCALE(4),TISO(20),NISO,NMAT                                    00959
C                                                                           00960
      COMMON/CH/R,VBAR,FTE(350),FI(350),FXE(350),TNC ,FR
      COMMON/ELEM/NEBR,NECO,NETR,NXX,NXX1,NE1
      COMMON/ELEMENT/XLA,A,AKT
      COMMON/FACT/AF(50),RF(50),QF,AXP
      COMMON /FIT/AR(50)
      COMMON /FTACP/FUEL,FUELT,RSR,RSR1,TER,TUM,TUR,LBFY,LTFY,TER1
```

```
      COMMON/GEOX/NX,NNX,NXCOD,NNXCO,NXSRD,JB
      COMMON/GEOY/NY,NNY,NYCOD,NNYBS,NYBRD,NYTRD,NEBS,NYBSD
      COMMON/HCA1/HCAF(50)
      COMMON/RESULT/ETIM(50),AVET(50),HEAG(50),TRFI(50),FUFF(50)
      COMMON/ROCK/TM,DE,DH,RM,RS                                          00962
      COMMON/TRANS/QTOT,AMP,KG,VP,RE(7),TO                                00830
      DIMENSION LMX1(50),LMX2(50),LMX3(50),LMX4(50)
      DIMENSION TMX1(50),TMX2(50),TMX3(50),TMX4(50)
      DIMENSION TMX(50),LMX(50),TS(100),QS(100)                           00963
      DIMENSION ARF(112)
      DIMENSION NCORE(112)
      DIMENSION TMXE(112,29),TIMC(29)
      DIMENSION ALPT(6),QLAT(1000)
      DIMENSION FFI(1000)
      DIMENSION FII(1000)
      DIMENSION TIME1(50),NI1(50),CONV1(50),NCOV1(50),RFI1(50)
      DATA(FFI(I),I=1,1000)/1000*1.0/
      DATA TSUB,QSUB,FACT/3666.,65.E6,1913.5/
      DATA QLAT/1000*0.0/
      DATA (FII(I),I=1,1000)/1000* 1.0/
      DATA ALPT/5.65,4.18,3.55,3.10,2.75,2.47/
      DATA(TS(I),I=1,100)/100*0./
      DATA RFI,RFTE,RFFI,RFXE/4*0./
      DATA SRSI,SRI/2*0.0/
C *    *    *    *    *    *    *
C TRANS
C *    *    *    *    *    *    *
C     F.P.   IS RELEASED FROM THE FAILED FUEL-PARTICLES ONLY
C     FR = FUEL FAILURE FRACTION
C     FFI(N) = FRACTION OF ISOTOPE INVENTORY IN ELEMENT N DUE TO THE FAILED FUE
C     RSI = SUM OF RELEASE FROM ALL ELEMENTS IN A TIME INCREMENT (FAILED FUEL
C     INITIAL VALUES
      NC=0
      FR =1.0            $ RTE=0.0            $ QVG(1) =0.0
      VT =0.0            $ RXE=0.0            $ TAVE(1)=0.0
      RI =0.0            $ RSI=0.0
C     VARIABLES AND CONDITIONS
      TNC   =TINC/3600.
      DO 520 N=1,NUMNP
      IF (TN(N).LT.TSUB.OR.QLAT(N).GT.QSUB) GO TO 520
      QLAT(N) =QLAT(N)+2.*FACT*(TN(N)-TSUB)
      TN(N)=2.*TSUB-TN(N)
  520 CONTINUE
      DO 530 N=1,50
  530 TMX(N)  =0.0       $ COUNT=FSUM   =0.0
      DO 550 N=1,NUMEL   $ M    =MATL(N)
      IF ( M,NE,1) GO TO 401
      NC=NC+1
      NCORE(NC)=N
      IA    =(N-NXX1)/NNX+1
      IR    =(N-NE1 )-IA*NNX
      IF (FFI(1).EQ.1)  FFI(N)=AF(IA)*RF(IR)
      ARF(NC)=AF(IA)*RF(IR)
  701 I     =IX(N,1)                  $ J    =IX(N,3)
      DY    =(Y(J)-Y(I))*0.5          $ DX   =(X(J)-X(I))*0.5
      V2    =(X(J)**2-(X(J)-DX)**2)*DY*3.14159
      V1    =((X(I)+DX)**2-X(I)**2)*DY*3.14159
      TBAR =(TN(I)+TN(J-1))*V1+(TN(J)+TN(I+1))*V2
      VBAR =(V1+V2)*2.                $ TBAR   =TBAR/VBAR
      TNI(N)=TBAR                     $ VT     =VT+VBAR
      TMXE(NC,NIT)=TBAR
      CALL POWER (N,I,J,J,QX)
```

```
      QVG(1)=QVG(1)+QX*VBAR              $ TAVE(1)=TAVE(1)+TBAR*VBAR
C**
C     FUEL FAILURE FRACTION
      AKDT1 =AKT/(TBAR)
      R = A*EXP(AKDT1)
C     FUEL=1,0 FOR BISO  $  FUEL=2,0 FOR TRISO
      IF (FUEL.EQ.1.) 1,10
    1 CONTINUE
C     BISO*****************************************************************
C     LBFY=1,2 LINEAR MODEL
C     LBFY=3,4 NONLINEAR MODEL
      IF (LBFY.GE.3) 31,32
   31 IF (TBAR.LT.TER)2,3
C     NON-LINEAR MODEL
C     GASSAR 7/18/75  BISO 3 YR FUEL T-1693-2073-2273 RSR=0,0054
C     GASSAR 7/18/75  BISO 4 YR FUEL T-1683-2073-2273 RSR=0,0071
    2 FR=0,0
      GO TO 40
    3 IF (TBAR.LT.TUM) 4,5
    4 FR=EVAL(TBAR,5,AR,5)
      FR=EXP(FR)
      GO TO 50
    5 IF (TBAR.LT.TUR) 6,7
    6 FR=(TBAR-TUM)/(TUR-TUM)
      FR=RSR1+FR*(1.-RSR1)
      GO TO 50
    7 FR =1,0
      GO TO 40
   32 FR=1.0
C     LINEAR MODEL
C     GASSAR 7/18/75  BISO 1 YR FUEL T-FAIL 2073-2273 RSR=0,0018
C     GASSAR 7/18/75  BISO 2 YR FUEL T-FAIL 2073-2273 RSR=0,0038
      IF (TBAR.LT.TUR) FR=(TBAR-TER)/(TUR-TER)
      IF (TBAR.LT.TER) FR=0,0
      GO TO 40
   10 CONTINUE
C     TRISO****************************************************************
C     FUELT=2. FOR LINEAR MODEL  $  FUELT=1. FOR NONLINEAR MODEL
      IF (FUELT.EQ.1.) 11,12
   11 CONTINUE
C     NON-LINEAR MODEL
      FR =1,0
      IF (TBAR.LT.TER1) 38,39
   38 FR=0,0              $ GO TO 40
   39 IF (TBAR.LE.TER) 13,14
   13 FR=EVAL(TBAR,5,AR,5)
      FR=EXP(FR)
      GO TO 50
   14 IF (TBAR.LT.TUR) FR=(TBAR-TER)/(TUR-TER)
      FR=RSR1+FR*(1.-RSR1)
      GO TO 50
   12 CONTINUE
C     LINEAR MODEL
      FR =1,0
      IF (TBAR.LT.TUR) FR=(TBAR-TER)/(TUR-TER)
      IF (TBAR.LT.TER) FR=0,0
   40 CONTINUE
      FR =RSR + FR*(1.0-RSR)
   50 COUNT =COUNT + 1.0                $ FSUM   =FSUM+FR
C     RELEASE FROM THE FAILED FUEL = FFI(N)*(1.0-B)*FR
      B  =FFI(N)*EXP( -R*TINC/3600.)
C     SUM RELEASE FOR ALL ELEMENTS
```

```
      RSI=RSI+(FFI(N)-B )*VBAR*FR
C     REVISE INVENTORY IN EACH FUELED REGION
      R     =R+XLA
C     LOSS INCLUDES RELEASE PLUS DECAY
      R  =FFI(N)*EXP(- R*TINC/3600.)    $ FFI(N)=R+(1.-FR)*(FFI(N)-B)
  401 CONTINUE
      NS    =NN(N)-2
      DO 540 L=1,NS
      I     =IX(N,L)
      J     =IX(N,L+1)
      K     =IX(N,L+2)
      IF (TN(I).LE.TMX(M))  GO TO 561
      TMX(M)  = TN(I)                   $ LMX(M)  =I
  561 IF (TN(J).LE.TMX(M))  GO TO 562
      TMX(M)  = TN(J)                   $ LMX(M)  =J
  562 IF (TN(K).LE.TMX(M))  GO TO 540
      TMX(M)  = TN(K)                   $ LMX(M)  =K
  540 CONTINUE
  550 CONTINUE
      MM=NTINC-1
      IF (NIT.EQ.MM) 541,542
  541 CONTINUE
      PRINT 5008, (NCORE(I),ARF(I),I=1,112)
  542 CONTINUE
      TIMC(NIT)=TIME/3600.
      ICOUNT=IFIX(COUNT)
      TAVE(1) = TAVE(1)/VT
      AVET(NIT)=TAVE(1)
      HEAG(NIT)=QVG(1)
      FFI(1)=0.0
      RFI=RFI+RSI/VT
      TRFI(NIT)=RFI
  667 CONTINUE
      FSUM=FSUM/COUNT
      FUFF(NIT)=FSUM
      TMX1(NIT)=TMX(1)
      TMX2(NIT)=TMX(2)
      TMX3(NIT)=TMX(3)
      TMX4(NIT)=TMX(4)
      LMX1(NIT)=LMX(1)
      LMX2(NIT)=LMX(2)
      LMX3(NIT)=LMX(3)
      LMX4(NIT)=LMX(4)
      TIME1(NIT)=TIME/3600.
      NI1(NIT)=NI
      CONV1(NIT)=CONVRG
      NCOV1(NIT)=NCOV
      RFI1(NIT)=RFI
      IF (TIME.EQ.1440.) TINC=360.
      IF (TIME.EQ. 1800.) TINC=1800.
      IF (TIME.EQ.25200.) TINC=3600.
      TTI=(TIME+TINC)/3600.    $TTJ=TTI
      ETIM(NIT)=TIME1(NIT)
      TL    =ALOG10(TTI)
      AXP    =-1.75+TL*(-.248+TL*(.0059+TL*(-.00465-.0021*TL)))
      QF=10.**AXP
      QF=QF*1.1
      TIMESE=TTJ*3600.
      IF(NIT.EQ.NTINC) GO TO 6000
 5000 RETURN                                                    01010
 6000 CONTINUE
      PRINT 4000
```

```
      PRINT 4001,(NIT,ETIM(NIT),AVET(NIT),TMX1(NIT),TRFI(NIT),FUFF(NIT),
     1HEAG(NIT),NIT=1,NTINC)
      PRINT 7000
      PRINT 7001,(NIT,TIME1(NIT),LMX1(NIT),TMX1(NIT),LMX2(NIT),TMX2(NIT)
     1,LMX3(NIT),TMX3(NIT),LMX4(NIT),TMX4(NIT),NIT=1,NTINC)
      PRINT 8000
      PRINT 8001,(NIT,TIME1(NIT),NI1(NIT),CONV1(NIT),NCOV1(NIT),RFI1(NIT
     1),HCAF(NIT),NIT=1,NTINC)
      PRINT 3000,ICOUNT
      DO 5007,I=1,112
      PRINT 5006,I,NCORE(I),ARF(I)
      PRINT 5002,(TMXE(I,J),J=1,29)
      PRINT 2000
      WRITE (15),ARF(I),(TIMC(J),TMXE(I,J),J=1,29)
 5007 CONTINUE
  102 FORMAT (1F5.1,5X,2(F6.0,4X))
  103 FORMAT(5(E15.3))
 2000 FORMAT (//)
 3000 FORMAT (/* NO. OF CORE-ELEMENT =  *I5/)
 4000 FORMAT (1H1,*   NIT        TIME-HR      AVE CORE TEMP    MAX CORE TE
     1MP  FRACTION REL     FUEL F.F.     HEAT GEN. *///)
 4001 FORMAT(I7,3X,6(E15.7))
 5001 FORMAT (*      TIME = * F5.2/)
 5002 FORMAT (1H ,16(F8.0))
 5006 FORMAT (1H ,2(I3,2X),F10.5//)
 5008 FORMAT(1H ,7(I3,2X,F10.4,2X,))
 7000 FORMAT (1H1, * NIT  TIME-HR   NODE  MATL-1 MAX TEMP NODE      MATL-
     12-MAX TEMP    NODE  MATL-3  MAX TEMP   NODE  MATL-4 MAX TEMP *)
 7001 FORMAT (I4,3X,F5.2,2X,2(I6,5X,E12.5),5X,2(I6,5X,E12.5))
 8000 FORMAT (1H1, * NIT  TIME-HR    ITERATION  MAX CONVERGENCE AT NODE
     1NUMBER   RELEASE FRACTION              HCAP *)
 8001 FORMAT (I3,3X,F5.2,4X,I7,5X,E15.7,5X,I4,2(9X,E15.7))
      IF(NIT.EQ.NTINC)CALL EXIT
      END                                                              01011
$FM
  GASSAR 7/18/75 TRISO 1 YR FUEL T-FAIL 1940-2273 RSR=0.0015 I=133
              1  -10                                                  01013
$EJ
```

APPENDIX B

MAXIMUM HEAT REMOVAL BY NATURAL CIRCULATION OF HELIUM AT ONE ATMOSPHERE PRESSURE

An upper limit for the heat removal can be based on the assumption that the entire buoyancy force is balanced by the pressure drop in the reactor coolant channels. Further, a loop for the natural circulation is assumed such that the hot helium can be cooled to room temperature and establish a column of 300 K gas which must be balanced by the hot helium, plus the pressure drop in the channels.

Buoyancy Pressure Drop

At about 10 h, the core is above 2000 K average temperature, and the ^{131}I release is nearly complete. The pressure difference due to buoyancy for two isothermal columns at different temperatures is:

$$\Delta P = \frac{PH}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right), \qquad \text{(B-1)}$$

where P = average pressure,

H = height of columns,

R = gas constant, and

T_1, T_2 = the temperatures of the columns.

At 1 atm pressure and 10-m column height (Fig. 1), the pressure difference is 1.4 N/m^2 (2×10^{-4} psi). Equating this with the frictional pressure drop through the core with the Reynolds number in the laminar range, the maximum flow rate that can be sustained is 0.26 kg/s. With a temperature difference of 1700 K, 2.3 MW can be removed from the core which is 0.07% of full power, or approximately 7% of the decay power at 10 h. The above is an upper limit for a number of reasons not mentioned. This, the fact that the percentage is much lower at earlier times in the heatup, and the fact that to neglect natural circulation is conservative, justifies omitting it in the analysis model. However, natural circulation may still be a significant factor in determining the maximum temperature of metallic components external to the core.

APPENDIX C

PARAMETERS USED TO OBTAIN EFFECTIVE CONDUCTANCE FROM REFLECTORS TO PCRV

I. General equation:

$$\frac{1}{h_e} = \frac{1}{h_g} + \frac{1}{h_i} + \frac{1}{h_p}. \qquad \text{(C-1)}$$

II. $$h_g = \frac{\lambda_{He}}{\delta_{He}} + \varepsilon'\sigma(T_1^2 + T_2^2)(T_1 + T_2). \quad \text{(C-2)}$$

λ_{He} = helium conductivity = $0.00397\ (T_1)^{0.646}$.

δ_{He} = helium gap thickness,

= 2.64 m (104 in.) - top,

= 1.96 m (77 in.) - bottom,

= 0.38 m (15 in.) - side.

ε' = effective emissivity,

$$\frac{1}{\varepsilon'} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1, \quad \text{(C-3)}$$

ε_1 = graphite = 0.8,

ε_2 = steel = 0.3.

σ = Stefan-Boltzman constant = 5.67×10^{-8} $W/m^2 \cdot K^4$.

T_1 = local temperature of the outer surface of the graphite reflectors.

T_2 = local temperature of the PCRV.

III. $$h_i = \frac{\lambda_i}{\delta_i}. \quad \text{(C-4)}$$

λ_i = conductivity of thermal barrier = $a + b\ T_i$ $(W/m^\circ K)$.

a = 0.26 - top and side,

= 1.08 - bottom.

b = 2.6×10^{-4} - top and side,

= 6.49×10^{-4} - bottom.

T_i = average temperature of thermal barrier (°F).

The average temperature of the thermal barrier can be estimated from a local heat balance at the previous iteration. The result is:

$$T_i = \frac{1}{2}(T_2 + T_s), \quad \text{(C-5)}$$

where T_s = surface temperature of the thermal barrier

$$T_s = \left[T_2 + \left(\frac{h_g}{h_i}\right) T_1\right] / \left(1 + \frac{h_g}{h_i}\right). \qquad \text{(C-6)}$$

δ_i = thickness of the thermal barrier,
= 0.10 m (4 in.) - top and side,
= 0.25 m (10 in.) - bottom.

IV. h_p = PCRV cooling water heat transfer coefficient,
= 204 $W/m^2 \cdot K$ - top and bottom,
= 273 $W/m^2 \cdot K$ - side.

APPENDIX D

THE MATHEMATICAL FORMATION OF A MODEL DECAY SCHEME

I. INTRODUCTION

Fission products existing in an HTGR core include many atomic species--some of which are stable, while the others are radioactive. To evaluate the time-dependent release of the fission products from the reactor core, one has to know the genetic relationships of the nuclides, in addition to many other factors such as the core temperature, the fuel age, etc. The genetic relationships for the fission products that are of importance to safety analysis[10] can be found in Ref. 9. Since there are so many different genetic relationships which have to be considered in evaluating the time-dependent fission product release, it is desirable to seek a model decay scheme that represents all the genetic relationships of interest. We have constructed a model decay scheme which is capable of delegating all those genetic relationships whose transformations do not involve neutron absorption processes. When it is incorporated into the AYER code,[2] this model will be able to compute the time-dependent release of fission products in any one of the many different decay chains in consideration. The mathematical formation of the model decay scheme is presented in this appendix.

II. THE DESCRIPTION OF THE MODEL

The schematic representation of the model decay chain is shown in Fig. D-1. The model consists of eight types of elements which are denoted by S_1, S_2, ..., S_8. It is assumed that every type of element can decay by two processes with a pair of branching ratios, $B_{i,i+1}$ and $B_{i,i+2}$, except the last two. The atoms of S_7 decay directly to form S_8 which is a stable element. By assigning the appropriate pair (or pairs) of branching ratios to have values from zero to one, the model will become identical to one of the many decay chains that are of importance to HTGR safety analysis.

III. MATHEMATICAL FORMATION

To obtain a set of equations which will describe the time-dependent release of the fission products in the model decay chain, the following assumptions are used.

- No neutron absorption process is associated with the decay chain.
- The fission products are released only from those fuel particles whose pyrocarbon and/or silicon carbide coatings have failed.
- The temperature distribution in the reactor core and the core-temperature history are determined by the AYER code, which is based on the finite element method[11] as the computation scheme. The process of releasing fission products from each of the finite volume-elements is considered as an independent event. During a time interval, the total amount of fission products released is the sum of the releases from all the finite volume-elements in the reactor core.

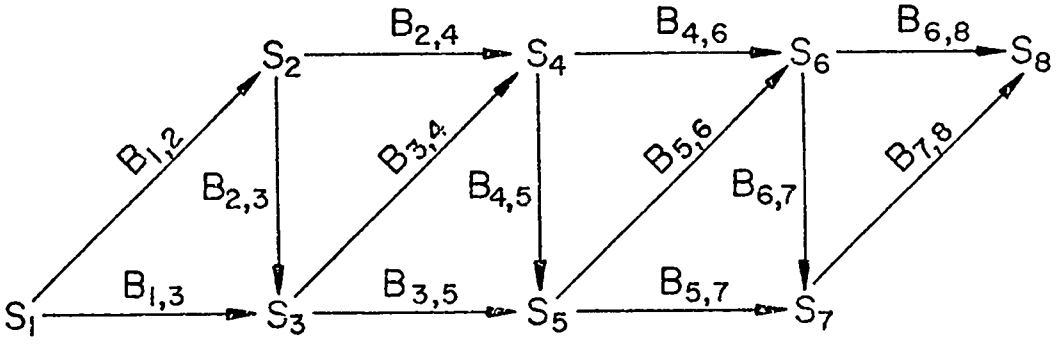

Fig. D-1
The schematic model decay chain.

- At a given time, the atoms of each element in the model decay chain may be partitioned into two groups. Those atoms which have been remaining inside the coatings of the fuel particles form one group; those which have been released and exist outside the coatings form the other. For the jth element in the chain, the number of atoms of the "inside group" will be denoted by $N_j(t)$, while the number of atoms of the "outside group" will be represented by $R_j(t)$.

With these assumptions, the equations describing the rate of change for elements in the chain within a given volume-element may be derived. For each type of nuclei there are two rate equations--one for the inside group and the other for the outside group. For the starting element of the chain, the rate equations are:

$$\frac{dN_1(t)}{dt} = - B_{1,2}\ \lambda_1\ N_1(t) - B_{1,3}\ \lambda_1\ N_1(t) - r_1(t)\ f(t)\ N_1(t), \qquad \text{(D-1)}$$

and

$$\frac{dR_1(t)}{dt} = - B_{1,2}\ \lambda_1\ R_1(t) - B_{1,3}\ \lambda_1\ R_1(t) + r_1(t)\ f(t)\ N_1(t), \qquad \text{(D-2)}$$

where λ_1 and $r_1(t)$ are the decay constant and release constant for the element, respectively; $f(t)$ is the failed fuel fraction in the volume-element considered at time t.

The other notations have been defined above. Because the release constant and failed fuel fraction are functions of the local core temperature which is a function of time, so they are also functions of time. The rate equations for the second element are:

$$\frac{dN_2(t)}{dt} = B_{1,2}\ \lambda_1\ N_1(t) - B_{2,3}\ \lambda_2\ N_2(t) - B_{2,4}\ \lambda_2\ N_2(t) - r_2(t)\ f(t)\ N_2(t), \qquad \text{(D-3)}$$

and

$$\frac{dR_2(t)}{dt} = B_{1,2}\,\lambda_1\,R_1(t) - B_{2,3}\,\lambda_2\,R_2(t) - B_{2,4}\,\lambda_2\,N_2(t)$$

$$+ r_2(t)\,f(t)\,N_2(t). \qquad \text{(D-4)}$$

The first term at the right-hand side of Eq. (D-3) is the rate of production for $N_2(t)$ due to one of the branching decays of $N_1(t)$. The second and third terms are the depletion rate for $N_2(t)$ by disintegration. The last term represents the rate of decrease of $N_2(t)$ through release. The terms in Eq. (D-4) have similar meanings except that the last term now denotes the rate of increase of $R_2(t)$ due to release from failed fuels.

Likewise, for any other element in the model decay chain the rate equations are:

$$\frac{dN_j(t)}{dt} = B_{(j-2),j}\,\lambda_{(j-2)}(t)\,N_{(j-2)}(t) + B_{(j-1),j}\,\lambda_{(j-1)}\,N_{(j-1)}(t)$$

$$- B_{j,(j+1)}\,\lambda_j\,N_j(t) - B_{j,(j+2)}\,\lambda_j\,N_j(t) - r_j(t)\,f(t)\,N_j(t), \qquad \text{(D-5)}$$

and

$$\frac{dR_j(t)}{dt} = B_{(j-2),j}\,\lambda_{(j-2)}\,R_{(j-2)}(t) + B_{(j-1),j}\,\lambda_{(j-1)}\,R_{(j-1)}(t)$$

$$- B_{j,(j+1)}\,\lambda_j\,R_j(t) - B_{j,(j+2)}\,\lambda_j\,R_j(t) + r_j(t)\,f(t)\,N_j(t); \qquad \text{(D-6)}$$

with $j = 3, 4, \ldots, 8$.

The constraints on the branching ratios and decay constants are

$$B_{7,8} = 1,\ B_{7,9} = 0,\ \text{for } j = 7 \qquad \text{(D-7)}$$

and

$$B_{8,9} = B_{8,10} = 0,\ \lambda_8 = 0,\ \text{for } j = 8. \qquad \text{(D-8)}$$

The meaning of the terms in Eqs. (D-5) and (D-6) is precisely the same as that just described for Eqs. (D-3) and (D-4), except there are now two production terms due to the activities of both precursors. Thus, there are 16 rate equations for the model decay scheme. These equations, plus the appropriate initial conditions, will be able to determine the 16 unknowns, $N_j(t)$, $R_j(t)$ ($j = 1, 2, \ldots, 8$). The solutions to the system equations which are capable of determining the time-dependent fission product release are given in the next section.

IV. SOLUTIONS FOR RATE EQUATIONS

The rate equations for the model decay scheme are a set of 16 coupled equations. Their solutions can be obtained in sequence. For a dual decay process, the sum of the pair of branching ratios is unity; that is

$$B_{j,(j+1)} + B_{j,(j+2)} = 1; \; j = 1, 2, \ldots, 7. \tag{D-9}$$

Therefore, the system of rate equations reduces to

$$\frac{dN_1(t)}{dt} = - \lambda_1 N_1(t) - r_1(t) f(t) N_1(t), \tag{D-10}$$

$$\frac{dR_1(t)}{dt} = - \lambda_1 R_1(t) + r_1(t) f(t) N_1(t); \tag{D-11}$$

$$\frac{dN_2(t)}{dt} = B_{1,2} \lambda_1 N_1(t) - \lambda_2 N_2(t) - r_2(t) f(t) N_2(t), \tag{D-12}$$

$$\frac{dR_2(t)}{dt} = B_{1,2} \lambda_1 R_1(t) - \lambda_2 R_2(t) + r_2(t) f(t) N_2(t); \tag{D-13}$$

$$\frac{dN_j(t)}{dt} = B_{(j-2),j} \lambda_{(j-2)} N_{(j-2)}(t) + B_{(j-1),j} \lambda_{(j-1)} N_{(j-1)}(t)$$

$$- \lambda_j N_j(t) - r_j(t) f(t) N_j(t), \tag{D-14}$$

$$\frac{dR_j(t)}{dt} = B_{(j-2),j}\,\lambda_{(j-2)}\,R_{(j-2)}(t) + B_{(j-1),j}\,\lambda_{(j-1)}\,R_{(j-1)}(t) - \lambda_j\,R_j(t) + r_j(t)\,f(t)\,N_j(t); \quad \text{(D-15)}$$

where $j = 3, 4, \ldots, 8$.

The solutions to the simplified equations can be obtained as follows:

a. By combining Eqs. (D-10), (D-11), (D-12), (D-13), (D-14), and (D-15) in pairs, a set of equations in which the release constants and failed fuel fraction are absent is found. The combined equations are:

$$\frac{d}{dt}\,[N_1(t) + R_1(t)] = -\,\lambda_1[N_1(t) + R_1(t)], \quad \text{(D-16)}$$

$$\frac{d}{dt}\,[N_2(t) + R_2(t)] = B_{1,2}\,\lambda_1[N_1(t) + R_1(t)] - \lambda_2[N_2(t) + R_2(t)], \quad \text{(D-17)}$$

$$\frac{d}{dt}\,[N_j(t) + R_j(t)] = B_{(j-2),j}\,\lambda_{(j-2)}\,[N_{(j-2)}(t) + R_{(j-2)}(t)] + B_{(j-1),j}\,\lambda_{(j-1)}[N_{(j-1)}(t) + R_{(j-1)}(t)] - \lambda_j[N_j(t) + R_j(t)]; \quad \text{(D-18)}$$

where $j = 3, 4, \ldots, 8$.

These combined equations describe the disintegration processes for the atoms of the elements in the model decay chain, disregarding whether they exist inside or outside the coatings of the fuel particles. Assuming the initial values at $t = 0$ given by

$$N_j(t=0) = N_j(0),$$

$$R_j(t=0) = 0;$$

$$j = 1, 2, \ldots, 8, \quad \text{(D-19)}$$

the equations can be integrated successively.*

*In the present case, it is assumed that the values of all the decay constants are distinguishable.

The results are

$$N_j(t) + R_j(t) = \sum_{k=1}^{j} C_{j,k} \exp(-\lambda_k t);\ j = 1, 2, \ldots, 8. \tag{D-20}$$

The coefficients, $C_{j,k}$ are given by the recursion formulas.
For $j = k$,

$$C_{j,j} = N_j(0) - \sum_{i=1}^{(j-1)} C_{j,i};\ j = 1, 2, \ldots, 8; \tag{D-21}$$

for $j > k$,

$$C_{j,k} = \frac{1}{(\lambda_j - \lambda_k)} \left[B_{(j-1),j}\ \lambda_{(j-1)}\ C_{(j-1),k} + B_{(j-2),j}\ \lambda_{(j-2)}\ C_{(j-2),k}\ U(j-2-k) \right];$$

$$j = 2, 3, \ldots, 8,\ k = 1, 2, \ldots, (j-1); \tag{D-22}$$

where the unit step function, $U(Z)$ is defined as

$$U(Z) \equiv \begin{cases} 0, \text{ when } Z < 0; \\ 1, \text{ when } Z \geq 0. \end{cases} \tag{D-23}$$

b. By integrating equations (D-10), (D-12), and (D-14), successively, one finds

$$N_j(t) = N_j(0) \exp\left[- \lambda_j t - \int_o^t d\tau\ r_j(\tau)\ f(\tau) \right] + \int_o^t dt'\ B_{(j-1),j}\ \lambda_{(j-1)}\ N_{(j-1)}(t')\ e^{a} + \int_o^t dt'\ B_{(j-2),j}\ \lambda_{(j-2)}\ N_{(j-2)}(t')\ e^{a}$$

$$j = 1, 2, \ldots, 8, \tag{D-24}$$

where

$$a = [-\lambda_j(t-t') - \int_{t'}^{t} d\tau\, r_j(\tau)\, f(\tau)]. \qquad \text{(D-25)}$$

The constraints on Eq. (D-24) are that if j = 1, both integral terms which integrate over the variable t' vanish; and if j = 2, only the first of these integrals exists. The physical meaning for the right-hand side of Eq. (D-24) can be visualized as follows. The first term represents a portion of $N_j(0)$ which has survived disintegration and is being released up to time t. In the second term, the quantity $[B_{(j-1),j}\ \lambda_{(j-1)}\ N_{(j-1)}(t')\ dt']$ is the amount of jth element created in a time interval between t = t' and t = t' + dt' due to the activity of its precursor, the (j-1)th element. The multiplication of this quantity with the "survival probability function,"

$$\exp\ [-\lambda_j(t-t') - \int_{t'}^{t} d\tau\, r_j(\tau)\, f(\tau)],$$

gives the portion of these created atoms which have neither decayed nor migrated during the period from the "birth time" t' to time t. Consequently, the integration of the product over all the birth time from t' = 0 to t' = t yields the total amount of the created and survived jth type atoms due to the activity of the precursor. The third term has the same physical meaning as the second term except that the precursor for the third term is the (j-2)th element in the decay chain. Therefore, the sum of these three terms represents the number of jth type atoms which still exist inside the coatings of the fuel particles at time t.

c. Using Eq. (D-24) for $N_j(t)$, Eq. (D-20) can be written as

$$R_j(t) = \sum_{k=1}^{j} C_{j,k} \exp(-\lambda_k t) - N_j(t);\ j = 1, 2, \ldots, 8. \qquad \text{(D-26)}$$

The set of equations in Eq. (D-26) describes the amount of each element in the model decay chain which has been released, but has not decayed up to

time t. The coefficients $C_{j,k}$ are given in Eqs. (D-21) and (D-22), while the N_j's are given by Eq. (D-24). Incorporating these results with the AYER code, the time-dependent release of fission products that are important to safety analysis can be evaluated numerically. A process for this numerical evaluation is currently being investigated. Two numerical examples which consider only the disintegration processes for the atoms of elements in a decay chain are given in the next section.

V. NUMERICAL EXAMPLES

The model decay scheme developed above possesses two features--one indicates the release process and the other relates to the decay process. As an initial step in the verification of the model, the following special case is being considered. In the special case, only the decay aspect of the model is investigated. This is accomplished by setting the failed fuel fraction equal to zero. As a result of this condition and the assumption (b) of Sec. III, Eq. (D-24) reduces to Eq. (D-20) with $R_j(t)$ equal to zero. This is

$$N_j(t) = \sum_{k=1}^{j} C_{j,k} \exp(-\lambda_k t); \quad j = 1, 2, \ldots, 8, \tag{D-27}$$

where the coefficients $C_{j,k}$ are again given by Eqs. (D-21) and (D-22). To reduce these results to those given in Ref. 12, further changes are necessary:

a. Branching processes are absent.

b. Initially, only the first element of the decay chain is present. That is

$$B_{1,2} = B_{2,4} = B_{3,4} \ldots + B_{7,8} = 1, \tag{D-28}$$

$$B_{1,3} = B_{2,4} = B_{3,5} \ldots = B_{6,8} = 0; \tag{D-29}$$

and

$$N_1(0) = N_0, \; N_2(0) = N_3(0) = \ldots = N_8(0) = 0. \tag{D-30}$$

Under these conditions, the coefficients $C_{j,k}$ which are given in Eqs. (D-21) and (D-22) simplify considerably:

For $j = k$,

$$C_{j,j} = N_j(0)\,\delta_{1,j} - \sum_{i=1}^{j-1} C_{j,i};\quad j = 1, 2, \ldots, 8; \tag{D-31}$$

For $j > k$,

$$C_{j,k} = \frac{\lambda_{(j-1)}}{(\lambda_j - \lambda_k)}\, C_{(j-1),k};$$

$$j = 2, 3, \ldots, 8,\ k = 1, 2, \ldots, (j-1) \tag{D-32}$$

where the $\delta_{1,j}$ is the Kronecker delta. These simplified results are equivalent to those of Ref. 12. Two numerical examples were computed using the model decay scheme with input data taken from Ref. 12. The computed results are plotted in Figs. D-2 and D-3. These are identical to Figs. 10-6 and 10-7 of Ref. 12, except that the ordinate in Figs. D-2 and D-3 is the fractional number of initial atoms rather than number of atoms which was used in Ref. 12.

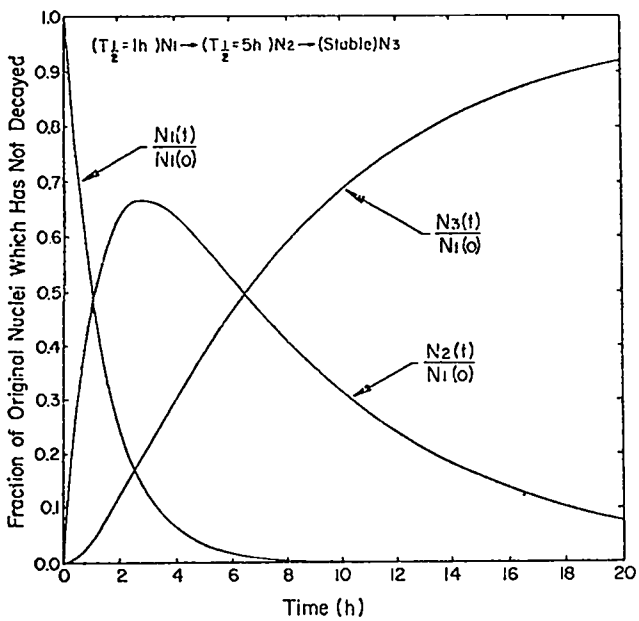

Fig. D-2.
A linear decay chain with three elements; only the parent is present initially.

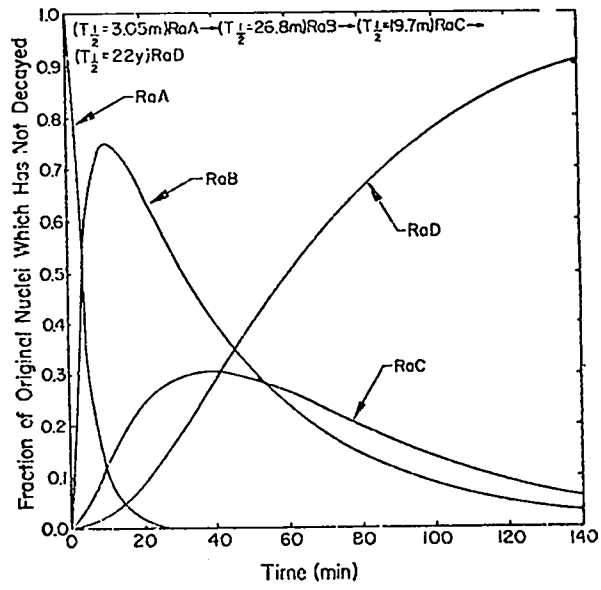

Fig. D-3.
A linear decay chain with four elements; only the parent is present initially.

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