Family Mixing and the Origin of Mass

The difference between weak eigenstates and mass eigenstates

Stuart Raby

The Standard Model of elementary particle physics contains two disjoint sectors. The gauge sector describes the interactions of quarks and leptons (fermions, or spin-1/2 particles) with the spin-1 gauge bosons that mediate the strong, weak, and electromagnetic forces. This sector has great aesthetic appeal because the interactions are derived from local gauge symmetries. Also, the three families of quarks and leptons transform identically under those local symmetries and thus have the same basic strong, weak, and electromagnetic interactions.

The Higgs sector describes the interactions of the quarks and leptons with the spin-0 Higgs bosons h^+ and h^0. This sector is somewhat ad hoc and contains many free parameters. The Higgs bosons were originally introduced to break the weak isospin gauge symmetry of the weak interactions by giving mass to the weak gauge bosons, the W and the Z^0. The W and the Z^0 must be very heavy to explain why the weak force is so weak. But in the Standard Model, interactions with those Higgs bosons are also responsible for giving nonzero masses to the three families of quarks and leptons. Those interactions must yield different masses for the particles from different families and must cause the quarks from different families to mix, as observed in experiment. But neither the nine masses for the quarks and charged leptons nor the four parameters that specify the mixing of quarks across families are determined by any fundamental principle contained in the Standard Model. Instead, those thirteen parameters are determined from low-energy experiments and are matched to the free parameters in the Standard Model Lagrangian.

By definition, weak eigenstates are the members of the weak isospin doublets that transform into each other through interaction with the W boson (see Figure 5 on page 38). Mass eigenstates are states of definite mass created by the interaction with Higgs bosons. Those states describe freely propagating particles that are identified in detectors by their electric charge, mass, and spin quantum numbers. Since the Higgs interactions cause the quark weak eigenstates to mix with each other, the resulting mass eigenstates are not identical to the weak eigenstates.

Each set of eigenstates provides a description of the three families of quarks, and the two descriptions are related to each other by a set of unitary rotations. Most experimentalists are accustomed to seeing the Standard Model written in the mass eigenstate basis because the quarks of definite mass are the ingredients of protons, neutrons, and other metastable particles that the experimentalists measure. In the mass eigenstate basis, the Higgs interactions are diagonal, and the mixing across families appears in the gauge sector. In other words, the unitary rotations connecting the mass eigenstate basis to the weak eigenstate basis appear in the gauge interactions. Those rotation matrices could, in principle, appear in all the gauge interactions of quarks and leptons; but they do not. The Standard Model symmetries cause the rotation matrices to appear only in the quark charge-changing currents that couple to the W boson.

The specific product of rotation matrices that appears in the weak charge-changing currents is just what we call the CKM matrix, the unitary 3×3 mixing matrix deduced by Cabibbo, Kobayashi, and Maskawa. The elements in the CKM matrix have been determined by measuring, for example, the strengths of the strangeness-changing processes, in which a strange quark from the second family of mass states transforms into an up quark from the first family. So far, family mixing has not been observed among the leptons, with the possible exception of neutrino oscillations. If oscillations are confirmed, the mixing angles measured in the neutrino experiments will become part of a CKM mixing matrix for the leptons.

This sidebar derives the form of the CKM matrix and shows how it reflects the difference between the rotation matrices for the up-type quarks ($Q = +2/3$) and those for their weak partners, the down-type quarks ($Q = -1/3$). This difference causes the family mixing in weak-interaction processes and is an example of the way in which the Higgs sector breaks the weak symmetry. We will also show that, because the neutrino masses are assumed to be degenerate (namely, zero), in the Standard Model, the rotation matrices for the neutrinos can be defined as identical to those for their weak partners, and therefore the CKM matrix for the leptons is the identity matrix. Thus, in the minimal Standard Model, in which neutrinos are massless, no family mixing can occur among the leptons, and individual-lepton-family number is conserved.

This discussion attributes the origin of mixing to the mismatch between weak eigenstates and mass eigenstates caused by the Higgs sector. A more fundamental understanding of mixing would require understanding the origin of fermion masses and the reason for certain symmetries, or approximate symmetries, to hold in nature. For example, a fundamental theory of fermion masses would have to explain why muon-family number is conserved, or only approximately conserved. It would also have to explain why the $K^0 - \overline{\mathrm{K}}^0$ mixing amplitude is on the order of $G_{\mathrm{F}}{}^2$ and not larger. The small amount of family mixing observed in nature puts severe constraints on any theory of fermion masses. Developing such a theory is an outstanding problem in particle physics, but it may require a significant extension of the Standard Model.

To discuss mixing as it appears in the Standard Model, it is necessary to explicitly write down the parts of the Standard Model Lagrangian that contain the Yukawa interactions between the fermions and the Higgs bosons (responsible for fermion masses) and the weak gauge interaction between the fermions and the W boson (responsible for charge-changing processes such as beta decay). But first, we must define some notation. As in the sidebar "Neutrino Masses" on page 64, we describe the fermion states by two-component left-handed Weyl spinors. Specifically, we have the fields u_i, d_i, u_i^c, d_i^c, e_i, ν_i, and e_i^c, where the family index i runs from one to three. The u_i are the fields for the three up-type quarks u, c, and t with electric charge $Q = +2/3$, the d_i are the fields for the three down-type quarks d, s, and b with $Q = -1/3$, the e_i stand for the three charged leptons e, μ, and τ with $Q = -1$, and the ν_i stand for the three neutrinos ν_e, ν_μ, and ν_τ with $Q = 0$. The fields u_i and u_i^c, for example, are defined as follows:

u_i annihilates the left-handed up-type quark u_{L} and creates the right-handed up-type antiquark $\bar{u}_{\mathrm{R}}$ in family i, and

u_i^c annihilates the left-handed up-type antiquark $\bar{u}_{\mathrm{L}}$ and creates the right-handed up-type quark u_{R} in family i.

To describe the Hermitian conjugate fields $u_i^\dagger$ and $u_i^{c\dagger}$, interchange the words annihilate and create used above. Thus u_i, u_i^c, and their Hermitian conjugates describe the creation and annihilation of all the states of the up-type quarks. The down-type quark fields and the charged lepton fields are similarly defined. For the neutrinos, only the fields ν_i containing the states ν_{L} and $\bar{\nu}_{\mathrm{R}}$ are observed; the fields ν_i^c are not included in the Standard Model. In other words, the Standard Model includes right-handed charged leptons, but it has no right-handed neutrinos (or left-handed antineutrinos).

The Pauli Matrices for Spin-1/2 Particles

The Pauli spin matrices generate all rotations of spin-1/2 particles. Spin-1/2 particles have only two possible spin projections along, say the 3-axis: spin up, or $s_3 = +1/2$, and spin down, or $s_3 = -1/2$. The step-up operator σ^+ raises spin down to spin up, the step-down operator σ^- lowers spin up to spin down, and σ_3 gives the value of the spin projection along the 3-axis. The basis set for the spin quantized along the 3-axis is given by

$$\begin{pmatrix}1\\0\end{pmatrix} \text{ and } \begin{pmatrix}0\\1\end{pmatrix} \ ,$$

and the matrices are given by

$$\sigma^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad \sigma^2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \ .$$

Defining the matrices $\sigma^\pm$ as

$$\sigma^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2) \ ,$$

one arrives at the following commutation relations:

$$[\sigma^3, \sigma^\pm] = \pm 2\sigma^\pm \text{ , and}$$
$$[\sigma^+, \sigma^-] = \sigma^3 \ .$$

The Weak Eigenstate Basis. We begin by defining the theory in terms of the weak eigenstates denoted by the subscript 0 and the color red. Specifically, the weak gauge coupling to the W is given by

$$\mathscr{L}_{\text{weak}} = +\frac{g}{\sqrt{2}}(W^+_\mu J^\mu + W^-_\mu J^{\mu\dagger}) \ , \qquad (1)$$

where the charge-raising weak current J^μ is defined as

$$J^\mu = \sum_i u_{0i}{}^\dagger \bar{\sigma}^\mu d_{0i} + \nu_{0i}{}^\dagger \bar{\sigma}^\mu e_{0i} \ , \qquad (2)$$

and the charge-lowering current $J^{\mu\dagger}$ is defined as

$$J^{\mu\dagger} = \sum_i d_{0i}{}^\dagger \bar{\sigma}^\mu u_{0i} + e_{0i}{}^\dagger \bar{\sigma}^\mu \nu_{0i} \ . \qquad (3)$$

The constant g in Equation (1) specifies the strength of the weak interactions, and the $\bar{\sigma}^\mu$ is a four-component space-time vector given by $(1, -\sigma^j)$, where the σ^j are the standard Pauli spin matrices for spin-1/2 particles with $j = x, y, z$, the spatial directions. These 2×2 matrices act on the spin components of the spin-1/2 fields and are totally independent of the family index i. Each term in the charge-raising and charge-lowering currents connects states from the same family, which means the weak interactions in Equation (1) are diagonal in the weak eigenstate basis. In fact, those interactions define the weak eigenstates.

To understand the action of the currents, consider the first term, $u_0{}^\dagger \bar{\sigma}^\mu d_0$, in the charge-raising current J^μ. It annihilates a left-handed down quark and creates a left-handed up quark ($d_{0L} \to u_{0L}$) and, thereby, raises the electric charge by one unit. Electric charge is conserved because the W^+ field creates a W^-(see top diagram at right). The first term in the charge-lowering current $J^{\mu\dagger}$ does the reverse: $d_0{}^\dagger \bar{\sigma}^\mu u_0$ annihilates a left-handed up quark and creates a left-handed down quark ($u_{0L} \to d_{0L}$) and, thereby, lowers the electric charge by one unit; at the same time, the W^- field creates a W^+ (see bottom diagram at right). Thus, the members of each pair u_{0i} and d_{0i} transform into each other under the action of the charge-raising and charge-lowering weak currents and therefore are, by definition, a weak isospin doublet. The quark doublets are (u_0, d_0), (c_0, s_0), and (t_0, b_0), and the lepton doublets are (ν_{e0}, e_0), ($\nu_{\mu 0}$, μ_0), and ($\nu_{\tau 0}$, τ_0). The first member of the doublet has weak isotopic charge $I_3{}^{\text{w}} = +1/2$, and the second member has $I_3{}^{\text{w}} = -1/2$.

Finally, note that J^μ and $J^{\mu\dagger}$ are left-handed currents. They contain only the fermion fields f_0 and not the fermion fields $f_0{}^{\text{c}}$, which means that they create and annihilate only left-handed fermions $f_{0\text{L}}$ (and right-handed antifermions $\bar{f}_{0\text{R}}$). The right-handed fermions $f_{0\text{R}}$ (and left-handed antifermions $\bar{f}_{0\text{L}}$) are simply impervious to the charge-changing weak interactions, and therefore, the $f_0{}^{\text{c}}$ are weak isotopic singlets. They are invariant under the weak isospin transformations.

Weak isospin symmetry, like strong isospin symmetry from nuclear physics and the symmetry of rotations, is an SU(2) symmetry, which means that there are three generators of the group of weak isospin symmetry transformations. Those generators have the same commutation relations as the Pauli spin matrices. (The Pauli matrices, shown at left, generate all the rotations of spin-1/2 particles.) The J^μ and $J^{\mu\dagger}$ are the raising and lowering generators of weak isospin analogous to σ^+ and σ^-. The generator analogous to $1/2\sigma_3$ is $J_3{}^\mu$ given by

$$J_3{}^\mu = 1/2 \sum_i u_{0i}{}^\dagger \bar{\sigma}^\mu u_{0i} - d_{0i}{}^\dagger \bar{\sigma}^\mu d_{0i} + \nu_{0i}{}^\dagger \bar{\sigma}^\mu \nu_{0i} - e_{0i}{}^\dagger \bar{\sigma}^\mu e_{0i} \ , \qquad (4)$$

and the time components of these three currents obey the commutation relations $[J^0, J^{0\dagger}] = 2J_3{}^0$. In general, the time component of a current is the charge density, whereas the spatial component is the flux. Similarly, $J_3{}^0$ is the weak isotopic charge density. It contains terms of the form $f_0{}^\dagger f_0$, which are number operators N_f that count the number of f particles minus the number of $\bar{f}$ antiparticles present. When this density is integrated over all space, it yields the weak isotopic charge $I_3{}^{\text{w}}$:

$$\int J_3{}^0(x) d^3x = I_3{}^{\text{w}} \ .$$

Now, let us consider the Higgs sector. The fermion fields interact with the Higgs weak isospin doublet (h^+, h^0) through the Yukawa interactions given by

$$\mathscr{L}_{\text{Yukawa}} = \sum_{i,j} u^{\text{c}}_{0i} (Y_{\text{up}})_{ij} [u_{0j}\, h^0 - d_{0j}\, h^+] + d^{\text{c}}_{0i} (Y_{\text{down}})_{ij} [u_{0j}\, (h^+)^\dagger + d_{0j}\, (h^0)^\dagger] + e^{\text{c}}_{0i} (Y_{\text{lepton}})_{ij} [\nu_{0j}\, (h^+)^\dagger + e_{0j}\, (h^0)^\dagger] \ ,$$

where Y_{up}, Y_{down}, and Y_{lepton} are the complex 3×3 Yukawa matrices that give the strengths of the interactions between the fermions and the Higgs bosons. Because the Higgs fields form a weak isospin doublet, each expression in brackets is an inner product of two weak doublets, making an isospin singlet. Thus, each term in the Lagrangian is invariant under the local weak isospin symmetry since the conjugate fields (for example, $u^{\text{c}}{}_0$) are weak singlets. The lepton terms in Equation (5) are introduced in the sidebar “Neutrino Masses” (page 64), where masses are shown to arise directly from the Yukawa interactions because h^0 has a nonzero vacuum expectation value $\langle h^0 \rangle = v/\sqrt{2}$ that causes each type of fermion to feel an everpresent interaction. These interactions yield mass terms given by

$$\mathscr{L}_{\text{Yukawa}} \to \mathscr{L}_{\text{mass}} = u^{\text{c}}_{0i} (Y_{\text{up}})_{ij}\, u_{0j} \langle h^0 \rangle + d^{\text{c}}_{0i} (Y_{\text{down}})_{ij}\, d_{0j} \langle h^{0\dagger} \rangle + e^{\text{c}}_{0i} (Y_{\text{lepton}})_{ij}\, e_{0j} \langle h^{0\dagger} \rangle \ . \quad (6)$$

Notice that each term in $\mathscr{L}_{\text{mass}}$ contains a product of two fermion fields $f_0{}^{\text{c}} f_0$, which, by definition, annihilates a left-handed fermion and creates a right-handed fermion. Thus, these Yukawa interactions flip the handedness of fermions, a prerequisite for giving nonzero masses to the fermions. These terms resemble the Dirac mass terms introduced in the sidebar “Neutrino Masses,” except that the matrices Y_{up}, Y_{down}, and Y_{lepton} are *not* diagonal. Thus, in the weak eigenstate basis, the masses and the mixing across families occur in the Higgs sector.

The Mass Eigenstate Basis and the Higgs Sector. Let us examine the theory in the mass eigenstate basis. We find this basis by diagonalizing the Yukawa matrices in the mass terms of Equation (6). In general, each Yukawa matrix is diagonalized by two unitary 3×3 transformation matrices. For example, the diagonal Yukawa matrix for the up quarks $\hat{Y}_{\text{up}}$ is given by

$$\hat{Y}_{\text{up}} = V_u^{\text{R}}\, Y_{\text{up}} V_u{}^{\text{L}\dagger} \ , \qquad (7)$$

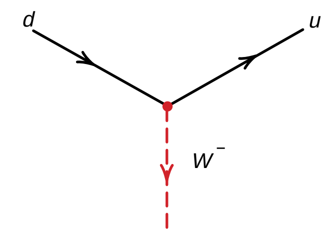
where matrix V_u^{R} acts on the right-handed up-type quarks in the fields $u^{\text{c}}{}_0$, and matrix $V_u{}^{\text{L}}$ acts on the left-handed up-type quarks in u_0. The diagonal elements of $\hat{Y}_{\text{up}}$ are $(\lambda_{\text{u}}, \lambda_{\text{c}}, \lambda_{\text{t}})$, the Yukawa interaction strengths for all the up-type quarks: the up, charm, and top, respectively. Matrices $\hat{Y}_{\text{down}}$ and $\hat{Y}_{\text{lepton}}$ are similarly diagonalized. If u_0 and $u^{\text{c}}{}_0$ are the fields in the weak eigenstate, the fields in the mass eigenstate, u^{c} and u, are defined by the unitary transformations

$$u^{\text{c}}{}_0 = u^{\text{c}}\, V_u^{\text{R}} \quad \text{and} \quad u_0 = V_u{}^{\text{L}\dagger}\, u \ . \qquad (8)$$

Since the Vs are unitary transformations, $V^\dagger V = VV^\dagger = I$, we also have

The charge-raising weak interaction in the first family

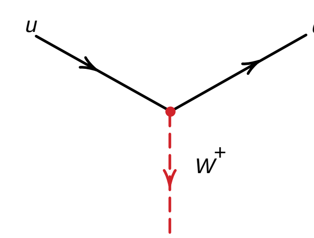
$$(W^+_\mu J^\mu)_{\text{first family}} = W^+_\mu\, u_0{}^\dagger \bar{\sigma}^\mu d_0 \ .$$



A down quark changes to an up quark with the emission of a W^-.

The charge-lowering weak interaction in the first family

$$(W^-_\mu J^{\mu\dagger})_{\text{first family}} = W^-_\mu\, d_0{}^\dagger \bar{\sigma}^\mu u_0 \ .$$



An up quark changes to a down quark with the emission of a W^+.

$$u^c = u^c_0\, V_u^{R\,\dagger} \quad \text{and} \quad u = V_u^{\,L} u_0 \ .$$

In this new mass basis, $\mathcal{L}_{\text{mass}}$ in Equation (6) takes the form

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= \sum_i u^c{}_i\, \hat{Y}^i_{\text{up}}\, u_i <h^0> + d_i^c\, \hat{Y}^i_{\text{down}}\, d_i <h^0> + e^c{}_i\, \hat{Y}^i_{\text{lepton}}\, e_i <h^0> \\ &= \sum_i u^c{}_i\, \hat{M}^{\,i}_{\text{up}}\, u_i + d_i^c\, \hat{M}^{\,i}_{\text{down}}\, d_i + e^c{}_i\, \hat{M}^{\,i}_{\text{lepton}}\, e_i \ , \end{aligned} \tag{9}$$

where the matrices $\hat{M}^i = \hat{Y}^i \text{v}/\sqrt{2}$ are diagonal, and the diagonal elements are just the masses of the fermions. In particular, we can write out the three terms for the up-type quarks u, c, and t:

$$\begin{aligned}\sum_i u^c{}_i\, \hat{M}^{\,i}_{\text{up}}\, u_i &= \lambda_u\, u^c\, u <h^0> + \lambda_c\, c^c\, c <h^0> + \lambda_t\, t^c\, t <h^0> \\ &= \lambda_u\, \text{v}/\sqrt{2}\, u^c\, u + \lambda_c\, \text{v}/\sqrt{2}\, c^c\, c + \lambda_t\, \text{v}/\sqrt{2}\, t^c\, t \\ &= m_u\, u^c\, u + m_c\, c^c\, c + m_t\, t^c\, t \ , \end{aligned} \tag{10}$$

with the masses of the up, charm, and top quarks given by

$$m_u = \lambda_u \text{v}/\sqrt{2}\,,\ \ m_c = \lambda_c \text{v}/\sqrt{2}\,,\ \text{ and } m_t = \lambda_t \text{v}/\sqrt{2}\ .$$

Thus, the Higgs sector defines the mass eigenstate basis, and the diagonal elements of the mass matrices are the particle masses.

Mixing in the Mass Eigenstate Basis. Now, let us write the weak gauge interaction with the W in the mass eigenstate. Recall that

$$\mathcal{L}_{\text{weak}} = +\frac{g}{\sqrt{2}}(W^+_\mu J^\mu + W^-_\mu J^{\mu\dagger}) \ ,$$

but to write the charge-raising weak current J^μ in the mass eigenstate, we substitute Equation (8) into Equation (2),

$$\begin{aligned} J^\mu &= \sum_i u_{0i}{}^\dagger \bar{\sigma}^\mu d_{0i} + \nu_{0i}{}^\dagger \bar{\sigma}^\mu e_{0i} \\ &= \sum_{i,k,j} u_i{}^\dagger (V_u{}^{\text{L}})_{ik} \bar{\sigma}^\mu (V_d{}^{\text{L}\dagger})_{kj} d_j + \nu_i{}^\dagger \bar{\sigma}^\mu e_i \\ &= \sum_{i,j} u_i{}^\dagger \bar{\sigma}^\mu (V_{\text{CKM}})_{ij} d_j + \nu_i{}^\dagger \bar{\sigma}^\mu e_i \ , \end{aligned} \tag{11}$$

and to rewrite the charge-lowering current $J^{\mu\dagger}$, we substitute Equation (8) into Equation (3):

$$\begin{aligned} J^{\mu\dagger} &= \sum_i d_{0i}{}^\dagger \bar{\sigma}^\mu u_{0i} + e_{0i}{}^\dagger \bar{\sigma}^\mu \nu_{0i} \\ &= \sum_{i,k,j} d_i{}^\dagger (V_d{}^{\text{L}})_{ik} \bar{\sigma}^\mu (V_u{}^{\text{L}\dagger})_{kj} u_j + e_i{}^\dagger \bar{\sigma}^\mu \nu_i \\ &= \sum_{i,j} d_i{}^\dagger\, \bar{\sigma}^\mu (V_{\text{CKM}})^\dagger_{ij}\, u_j + e_i{}^\dagger\, \bar{\sigma}^\mu e_i \ , \end{aligned} \tag{12}$$

where

$$V_{\text{CKM}} = V_u^{\text{L}}\, V_d^{\text{L}\dagger} \ . \tag{13}$$

Thus, the charge-raising and charge-lowering quark currents are *not* diagonal in the mass eigenstate basis. Instead, they contain the complex 3×3 mixing matrix V_{CKM}. This matrix would be the identity matrix were it not for the difference between the rotation matrices for the up-type quarks V_u^{L} and those for the down-type quarks V_d^{L}. It is that difference that determines the amount of family mixing in weak-interaction processes. For that reason, all the mixing can be placed in either the up-type or down-type quarks, and by convention, the CKM matrix places all the mixing in the down-type quarks. The weak eigenstates for the down-type quarks are often defined as d':

$$d' = V_{\text{CKM}}\, d = V_u^{\text{L}}\, V_d^{\text{L}\dagger}\, d = V_u^{\text{L}}\, d_0 \ , \tag{14}$$

in which case, the up-type weak partners to d' become u':

$$u' = V_u^{\text{L}}\, u_0 \equiv u \ .$$

When all the mixing is placed in the down-type quarks, the weak eigenstates for the up-type quarks are the same as the mass eigenstates. (We could just as easily place the mixing in the up-type quarks by defining a set of fields u' given in terms of the mass eigenstates u and V_{CKM}.) Independent of any convention, the weak currents J^μ couple quark mass eigenstates from different families. The form of the CKM matrix shows that, from the Higgs perspective, the up-type and down-type quarks look different. It is this mismatch that causes the mixing across quark families. If the rotation matrices for the up-type and down-type left-handed quarks were the same, that is, if $V_u{}^{\text{L}} = V_d{}^{\text{L}}$, the CKM matrix would be the identity matrix, and there would be no family mixing in weak-interaction processes. The existence of the CKM matrix is thus another example of the way in which the mass sector (through the Higgs mechanism) breaks the weak isospin symmetry. It also breaks nuclear isospin symmetry (the symmetry between up-type and down-type quarks), which acts symmetrically on left-handed and right-handed quarks.

Note that the mixing matrices V^{R} associated with the right-handed fermions do not enter into the Standard Model. They do, however, become relevant in extensions of the Standard Model, such as supersymmetric or left-right-symmetric models, and they can add to family-number violating processes.

Finally, we note that, because the neutrinos are assumed to be massless in the Standard Model, there is no mixing matrix for the leptons. In general, the leptonic analog to the CKM matrix has the form

$$V_{\text{lepton}} = V_\nu{}^{\text{L}} V_e^{\text{L}\dagger} \ .$$

But we are free to choose any basis for the neutrinos because they all have the same mass. By choosing the rotation matrix for the neutrinos to be the same as that for the charged leptons $V_\nu{}^{\text{L}} = V_e{}^{\text{L}}$, we have

$$\nu_0 = V_e{}^{\text{L}\dagger}\, \nu \quad \text{and} \quad e_0 = V_e{}^{\text{L}\dagger}\, e \ .$$

The leptonic part of, for example, the charge-raising current is

$$\sum_i \nu_{0i}{}^\dagger\, \bar{\sigma}^\mu e_{0i} = \sum_{i,k,j} \nu_i{}^\dagger (V_e{}^{\text{L}})_{ij} \bar{\sigma}^\mu (V_e{}^{\text{L}\dagger})_{jk} e_k = \sum_i \nu_i{}^\dagger\, \bar{\sigma}^\mu e_i \ ,$$

and the leptonic analog of the CKM matrix is the identity matrix. This choice of eigenstate would not be possible, however, if neutrinos have different masses. On the contrary, the neutrinos would have a well-defined mass eigenstate and there would likely be a leptonic CKM matrix different from the identity matrix. It is this leptonic mixing matrix that would be responsible for neutrino oscillations as well as for family-number violating processes such as $\mu \rightarrow e + \gamma$. ■