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Title: THE SHIFT OF PROMPT CRITICAL IN REFLECTED REACTORS AND THE LIMITATIONS OF THE MEAN PROMPT-NEUTRON LIFETIME MODEL

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THE SHIFT OF PROMPT CRITICAL IN REFLECTED REACTORS AND THE LIMITATIONS OF THE MEAN PROMPT-NEUTRON LIFETIME MODEL

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ABSTRACT

Prompt critical in a bare reactor is defined as the point at which the reactivity ρ of the reactor is equal to the effective delayed neutron fraction β. In a reflected reactor, however, it is shown that prompt critical will occur at a reactivity of $\rho \approx \beta(1-f)$ where f is the fraction of core neutrons that return to the core region after having leaked into the reflector.

Furthermore, it is also shown that the mean prompt-neutron lifetime model that has been traditionally used to characterize the dynamic response of reflected reactors may not always provide an adequate representation of the system for reactivities greater than 1\$.

And finally, the coupled, point-kinetic equations proposed by Avery[1] and further developed by Cohn[2] for simple reflected systems are recast into a more usable form that can be readily used to perform superprompt critical transient analyses.

I. INTRODUCTION

Prompt critical represents the point at which a neutron chain reaction can be sustained by prompt neutrons alone. From a mathematical standpoint, prompt critical occurs when the reciprocal time constant, α, associated with the decay or growth of prompt neutron chains is just equal to zero. In a bare reactor, this occurs when the reactivity ρ is equal to the effective delayed neutron fraction β.

Using the coupled, point-kinetic equations proposed by Avery[1] and further developed by Cohn[2] for simple reflected systems, it was shown by Kistner[3] that there will exist two distinct time constants associated with the decay or growth of prompt neutron chains; one of the time constants is always negative while the other time constant becomes positive at reactivities greater than

$$\rho \approx \beta(1-f) \tag{1}$$

where f is the fraction of core neutrons that return to the core region after having leaked into the reflector. By definition, this reactivity must correspond to the point of prompt critical in a reflected system.

In this manuscript we develop the above expression, as well as discuss the limitations of the mean prompt-neutron lifetime model derived from the same system of equations.

II. THEORY

A. Point-Kinetic Equations for a Reflected System

In 1958, Avery[1] presented a general point-kinetic model to describe the time-dependent behavior of multiplying systems comprised of an arbitrary number of regions, each characterized by a multiplication factor k_i and a neutron lifetime τ_i. For a two-region system consisting of a simple core surrounded by a non-multiplying, source-free reflector, Cohn[2] reduced Avery's model to the following set of coupled differential equations:

$$\frac{dN_c}{dt} = \frac{k_c(1-\beta)-1}{\tau_c}N_c + \frac{f_{rc}}{\tau_r}N_r + \sum\lambda_i C_i + S \tag{2}$$

$$\frac{dN_r}{dt} = \frac{f_{cr}}{\tau_c}N_c - \frac{N_r}{\tau_r} \tag{3}$$

$$\frac{dC_i}{dt} = \frac{k_c\beta_i N_c}{\tau_c} - \lambda_i C_i \quad \text{for i=1,2,...,m} \tag{4}$$

where

- N_c number of neutrons in the core region,
- N_r number of neutrons in the reflector region,
- k_c multiplication factor of the *bare* core,
- β effective delayed neutron fraction,
- τ_c neutron lifetime in the *bare* core,
- τ_r neutron lifetime in the reflector region,
- f_{cr} fraction of neutrons that leak from the core into the reflector,
- f_{rc} fraction of neutrons that leak from the reflector back into the core,
- f total fraction of core neutrons returned to the core after having leaked into the reflector, $= f_{rc}f_{cr}$,

- C_i concentration of the i[th] precursor group,
- β_i delayed neutron fraction of the i[th] precursor group,
- λ_i decay constant of the i[th] precursor group,
- m number of delayed neutron groups, and
- S intrinsic/external neutron source rate.

B. Overall Effective Multiplication Factor

Following the approach of Mowery and Romesburg,[4] we obtain an expression for the *overall* effective multiplication factor, k, of the *integral* system by solving for the equilibrium condition of the above system of equations. This leads to the following expression for the number of neutrons in the core region at equilibrium, N_{co},

$$N_{co} = \frac{\tau_c S}{1 - (k_c + f)} \tag{5}$$

and the number of neutrons in the reflector region at equilibrium, N_{ro}.

$$N_{ro} = \frac{f_{cr}\tau_r}{\tau_c} N_{co} \tag{6}$$

Hence, the total neutron population of the *integral* system at equilibrium, N_{to}, is given by

$$N_{to} = N_{co} + N_{ro} = \frac{(\tau_c + f_{cr}\tau_r)\, S}{1 - (k_c + f)} \tag{7}$$

By direct comparison with the source-multiplication equation for a bare reactor, we infer from the above expression that the *overall* effective multiplication factor for a reflected system is

$$k = k_c + f \tag{8}$$

From Eq. (7), we also define the system's *static* mean neutron lifetime τ_s as

$$\tau_s = \tau_c + f_{cr}\tau_r \tag{9}$$

The reciprocal of τ_s represents the average loss rate from the integral system in the equilibrium state, but, as will be shown later, differs slightly from the mean prompt-neutron lifetime that characterizes the kinetic behavior of the system.

C. Estimation of Kinetic Parameters

Before proceeding, we would like to stress the meaning of k_c, τ_c, τ_r, f_{rc}, f_{cr} and f and briefly describe how these parameters may be obtained from a series of transport and/or Monte Carlo reactor physics calculations.

As previously defined, k_c corresponds to the k eigenvalue obtained from a calculation in which only the *bare* core is modeled. In highly reflected systems, k_c will typically be on the order of 0.80.

The average neutron lifetime in the core, τ_c, is defined as the mean time between any type of neutron interaction that results in a loss of a neutron from the core. As with k_c, τ_c corresponds to the bare core and, in most cases, can be ascertained directly from the output summary of a Monte Carlo analysis of a bare core model. Some attention, however, must be given to the interpretation of the labels that are assigned to the quantities that are listed in the output summary of the code in order to extract the correct value for τ_c.

For example, in the case of MCNP, four different lifetimes are listed: 1) the fission lifetime t_f, 2) the capture (or nonfission absorption) lifetime t_a, 3) the leakage lifetime t_l, and 4) the total removal lifetime t_r. In the context of the kinetic equations, it is somewhat of a misnomer to refer to the first three of these four quantities as neutron lifetimes because they actually represent (at least in the case of MCNP) the average interaction time required for a single neutron to obtain a given end result (i.e., fission, nonfission capture, and leakage); t_f represents the average time from birth to interaction for a single neutron to cause a fission; t_a represents the average time from birth to interaction for a single neutron to be captured in a nonfission reaction; and t_l represents the average time from birth to interaction for a single neutron to leak from the system. The average neutron removal lifetime in the core, t_r, is related to these three quantities by

$$t_r = P_f t_f + P_a t_a + P_l t_l \tag{10}$$

where P_f, P_a, and P_l are the fraction of neutrons that interact by fission, capture, and leakage, respectively. In the case of MCNP, t_r is identically equal to τ_c.

Because τ_c represents the average neutron removal lifetime in the core, N_c/τ_c represents the total number of neutrons loss per unit time. Therefore, $P_f N_c/\tau_c$ represents the total fission rate, $P_a N_c/\tau_c$ represents the total nonfission capture rate, and $P_l N_c/\tau_c$ represents the total leakage rate. We can also represent these same interaction rates as N_c/τ_f, where τ_f is the average time between fission events, N_c/τ_a, where τ_a is the average time between nonfission captures, and N_c/τ_l, where τ_l is the average time between leakage events. Hence, we can define a mean fission lifetime, τ_f, to be τ_c/P_f, a mean capture lifetime, τ_a, to be τ_c/P_a, and a mean leakage lifetime, τ_l, to be τ_c/P_l. It is obvious from Eq. (10) that τ_f is not the same as t_f and so forth.

Using the *bare* core model, the fraction of core neutrons that leak from the core into the reflector, f_{cr}, can be established by integrating the positive leakage current over that portion of the core surface area that is reflected. For small, fully-reflected systems, this fraction will typically be on the order of 50 to 60%.

The *overall* effective multiplication factor, k, is determined from another k eigenvalue calculation using a model of the integral system (i.e., core plus reflector). Given k and k_c, the total fraction, f, of core neutrons that leak from the core into the reflector and then return to the core can be calculated from Eq. (8). Once f is known, we deduce the fraction of neutrons in the reflector that return to the core, f_{rc}, from the definition of f:

$$f_{rc} = \frac{f}{f_{cr}} \tag{11}$$

The average neutron lifetime in the reflector, τ_r, is obtained from an *integral* system model calculation using the equilibrium condition defined by Eq. (6). That is,

$$\tau_r = \left(\frac{\tau_c}{f_{cr}}\right)\frac{N_{ro}}{N_{co}} \tag{12}$$

where N_{ro} and N_{co} are the total number of neutrons in the reflector region and core region, respectively, at equilibrium.

N_{ro} and N_{co} are easily obtained by integrating the spatial-dependent, energy-dependent neutron fluxes over the respective volume of the two regions:

$$N_{ro} = \int_{reflector} \frac{\phi(E,r)}{v(E)}\,dEdr \tag{13}$$

and

$$N_{co} = \int_{core} \frac{\phi(E,r)}{v(E)}\,dEdr \tag{14}$$

where $v(E)$ is the neutron velocity corresponding to energy E.

D. The Shift in Prompt Critical

In a bare reactor, the decay or growth of prompt neutron chains is described by

$$N_p = Ae^{\alpha t} \tag{15}$$

in which α is defined by

$$\alpha = \frac{k(1-\beta)-1}{\tau} \tag{16}$$

When $k < 1/(1-\beta)$, α is negative and the prompt neutron chains decay with time; when $k > 1/(1-\beta)$, α is positive and the prompt neutron chains grow with time; when α is zero, the prompt neutron chains, once initiated, propagate indefinitely; hence, $\alpha = 0$ defines the condition of prompt critical.

In reflected reactors, the decay or growth of prompt neutron chains is described by

$$N_p = A_1e^{\alpha_1 t} + A_2e^{\alpha_2 t} \tag{17}$$

where α_1 and α_2 arise as the result of two different groups of prompt neutrons.[3]

The first decay mode in Eq. (17) is associated with the prompt neutrons that multiply contiguously within the core region on a time scale corresponding to the average lifetime of a prompt neutron in the *bare* core, τ_c. The second decay mode is associated with that group of prompt neutrons that leak from the core region into the reflector region and then re-enter the core region where they further propagate the prompt-neutron chains by inducing additional fissions. This process occurs on the time scale of the *mean* prompt-neutron lifetime of the integral system.

If both α_1 and α_2 are negative, then the prompt neutron chains decay with time. On the other hand, if either α_1 or α_2 is positive, then the prompt neutron chains grow with time and the system is superprompt critical. Therefore, we define prompt critical for a reflected reactor as the point at which either α becomes zero.

We determine the reactivity corresponding to prompt critical in a reflected reactor using the solution obtained by Kistner.[3] In his formulation, delayed neutrons and external/intrinsic source neutrons are ignored in the coupled point-kinetic equations. Hence, Eqs. (2), (3), and (4) reduce to

$$\frac{dN_c}{dt} = -\lambda_c N_c + \lambda_{rc} N_r \tag{18}$$

$$\frac{dN_r}{dt} = -\lambda_r N_r + \lambda_{cr} N_c \tag{19}$$

where

$$\lambda_c = \frac{1-k_c(1-\beta)}{\tau_c} \tag{20}$$

$$\lambda_r = \frac{1}{\tau_r} \tag{21}$$

$$\lambda_{rc} = \frac{f_{rc}}{\tau_r} \tag{22}$$

and

$$\lambda_{cr} = \frac{f_{cr}}{\tau_c} \tag{23}$$

The solution of the above system of equations is obtained by taking the Laplace transform of Eqs. (18) and (19) and solving for the roots of the subsidiary equation; that is, α_1 and α_2 correspond to the roots of the quadratic equation

$$\alpha^2 + (\lambda_c + \lambda_r)\alpha + \lambda_c\lambda_r - \lambda_{cr}\lambda_{rc} = 0 \tag{24}$$

From the quadratic formula, we obtain

$$\alpha_1 = -\left[\frac{\lambda_c + \lambda_r}{2}\right] - \sqrt{\left[\frac{\lambda_c + \lambda_r}{2}\right]^2 - \Delta} \tag{25}$$

$$\alpha_2 = -\left[\frac{\lambda_c + \lambda_r}{2}\right] + \sqrt{\left[\frac{\lambda_c + \lambda_r}{2}\right]^2 - \Delta} \tag{26}$$

where

$$\Delta = \lambda_c\lambda_r - \lambda_{cr}\lambda_{rc} \tag{27}$$

The first root is always negative, whereas the second root becomes zero when $\Delta = 0$. Thus, prompt critical occurs when

$$\lambda_c\lambda_r = \lambda_{cr}\lambda_{rc} \tag{28}$$

Based on the definitions in Eqs. (20) through (23), this expression reduces to

$$f = 1 - k_c(1 - \beta) \tag{29}$$

Using $k = k_c + f$, we rewrite the above prompt critical condition in terms of the *overall* effective multiplication factor of the system as

$$k = \frac{1 - \beta f}{1 - \beta} \tag{30}$$

or, in terms of the traditional definition of reactivity,

$$\rho = \frac{\beta(1-f)}{1-\beta f} \approx \beta(1-f) \tag{31}$$

From Eq. (31), we see that prompt critical in a reflected reactor is shifted downward by a factor of $(1-f)$. As will be shown in the following section, this factor also appears in the definition of the mean prompt-neutron lifetime.

E. The Reflected-Core Inhour Equation

In most reflected reactors, k is controlled by changing k_c by means of inserting or removing control rods. Nevertheless, there are many reactors still in operation (e.g., SPR at Sandia and SKUA at Los Alamos) that control k by adding or removing reflector, thereby altering f. Regardless of the method used to control the reactivity of the system, the definition of the overall effective multiplication factor is still applicable. However, for the purposes of this paper, we assume that the change in k is controlled strictly by a change in k_c and that f is a constant over the operating reactivity range of the reactor.

For this situation, we obtain the applicable inhour equation for a reflected reactor by setting the denominator of the transfer function equal to zero where the transfer function is

$$\delta N_c = \frac{\frac{\tau_c}{k_c}\left[N_{co} + \sum \frac{C_{io}\lambda_i}{s + \lambda_i} + \frac{f_{rc}N_{ro}}{(\tau_r s + 1)}\right]}{s\frac{\tau_c}{k_c} + \frac{f\tau_r s}{k_c(\tau_r s + 1)} + \sum \frac{\beta_i s}{s + \lambda_i} - \rho\frac{k}{k_c}} \tag{32}$$

This yields

$$\rho = \frac{k_c}{k}\left[\omega\frac{\tau_c}{k_c} + \frac{f\tau_r\omega}{k_c(\tau_r\omega + 1)} + \sum \frac{\beta_i\omega}{\omega + \lambda_i}\right] \tag{33}$$

where ρ is defined in the usual way as $(k-1)/k$.

Note that when f approaches zero (which implies that $k \rightarrow k_c$ since $k = k_c + f$), the above expression collapses to the inhour equation for a bare reactor. When f is greater than zero, an extra term associated with the reflector appears in the equation, and the reactivity of the system is reduced by a factor of k_c/k.

Under certain conditions, Eq. (33) can be rewritten in a form analogous to the inhour equation for a bare reactor. To accomplish this, though, it is first necessary to define a *reflected-core* reactivity, ρ_c, as

$$\rho_c = \rho\left(\frac{k}{k_c}\right) = \frac{k-1}{k_c} \tag{34}$$

which, using the relationship $k = k_c + f$, can also be written as

$$\rho_c = \frac{k-1}{k-f} \tag{35}$$

If we define k_{co} as the multiplication factor of the bare core when the integral system is at delayed critical, i.e.,

$$k_{co} = 1 - f \tag{36}$$

then ρ_c becomes

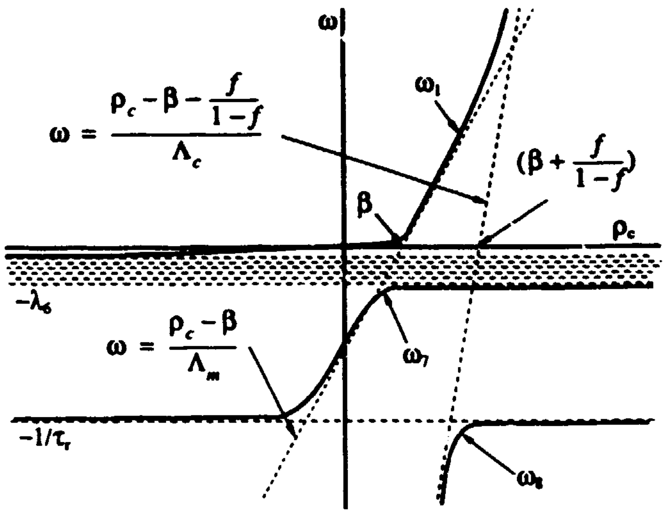

Figure 1. Qualitative plot of the roots of the reflected-core inhour equation. (Not drawn to scale).

$$\rho_c = \frac{k_c - k_{co}}{k_c} \tag{37}$$

which corresponds to the reactivity as defined by Cohn.[2]

Hence, Eq. (33) now becomes

$$\rho_c = \omega\frac{\tau_c}{k_c} + \frac{f\tau_r\omega}{k_c(\tau_r\omega + 1)} + \sum\frac{\beta_i\omega}{\omega + \lambda_i} \tag{38}$$

If the neutron lifetime in the reflector is small enough, we can combine the first two terms on the right-hand-side of Eq. (38) to yield the mean prompt-neutron lifetime model originally derived by Cohn.[2] However, as discussed in the following section, the mean prompt-neutron lifetime model should be used with caution because it may not always yield an adequate representation of the dynamic response of a reflected reactor at reactivities greater than 1\$.

F. Solution and Limitations of the Mean Prompt-Neutron Lifetime Model

If we assume the standard six groups of delayed neutrons, then Eq. (38) will have eight roots. A qualitative plot of these eight roots is shown in Fig. (1). The exact values of these eight roots, however, can be quite sensitive to the values of the prompt-neutron lifetime in the core and in the reflector. Furthermore, the appropriateness of the mean prompt-neutron lifetime model is also strongly dependent on which root of the reflected-core inhour equation is of interest and, in the case of the first root, on the reactivity of the system.

Case I. <u>Root 1 Below Prompt Critical</u>. Root 1 corresponds to the asymptotic inverse period of the reactor. For negative reactivities, this root will vary between 0.0 and $-\lambda_1$ (where λ_1 is the mean decay constant of the shortest lived delayed neutron group; this is approximately 0.01 s^{-1} for the common fissionable isotopes). For positive reactivities ranging from 0.0 to ~ 0.9\$, root 1 varies from 0.0 to a value on the order of 10 s^{-1}. Because the neutron lifetimes in most common reflector materials range from 10 µs (e.g., steel) to 1 ms (e.g., graphite), the product $\tau_r\omega_1 << 1.0$. Therefore, Eq. (38) reduces to the following equation

$$\rho_c \approx \Lambda_m\omega + \sum\frac{\beta_i\omega}{\omega + \lambda_i} \tag{39}$$

where the mean prompt-neutron *generation* time, Λ_m, is defined as the mean prompt-neutron lifetime,[a] i.e.,

$$\tau_m = \tau_c + f\tau_r \tag{40}$$

divided by k_c.

In the vicinity of delayed critical, k is approximately 1.0 and so $k_c \approx (1-f)$. Therefore,

$$\Lambda_m = \frac{\tau_c + f\tau_r}{1-f} \tag{41}$$

Equations (39) and (41) constitute the mean prompt-neutron lifetime time model for a reflected-core reactor.

As can be observed, Eq. (39) is now identical *in form* to the inhour equation for a bare reactor. However, it must be stressed that the neutron generation time in Eq. (39) represents a *mean* value as defined by Eq. (41) and that the reactivity ρ_c does not correspond to the traditional definition of reactivity [see Eq. (34)]. Nevertheless, because the form of the inhour equation is the same for both a bare reactor and a reflected reactor, the reactivity corresponding to a given inverse period must also be the same providing the characteristic neutron generation time in both systems is the same.

For example, if we compare a bare reactor having a prompt-neutron generation time of 50 µs with a reflected reactor having an equal mean prompt-neutron generation time and a return fraction of 20%, then, in accordance to the inhour equation for both the bare reactor and the reflected reactor, a 10 s asymptotic period will yield a reactivity of approximately 0.40\$. In the reflected reactor, however, 0.40\$ corresponds to ρ_c – not ρ. If converted to the

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[a] We note that the mean lifetime defined by Eq. (40) differs from the *static* mean lifetime previously defined in Eq. (9). The significance of this difference, however, is not well understood at this time.

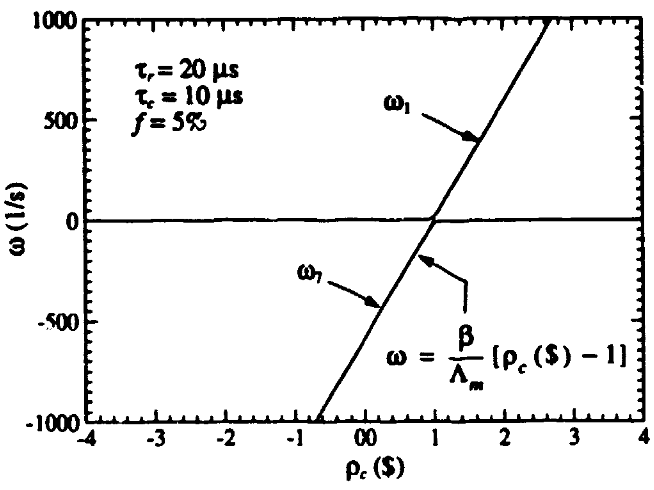

Figure 2. Plot of the first and seventh root of the reflected-core inhour equation.

traditional definition of reactivity, a 10 s asymptotic period in the reflected reactor would actually correspond to 0.32$ reactivity. It follows, therefore, that the absolute value of k necessary to produce a 10 s period in a reflected system is smaller than in a comparable bare system.

Case II. Root 1 Above Prompt Critical. Although the condition of $\tau_r\omega_1 << 1.0$ is satisfied for root 1 in the vicinity near prompt critical and below, it is not necessarily satisfied for reactivities greater than 1$. For those situations in which the neutron lifetime in the reflector is relatively small, it is likely that the condition $\tau_r\omega_1 << 1.0$ will still be satisfied at reactivities significantly greater than 1$. When this occurs, then the mean prompt-neutron lifetime model will be applicable and the first root will closely follow the asymptote

$$\omega = \frac{\rho_c - \beta}{\Lambda_m} \tag{42}$$

over the normal reactivity operating range of the system. An example of this situation is shown in Fig. (2) in which the exact solutions[a] for roots 1 and 7 are compared to the asymptote corresponding to the mean prompt-neutron lifetime model. As can be readily observed, both roots hug the asymptote very snugly over the range shown, thereby, confirming that the mean prompt-neutron lifetime model is applicable for the situation pictured.

[a] A Fortran code was written to solve for the roots of the inhour equation using a numerical scheme. In the context of this paper, therefore, *exact* means *to within machine accuracy.*

Before continuing, it is worth mentioning that the above asymptote crosses the reactivity axis at $\rho_c = \beta$ which defines prompt critical in the ω vs. ρ_c plane. Using Eq. (34) and the approximation that $k_c \approx 1 - f$ in the vicinity of prompt critical, we again obtain the result that prompt critical in a reflected reactor occurs at a reactivity of approximately $\rho \approx \beta(1 - f)$.

Case III. Roots 2 through 6. As with the bare-core inhour equation, roots 2 through 6 are completely bound by the decay constants corresponding to each of the six precursor groups. Therefore, root 2 ranges from $-\lambda_1$ to $-\lambda_2$, root 3 ranges from $-\lambda_2$ to $-\lambda_3$, and so forth. In general, the values of the λ's correspond to approximately 0.01, 0.03, 0.1, 0.3, 1.0, and 3.0 s^{-1}. Consequently, the condition $\tau_r\omega_i << 1.0$, where i equals 2 through 6, is easily satisfied. Hence, for these five roots, the mean prompt-neutron lifetime model is applicable.

Case IV. Root 7. The seventh root of the reflected-core inhour equation varies from $-\lambda_6$ to $-1/\tau_r$, and at reactivities in the vicinity of delayed critical, is asymptotic to Eq. (42). More often than not, the seventh root will not satisfy the condition $\tau_r\omega_7 << 1.0$ except when the reflector lifetime is very small and/or the reactivity is in the vicinity of prompt critical. As such, the mean prompt-neutron lifetime model will frequently be invalid for this particular root. An example of when the mean prompt-neutron lifetime model fails is shown in Fig. (3) where ω_7 (and ω_1) can be seen to deviate significantly from the mean prompt-neutron lifetime asymptote.

As readily observed from Figs. (2) and (3), the ω_7 root *below* prompt critical and the ω_1 root *above* prompt critical appear to be a mere continuation of each other. This, in fact, is the case. If one ignores the region very near prompt critical, it can be readily shown by direct comparison that the composite of ω_7 below prompt critical and ω_1 above prompt critical coincides almost exactly with α_2 in Kistner's model [see Eq. (26)]. (The comparison is not exact between the roots of the two models as a result of the inclusion of delayed neutrons in the exact solution.)

Consistency between Kistner's model and the exact solution of the reflected-core inhour equation is further demonstrated by expanding the radical in Eq. (26) and evaluating the resulting function at delayed critical. This yields

$$\alpha_{2o} = -\frac{\beta(1-f)}{\tau_c + f\tau_r} \tag{43}$$

which agrees with Eq. (42) evaluated at delayed critical.

Case V. Root 8. The eighth root of the reflected-core inhour equation varies from $-1/\tau_r$ to $-\infty$ and, at reactivities in the vicinity of delayed critical, is asymptotic to

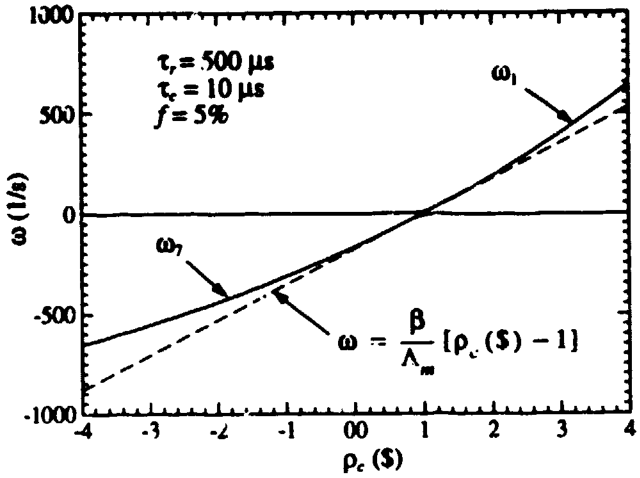

Figure 3. Plot of the first and seventh root of the reflected-core inhour equation.

$$\omega = \frac{\rho_c - \beta - \frac{f}{1-f}}{\Lambda_c} \tag{44}$$

where

$$\Lambda_c = \frac{\tau_c}{k_c} \approx \frac{\tau_c}{1-f} \tag{45}$$

is the prompt-neutron generation time of the *bare* core.

It should be noted that this root does not exist in the mean prompt-neutron lifetime model. It disappears as soon as it is assumed that $\tau_r\omega_8 << 1.0$ which, in most reactors, would rarely be satisfied because of the large magnitude of ω_8. For this reason, we are forced to solve for root 8 using Eq. (38) rather than Eq. (39) regardless of the validity of the mean prompt-neutron lifetime model.

III. REACTIVITY FORM OF POINT-KINETIC MODEL

Obviously, when the mean prompt-neutron lifetime model is not applicable for a given system, then the point-kinetic model represented by Eqs. (2), (3), and (4) should be used to predict the transient response of the system at reactivities greater than ρ_c = 1\$. However, the forms of Eqs. (2), (3), and (4) are not very convenient for obtaining numerical solutions since they are based on a time-dependent multiplication factor k_c rather than on a time-dependent reactivity.

With the use of Eqs. (34) and (35), Eqs. (2), (3), and (4) can be rewritten in terms of reactivity as

$$\frac{dN_c}{dt} = \frac{\rho_c - \beta - f(1-\beta)}{\tau_c} N_c + \frac{f_{rc}}{\tau_r} N_r + \sum \lambda_i C_i + S \tag{46}$$

$$\frac{dN_r}{dt} = \frac{f_{cr}}{\tau_c} N_c - \frac{N_r}{\tau_r} \tag{47}$$

$$\frac{dC_i}{dt} = \frac{(1-f)\beta_i N_c}{\tau_c} - \lambda_i C_i \tag{48}$$

where we have assumed that the term $(1-f)/(k-f)$ that would normally appear in the denominator of the first term on the right-hand-side of Eq. (46) is approximately equal to 1.0 in the vicinity of delayed critical and that $k_c \approx 1 - f$ in Eq. (4).

Note that Eqs. (46) through (48) collapse to the traditional point-kinetic equations for a bare system when the return fraction from the reflector, f_{rc}, is set equal to zero. This, by definition, forces f to equal zero [see Eq. (11)] and forces ρ_c to collapse to the traditional definition of reactivity [see Eq. (35)].

In the context of this model, the temperature feedback coefficient associated with the core region is defined as

$$\alpha_c = -\frac{d\rho_c}{dT_c} \tag{49}$$

and the core temperature is coupled to the neutron population (i.e., core power) by Newton's Law of Cooling:

$$\frac{dT_c}{dt} = K_c N_c - \gamma(T_c - T_{co}) \tag{50}$$

where

K_c reciprocal of the total heat capacity of the core,
γ reciprocal heat transfer time constant, and
T_{co} initial temperature of core at $t = 0$.

IV. CONCLUSIONS

Based on the solution of the reflected-core inhour equation, we make the following conclusions:

1. The reactivity ρ_c measured in reflected reactors using small positive or negative periods is a factor of k/k_c larger than the reactivity ρ defined in the traditional manner.

2. As a consequence of the aforementioned shift in reactivity, prompt critical in a reflected reactor occurs at a *reflected-core* reactivity of $\rho_c = \beta$, which is equivalent to a reactivity of $\rho \equiv \beta(1-f)$ where ρ is defined in the traditional manner.

3. The validity of the mean prompt-neutron lifetime

model is *root* dependent. For negative and small positive reactivities, the inverse asymptotic period ω_1 is small enough to satisfy the condition $\tau_r\omega_1 << 1.0$. However, for reactivities above prompt critical, this condition is only satisfied when τ_r is relatively small or the reactivity is very close to prompt critical.

4. When the condition $\tau_r\omega_1 << 1.0$ is not satisfied at reactivities above prompt critical, the asymptotic inverse period will vary in a *nonlinear* fashion with reactivity. Consequently, the dynamic response of a reflected-core pulse reactor may not be adequately represented during super-prompt critical operations using the bare core point-kinetic model.

5. For reflected-core systems in which the mean prompt-neutron lifetime model is not applicable, the relationship between asymptotic inverse period and super-prompt critical reactivity can be well represented by the second decay constant obtained in the two-region model developed by Kistner.

NOMENCLATURE

- k effective multiplication factor of integral system
- k_c multiplication factor of *bare* core
- ρ traditional reactivity $= (k-1)/k$
- ρ_c reflected-core reactivity $= (k-1)/k_c$
- N_c number of neutrons in the core region
- N_r number of neutrons in the reflector region
- N_t $= N_c + N_r$
- N_p number of prompt neutrons in integral system
- k_c multiplication factor of the *bare* core
- β effective delayed neutron fraction
- ν_t total number of neutrons born per fission
- τ_c neutron lifetime of the *bare* core
- τ_r neutron lifetime in the reflector region
- τ_f mean time between fission events in the bare core $= \nu_t\tau_c/k_c$
- τ_s *static* mean neutron lifetime of the integral system $= \tau_c + f_{cr}\tau_r$
- τ_m *dynamic* mean neutron lifetime of the integral system $= \tau_c + f\tau_r$
- Λ_m mean prompt-neutron generation time of integral system $= (\tau_c + f\tau_r)/k_c$
- Λ_c mean prompt-neutron generation time of the bare core $= \tau_c/k_c$
- f_{cr} fraction of neutrons that leak from the core into the reflector
- f_{rc} fraction of neutrons that leak from the reflector back into the core
- f total fraction of core neutrons that are returned to the core after having leaked from the core $= f_{rc}f_{cr}$
- C_i concentration of the i^{th} precursor group
- β_i delayed neutron fraction of the i^{th} group
- λ_i decay constant of the i^{th} precursor group
- S intrinsic/external neutron source rate
- ϕ neutron flux
- v neutron velocity
- w_i i^{th} root of inhour equation
- ω_1 asymptotic inverse period
- α prompt-neutron decay constant in one-region model
- α_1 prompt-neutron decay constant in two-region model associated with the *bare* core region $\cong \omega_8$ in the exact solution of the reflected-core inhour equation
- α_2 prompt-neutron decay constant in two-region model associated with the mean prompt-neutron lifetime of integral system $\cong \omega_7$ for negative reactivities and $\cong \omega_1$ for positive reactivities in the exact solution of the reflected-core inhour equation
- α_{20} α_2 evaluated at delayed critical
- α_c prompt temperature feedback coefficient of core
- K_c reciprocal of the total heat capacity of core
- γ reciprocal heat transfer time constant
- T_c average temperature of core
- T_{co} initial reference temperature of core

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