Dark Matter and Massive Neutrinos

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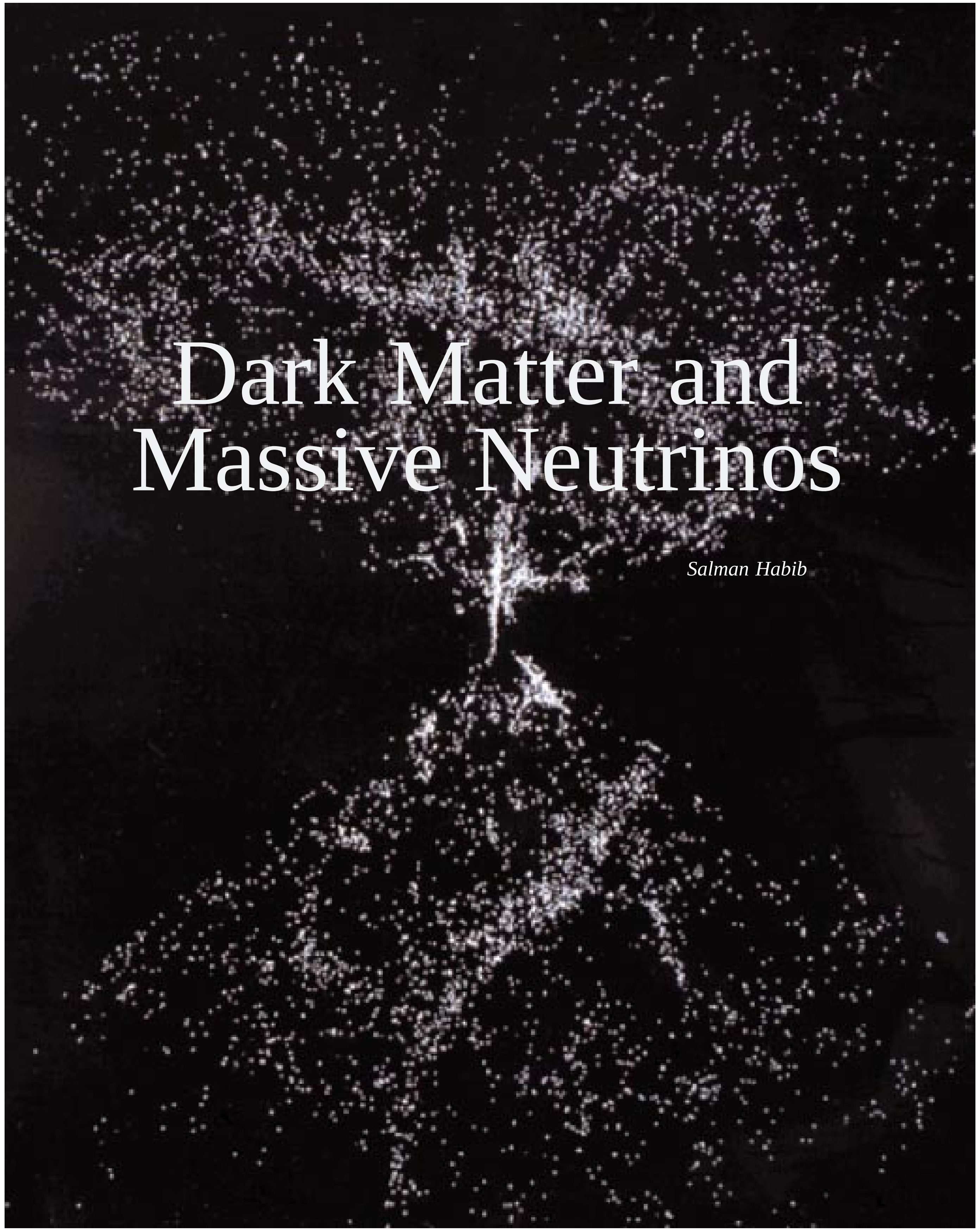

The Standard Model of particle physics does not include a natural mechanism to give mass to a neutrino. Neither does it provide a reason to exclude this possibility. Unlike photons and gravitons, which are constrained to be exactly massless because of gauge invariance, no such restriction applies to the neutrino. Determining neutrino masses has been a long-standing experimental challenge that, despite concerted efforts, has proved rather difficult. To date, there is no direct evidence for neutrino mass, though upper limits of $m_{\nu_e} < 15$ electron volts[1] (eV), $m_{\nu_\mu} < 170$ kilo-electron-volts (keV), and $m_{\nu_\tau} < 24$ million electron volts (MeV) have been placed.

It is natural to speculate what the impact on physics would be if neutrinos were in fact massive. As far as our everyday world is concerned, there would be almost no effect at all: nuclei would still undergo beta decay, elements would still transmute, and stars would still boil inside and explode because of neutrino heating. Solar and atmospheric neutrinos would still be missing, although physicists would be fairly certain as to where they went.

Turning to the Universe, however, massive neutrinos could effect a radical transformation. Next to the ubiquitous photons that compose the cosmic microwave background radiation (the radiation field that permeates the Universe), neutrinos are the second-most-abundant particle species. Were they to have even a small mass, it would lead to profound consequences for the evolution of the Universe. In this article, we explore the possible impact that neutrinos with mass would have on three central issues in modern cosmology: the dynamics of the Universe, structure formation, and dark matter.

Cosmology is the science of the evolution and structure of the Universe. The concerns of cosmology include the birth of the Universe, its present age, and its ultimate fate. Some of the most pressing questions of current interest relate to the material make-up of the Universe: How much mass is present? What is it made of? How is mass distributed in space and how did it get there? A massive neutrino might well play a key role in the resolution of these puzzles.

According to the accepted theoretical paradigm in cosmology—the Big Bang—the Universe began as a hot, dense plasma that was isotropic and homogeneous to a very high degree. Fifteen billion or so years later, however, it is quite inhomogeneous (except on very large scales). Today the Universe is filled with galaxies that are arranged in clusters and sheets that surround vast pockets of space. Cosmologists attempting to understand structure formation must confront this puzzle: how did density fluctuations originate in the early Universe, and how did these small inhomogeneities lead to the distribution of mass that is currently observed?

Running parallel to the questions surrounding structure formation is the enigma of dark matter. After many years of observations, astronomers and cosmologists have been forced to a curious conclusion: the Universe appears to be dominated by an unseen form of matter whose precise nature is unknown.

For decades, it has been accepted that the luminous matter visible to the astronomer's telescope—stars, dust, gas clouds, bright galaxies, even black holes—constitutes but a tiny fraction of the total mass of the Universe. The phrase "luminous matter" refers to any matter that emits, directly or indirectly, electromagnetic radiation (from radio waves to gamma rays) that can be detected on earth. It is in this sense that large black holes can be considered luminous, for they advertise their presence by x-rays that are emitted when material falls into the hole.

Dark matter is the unseen mass of the Universe. It is the antithesis of luminous matter, for it does not emit any detectable radiation, and its presence can be inferred only indirectly from the way it interacts with luminous matter. The three key questions relating to dark matter are what is it made of, how much is there, and how is it distributed?

Because it cannot be seen, we can only speculate as to what makes up dark matter. Many astronomers would argue that dark matter is simply stuff from the Universe's graveyard: brown dwarfs, dead stars, sparse gas clouds that never coalesced, even entire galaxies with low surface brightness. If this belief is true, dark matter would be ordinary *baryonic* matter—that is, matter composed of protons, neutrons, and electrons—that just fails to be detected.

Many theorists are convinced, however, that there is an exotic, *nonbaryonic* form of dark matter and that there is a lot more of it than ordinary matter. They hypothesize that the nonbaryonic dark matter is composed of particles that were created during the early, hot phase of the Universe but that still exist today. It is within this realm that the massive neutrino resides. While there are other plausible candidates for dark matter particles, such as axions and supersymmetric neutralinos, the neutrino is unique in that it is known to exist.

Because of improved observational capabilities, the last decade has seen a remarkable renaissance in astrophysics and cosmology. Telescopes such as the Keck and the Hubble Space Telescope, satellite experiments such as RELICT and the Cosmic Background Explorer (COBE), and large-scale redshift surveys such as CfA (conducted by the Harvard-Smithsonian Center for Astrophysics) and the Las Campanas Redshift Survey have changed the face of observational cosmology. As shown later on, the better quality of present-day data already allows us to rule out several plausible hypotheses concerning dark matter and structure formation.

In the coming decade, it is expected that data from new satellite missions that will measure the microwave background with unprecedented precision,

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[1] This is a conservative upper limit on the electron neutrino mass. See R. M. Barnett et al., *Physical Review D* **54**, 1 (1996) for a complete discussion of mass limits.

ombined with new, high-statistics edshift surveys that will probe the arge-scale structure of the Universe, will finally lead to a cohesive picture of he Universe on large-distance scales.

Although the primary purpose of this rticle is to explain current theories bout dark matter, structure formation, nd the dynamics of the Universe, we o so with a word of caution. At resent, the situation in cosmology is omewhat chaotic. Theorists are scrambling to keep pace with observational ata, much of which is not simple to nterpret and may contain significant ystematic errors. Some of the observaons discussed below are very recent nd their validity may not survive over me, but given the state of cosmology oday, they are all we have to work with at the moment.

Dark Matter: A Historical Problem

Astronomers tend to be a cautious ot, and with good reason. Ever since ohn Adams (in 1845) and Urbain-ean-Joseph Le Verrier (independently n 1846) inferred the existence of the lanet Neptune (through its gravitaonal effects on the orbit of Uranus), stronomers have appreciated that matter is often invisible to their elescopes.

It was therefore only a minor probem when Jacobus Kapteyn and James eans in 1922, and then Jan Oort a ecade later, deduced that our galaxy, he Milky Way, might contain at least wice as much mass as could be ccounted for by luminous matter lone. The missing mass would surely e found once more precise observaons were made and the systematic rrors identified and taken into account. hree-quarters of a century later, not nly is the mass still “missing,” but the action of galactic matter thought to be ark has increased.

The root of all such deductions is that mass is the sole source of the gravitaonal field, and gravity is the only force we know of that binds a galaxy together and creates structure on cosmic scales. If a galaxy as a whole rotates, only the force of gravity can prevent it from flying apart. Measuring how fast a galaxy spins, therefore, gives an estimate of the strength of the galactic gravitational field, from which we can deduce the total mass needed to create that field.

The visible portion of a typical galaxy extends out about 15 kiloparsecs (kpc) from the galactic center.^{2} Large clouds of atomic hydrogen, however, extend much farther, out to about 25 kpc. Measurements reveal that these clouds are in very high velocity orbits about the galactic center. A gravitational field strong enough to hold onto the hydrogen reaches well beyond the farthest stars in the galaxy.

Because the strength of gravity decreases inversely with the square of the distance from the source of the field, we are forced to conclude that the visible stars cannot be the *dominant* source of a galaxy’s gravitational field. Rather, a “dark halo” of unseen matter must exist beyond the luminous core and must constitute the bulk of a galaxy’s mass. This important deduction was first made by Kenneth Freeman in 1970.

How large is the halo? That is difficult to determine, because the hydrogen only extends so far, and until recently, there was no way to map out the gravitational field at arbitrarily large galactic radii. In fact, there does not exist a single spiral galaxy with a well-characterized halo or a well-determined mass! If we assume a “standard” galactic halo about 50 kpc in radius, then the total mass of a galaxy is roughly 10^{11} solar masses, or about a factor of 10 larger than estimates of the mass of the luminous core plus the hydrogen. Thus, by observing the dynamics of a galaxy, we conclude that roughly 90 percent of its structure consists of dark matter.

Current estimates, however, suggest that dark halos could go out much farther, to roughly 200 kpc. If these results hold up, then the amount of dark matter in galaxies could be several times greater than the earlier dynamical estimates.

Galaxies are not the only type of structure in the Universe. They often group together to form clusters, which may contain hundreds to thousands of galaxies and span a distance of a few megaparsecs. Similar to the stars in a single galaxy, the galaxies in a cluster are all bound together by a common gravitational field. By examining the dynamics of constituent galaxies, we can estimate the total mass of the cluster. This technique was first applied to rich clusters by Fritz Zwicky back in 1933.

The Coma cluster of galaxies is an oft-cited example. The dispersion of random galactic velocities about the mean in this cluster is roughly 900 kilometers per second (km/s). Such high velocities demand that the cluster mass be about 5×10^{15} solar masses in order for the system to be gravitationally bound. However, the total luminosity of the cluster is only 3×10^{13} solar luminosities. This would give a ratio of the total mass to luminous mass (M/L) of close to 200 to 1, an estimate that is consistent with determinations from other rich clusters. Because the presence of x-ray-emitting gas increases the luminous mass by roughly a factor of 2, M/L is really more like 100 to 1. Thus, approximately 99 percent of the mass of a cluster comes from dark matter, a value that is roughly consistent with recent estimates of the amount of dark matter bound up in individual galaxies.

The data implies that the M/L ratio begins to approach a maximum at distances of hundreds of kiloparsecs to a megaparsec, which suggests that the distribution of mass in a cluster has resulted from a redistribution of the mass in individual galaxies. Although that deduction is pleasing, it runs counter to a prejudice held by many theorists that M/L should continue to increase with increasing distance scales. These theorists claim that a Universe consisting of only 99 percent dark matter is still too light and that there is more mass in the Universe than implied by the observations discussed so far. To explore their reasons, we must make a small digression into Big Bang cosmology.

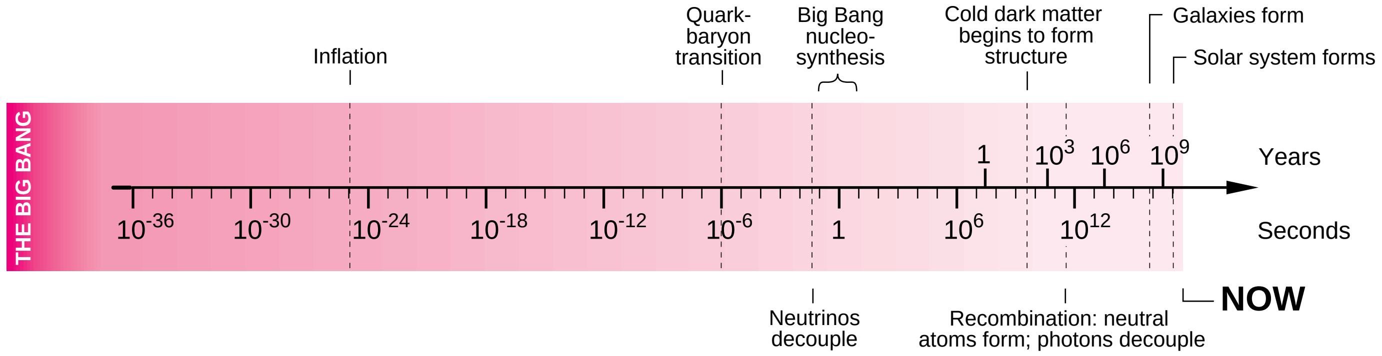

Figure 1. The Universe’s Time Line

The Big Bang model allows cosmologists to order events in the evolution of the Universe. This figure plots time on a logarithmic scale. Although cold dark matter begins to form structures within the first 100 years or so of the Universe’s history, those structures do not evolve into galaxies (or clusters of galaxies) until many millions of years later.

The Big Bang: Dark Matter and the Dynamics of the Universe

One of the seminal discoveries in the history of science is Edwin Hubble’s observation in 1929 that galaxies are receding from each other at a velocity v that is proportional to their distance l:

$$v = H(t)\, l \ .$$

The constant of proportionality, $H(t)$ (known as the Hubble constant), is actually a function of time. It is a difficult parameter to measure, but most cosmologists agree that its current value, $H(t_0) = H_0$, lies in the range of 55–75 km/s-Mpc. The uncertainty in the value of H_0 is contained in the parameter h, defined as $H_0 = 100\, h$ km/s-Mpc.

Hubble’s finding agreed with the velocity versus distance relationship predicted by Albert Einstein’s general theory of relativity. The expanding Universe was therefore taken to be strong evidence that general relativity correctly describes the dynamics of the Universe. Starting with present-day data, if the equations of general relativity are run backwards in time, the Universe becomes increasingly hotter and denser until the initial singularity, or a state of infinite density, is finally encountered. This is the moment of the Big Bang. If we run the clock forward from this moment (and use general relativity, particle and nuclear physics, electrodynamics, and thermodynamics to govern the interactions of matter, radiation, and geometry), we can construct a time line that orders the evolution of the Universe (see Figure 1).

The Big Bang model holds that the Universe began in a state of infinite density and temperature, followed by rapid expansion and cooling. About 10^{-30} seconds after its birth, quarks, leptons, and gauge bosons precipitated out much like ice crystals in a cooling pool of water. (Quarks and leptons are discussed in the primer, “The Oscillating Neutrino,” on page 28.) Within a few microseconds, protons and neutrons formed from the quarks, and within about one second, the synthesis of primordial nuclei—hydrogen, helium, and trace amounts of lithium—began.

Primordial nucleosynthesis was completed by the time the Universe was about three minutes old, but the Universe was still too hot for the nuclei to capture electrons and form neutral atoms. The Universe was filled with charged matter that continually scattered a background radiation field of energetic photons. Radiation and matter were in thermal equilibrium. Ten thousand years after the Big Bang, however, the expanding Universe had cooled to the point that atoms formed. This epoch, termed recombination, marks the time that radiation and matter decoupled. The Universe essentially became transparent to electromagnetic radiation, and radiation and matter began to follow separate evolutionary paths.

The primordial radiation field still permeates the Universe today and is essentially unchanged since the time of recombination. Because of the expansion of the Universe, the field has lost energy, and since a photon’s energy is proportional to frequency, the radiation has shifted down into the microwave band. The existence of cosmic microwave background radiation (CMBR) was predicted by George Gamow, Ralph Alpher, and Robert Herman in 1948, and it was finally detected by Arno Penzias and Robert Wilson in 1965.

The Universe is still expanding today and literally creating its own space. What, however, determines whether it will do so forever, or will eventually deflate? The crucial parameter turns out to be the average mass density of the Universe, ρ. Below a certain critical mass density, ρ_c, the Universe is “open” and will forever

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[2] A parsec is equal to 3.258 light-years, or about 3×10^{13} kilometers. In general, the nearest stars are on the order of a few parsecs from earth, the diameter of a spiral galaxy spans tens of thousands of parsecs (kiloparsecs, or kpc), and distances between galaxies are about a million parsecs (megaparsecs, or Mpc).

ontinue to expand. Above that critical ensity, the Universe is “closed” and will eventually recollapse into itself. At exactly the critical density, the Universe is said to be “flat.” It is still nfinite in space and time, but its rate f expansion asymptotically approaches ero. The critical density is easily erived; at the present time, it has the alue

$$\rho_c = \frac{3H_0^2}{8\pi G} \quad ,$$

where G is Newton’s gravitational onstant. Today,

$$\rho_c \sim 4 \times 10^{-30} \ \text{g/cm}^3 \quad ,$$

r a few hydrogen atoms per cubic meter. (We have assumed a value of $H_0 \approx 65$ km/s-Mpc.)

The critical, or closure, density rovides a natural base line with which o compare observed mass densities. Defining a parameter Ω as the ratio of ny density, ρ, to the critical density,

$$\Omega = \frac{\rho}{\rho_c} \quad ,$$

we have that at the critical density, $\Omega = 1$. The observed luminous, bary-nic matter leads to

$$\Omega_{\text{luminous}} = \frac{\rho_{\text{luminous}}}{\rho_c} \sim 0.003,$$

mere 0.3 percent of the closure ensity, whereas measurements of lusters produce values for Ω_{cluster} within the range of 0.1 to 0.3.

Most theorists, however, believe hat the Universe is at or extremely lose to the critical density (in spite of he apparent conflict with current bservations). The basis for their belief es in the resolution of what is called he “flatness” problem in cosmology.

At birth, the Universe is postulated o be infinitely dense (the initial singu-arity). For generic initial conditions, nce the Universe begins to expand, ravity under most circumstances auses it to recollapse instantly on itself or to expand at such an enormous rate that no structures could ever form. This is because the natural time scale in general relativity is the Planck time, which is only about 10^{-43} seconds! The fact that the Universe has existed for 10^{60} Planck times cannot be explained without very special initial conditions (or entirely new physics). Only if the Universe started exquisitely close to the critical density could it have survived for such a long time.

The theory of inflation, which has become an almost essential piece of today’s cosmology, was designed to deal with issues such as the flatness problem (also called the age problem). Inflation typically predicts deviations from the critical density on the order of only 1 part in 10^{60}. Inflation also accounts for the “horizon” problem, which stems from the observation that the cosmic microwave background is remarkably isotropic across the entire sky. This is a puzzle, because points in the sky separated in angle by more than roughly a degree have not been in causal contact since the Big Bang. Inflation provides a resolution to both problems by postulating a phase of rapid expansion of the Universe driven by a matter field called the inflaton. During inflation, the scale factor of the Universe grows by a factor of roughly 10^{43}. This growth occurs on a time scale as short as 10^{-32} seconds! In essence, inflation adds a long “history” to the Universe before the decoupling of radiation and matter, so that objects that appear to be acausally connected in the microwave sky actually interacted in the past. Finally, through quantum fluctuations of the inflaton field, inflation provides the Big Bang with a natural mechanism to generate primordial density perturbations. This is an important point that will be discussed in the section on structure formation.

Whether Ω is unity or on the order of a few tenths, it is apparent that the luminous fraction constitutes a very small proportion of the total mass of the Universe. The next natural question to ask is what is the composition of this mysterious dark matter?

Again, we turn to Big Bang cosmology. One of the major achievements to emerge from that paradigm is the theory of Big Bang nucleosynthesis (BBNS). This theory describes how the two lightest baryons (protons and neutrons) could fuse together and form the light elements that are observed today in the cosmos. Protons (^{1}H) and neutrons first fused together to form deuterium. Fusion reactions involving deuterium then created tritium and helium-3, which were then used in further reactions to build helium-4 and trace amounts of lithium-7. These six elements (^{1}H, ^{2}H, ^{3}H, ^{3}He, ^{4}He, and ^{7}Li) are the only long-lived nuclei to be produced early on in the history of the Universe. (Heavier elements, from beryllium to uranium, were produced millions of years after the Big Bang by stellar nucleosynthesis and by supernova explosions.)

BBNS theory has only one free parameter, η, which is the primordial ratio of baryons to photons (or equivalently, the ratio of matter to radiation in the early Universe). Relative abundances of the primordial elements can be predicted as a function of η and compared with observations. This comparison leads to the estimate

$$\eta = \frac{\text{number of baryons}}{\text{number of photons}} \sim 2.5 - 6 \times 10^{-10} \quad .$$

Now comes a remarkable bit of good fortune: an imprint of the primordial photon density still exists as the CMBR and has been measured with extreme precision by the COBE (satellite) and COBRA (rocket) experiments. Thus, from the estimates of η, we can estimate the primordial baryon density, that is, the total number of baryons that were produced during the Big Bang.

For many years, BBNS set a limit for the baryon density that was $\Omega_{\text{BBNS}} \sim 0.06$, a factor of 5 lower than the mass density estimate from clusters and only 6 percent of the value predicted by inflation. This was viewed as unequivocal evidence for a nonbaryonic, massive particle, and several candidates were proposed: massive neutrinos, axions, neutralinos, quark nuggets, and primordial black holes.

However, there may yet be further surprises in store. It turns out that the estimate of η depends sensitively on the primordial abundance of deuterium. Deuterium absorption lines were recently measured in primordial intergalactic clouds illuminated by a background quasar. The conclusion was that previous estimates for deuterium abundance were too high; consequently, the value of η almost doubled, and Ω_{BBNS} could now be as large as 0.1. This value is not far from the preferred value of the mass density ascribed to clusters ($\Omega_{\text{cluster}} \approx 0.3$).

Given the overall uncertainty of the various mass density measurements, it is dangerous to predict just how much of dark matter is nonbaryonic. However, this fraction is likely to be at least two-thirds of all dark matter ($\Omega_{\text{BBNS}} \sim 0.1$ and $\Omega_{\text{cluster}} \approx 0.3$), and it could be much higher if Ω eventually turns out to be unity. These results are summarized in Table I.

Table I. Comparison of Mass Densities

	Observation	Theory
Ω_{luminous}	0.003	—
Ω_{galaxy}	0.02–0.1	—
Ω_{cluster}	0.1–0.3	—
Ω_{baryonic}	—	0.01–0.1 (BBNS)
Ω_{total}	0.1–1	1 (inflation)

The Big Bang: Structure Formation

One of the striking features of the Universe today—as opposed to the early Universe—is its inhomogeneity. Like islands and archipelagos in some vast ocean, matter floating in space has condensed into stars, planets, gas clouds, galaxies, and galactic clusters. Even the clusters seem to be organized into larger structures, creating great walls and sheets of galaxies that surround enormous bubbles or voids of lower density. Observations indicate that the Universe is “lumpy” on distance scales up to several tens of megaparsecs.

In earlier redshift surveys such as the CfA, there was strong inhomogeneity on the largest scales probed ($\sim$50 h^{-1} Mpc). (Although this distance is on the order of 300 million light-years, the survey probed but a tiny fraction of the observable Universe, which is estimated to be about 3000 h^{-1} Mpc across.) However, much deeper surveys such as Las Campanas ($\sim$600 h^{-1} Mpc) provide evidence that on very large distance scales, the size of structures saturate and no longer increase. The Universe is apparently homogeneous on scales greater than about 100 h^{-1} Mpc (see Figure 2).

A major triumph of the standard Big Bang model has been the progress made in understanding structure formation as a result of the gravitational Jeans instability. It turns out that the evolution of small perturbations of a uniform background density can be studied in much the same way as the stability properties of an ordinary plasma. The Jeans instability comes about because gravity always attracts: above a certain wavelength, called the Jeans length, density fluctuations are unstable and grow exponentially. In an expanding Universe, this exponential growth is modified and slows down to a weak power law.

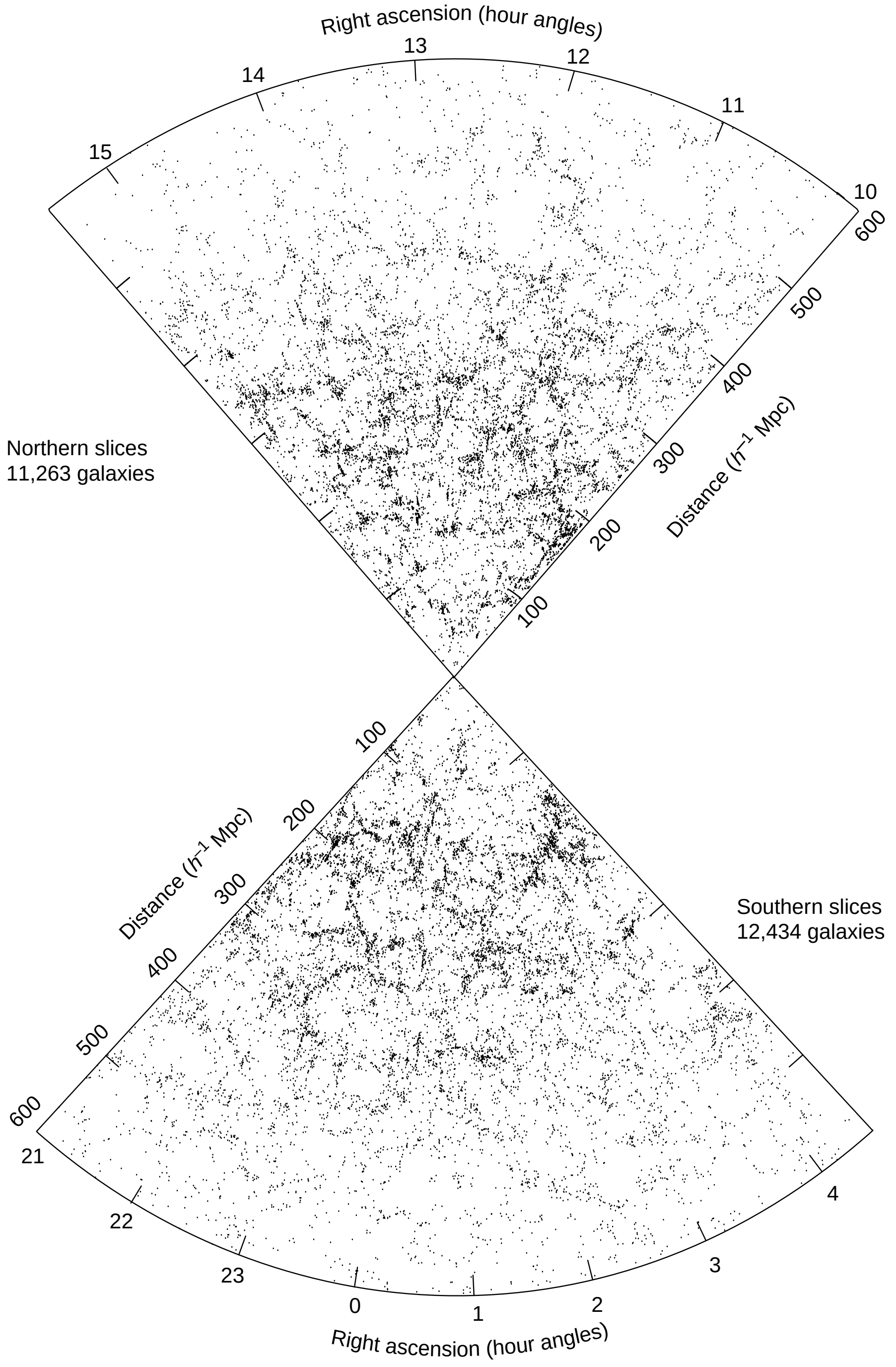
An important aspect of the Jeans instability is that it does not saturate at some finite value from nonlinear feedback, but rather increases in strength as the gravitational collapse proceeds. It stops increasing only when the structures formed have enough internal energy—for example, gas pressure in stars and kinetic energy in the solar system—to be able to resist collapsing further.

Another subtlety that has to be taken into account is the growth of density perturbations in the presence of thermal radiation. In the early history of the Universe, when matter is in the form of an ionized plasma and the energy density in radiation is much greater than that of matter, there is a strong coupling between radiation and matter. The radiation field itself does not collapse, and it prevents matter from collapsing because of the strong coupling. Only perturbations on scales longer than the Jeans wavelength, given by

$$\lambda_J = v_s \sqrt{\frac{\pi}{\rho G}} \quad ,$$

where v_s is the velocity of sound, continue to grow. Smaller-scale perturbations oscillate as damped acoustic waves. After recombination, the velocity of sound drops abruptly as the pressure support switches from radiation to neutral hydrogen. Consequently, density perturbations on much smaller scales can also begin to grow.

This picture of how initial density perturbations grow into structures is attractive, but it lacks a key ingredient: a source for the initial density perturbations that the Jeans instability would then amplify. The original Big Bang model does not have a physical mechanism to produce these perturbations. But the precise nature of the perturbation spectrum is very important, for it controls sensitively the types of



igure 2. Result of the Las Campanas Redshift Survey

his map of over 23,000 galaxies extends to approximately 600 h^{-1} Mpc, or about one-fifth of the observable Universe. Galaxies brighter than 19th magnitude were counted in arious “slices” of the sky. Each slice spanned about 90 to 120 degrees and was confined to a plane oriented at some angle (declination) with respect to the celestial equa- or. Data from three northern and three southern slices have been superimposed in his figure. The scale of the largest structures (the “voids” containing few galaxies) is oughly 50 h^{-1} Mpc, and there is no evidence for large-scale inhomogeneity on the cale of this survey. (The change in the galactic density beyond about 400 h^{-1} Mpc is n artifact. At great distances, the survey detects only the brighter galaxies.)

r more information about Las Campanas, see http://manaslu.astro.utoronto.ca:80/~lin/lcrs.html.

structures that form at later times. At present, the Universe has evolved numerous complex, scale-dependent structures, and the simplest primordial spectrum of density fluctuations that could potentially lead to what is observed today is the one put forth by Edward Harrison (in 1970) and Yakov Zeldovich (independently in 1972).

The Harrison-Zeldovich spectrum was based on very general theoretical considerations and has been used as the initial density perturbation spectrum in most analytical studies and simulations that attempt to track the Big Bang. This spectrum implies that the amplitude of primordial fluctuations in the gravitational potential does not depend on the spatial scale and, for a critical-density matter-dominated Universe, is also independent of time.

Significantly, the Harrison-Zeldovich spectrum also emerges from inflation theory. Quantum fluctuations of the inflaton field that drives the inflationary expansion provide a natural source of density perturbations that follow a Harrison-Zeldovich spectrum. Thus, aside from solving the flatness and horizon problems, inflation builds into the Big Bang model a natural mechanism for generating initial density perturbations.

Given the observational constraints and a primordial density perturbation spectrum, the question is whether the Jeans instability successfully produces the large-scale structures that are observed today. One point to address is the normalization—that is, the absolute amplitude—of the primordial density fluctuations. Simply choosing a spectrum does not determine its absolute scale. The normalization needs to be determined by experiment, but how do we measure the size of density fluctuations that were present 15 billion years ago? Remarkably, a window to the past exists that allows us to do just that: measuring anisotropies in the CMBR temperature provides a virtual time-machine to determine the lumpiness of the very early Universe.

The discovery of the microwave background was a stunning confirmation of the Big Bang, but detection of a temperature anisotropy in the field, or deviation from a perfectly uniform temperature, could have an even greater impact. Photons that make up the microwave field have been traveling unimpeded since the time of recombination. Because of intrinsic fluctuations in the temperature and gravitational potential of the Universe at the time the photons decoupled from matter, there is a very small anisotropy in the CMBR temperature observed today.

The anisotropy over large-distance scales was measured with very high precision by the COBE satellite, launched in 1989 (see Figure 3). (COBE’s angular resolution of 7 degrees corresponds to several hundred megaparsecs.) The COBE results, along with those of other experiments that probed the microwave background at higher angular resolution, set the normalization of the Harrison-Zeldovich spectrum (see Figure 4) and impose constraints on any proposed spectrum of initial density perturbations. One important consequence of the CMBR observations is that the observed large-scale structure of the Universe cannot have formed in the presence of ordinary baryonic matter alone.

In purely baryonic matter models, the growth of initial perturbations occurs only after recombination. As stated earlier, before that time, growth is suppressed by pressure that arises when radiation scatters from free electrons (Thomson scattering), resulting in the effective prevention of growth of perturbations on scales smaller than $\sim 180\ h^{-1}$ Mpc. To produce the observed large-scale structure requires that the perturbations leave an anisotropy in the microwave background temperature of roughly 1 part in 10,000 on the scales probed by COBE. But the measured fluctuations were much smaller, deviating from pure uniformity by only 1 part in 100,000. The microwave background, therefore, informs us that there was insufficient time for structure formation in a purely baryon-dominated Universe.

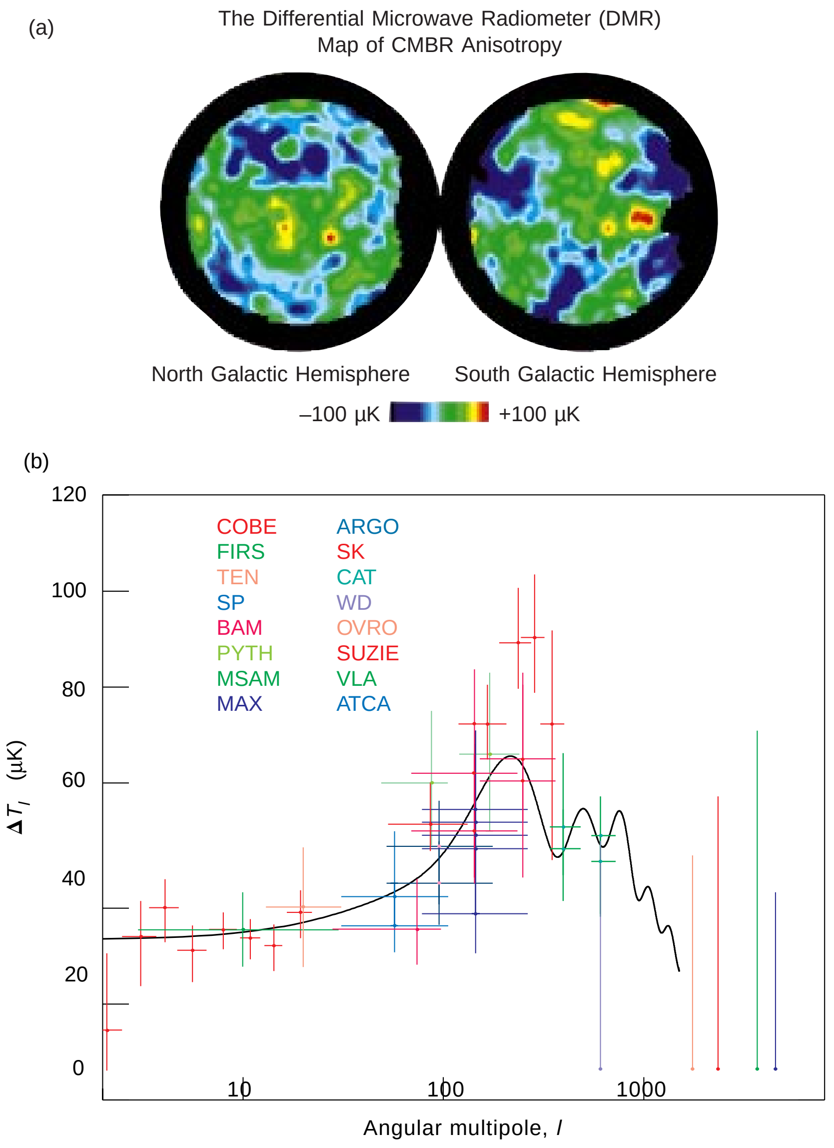
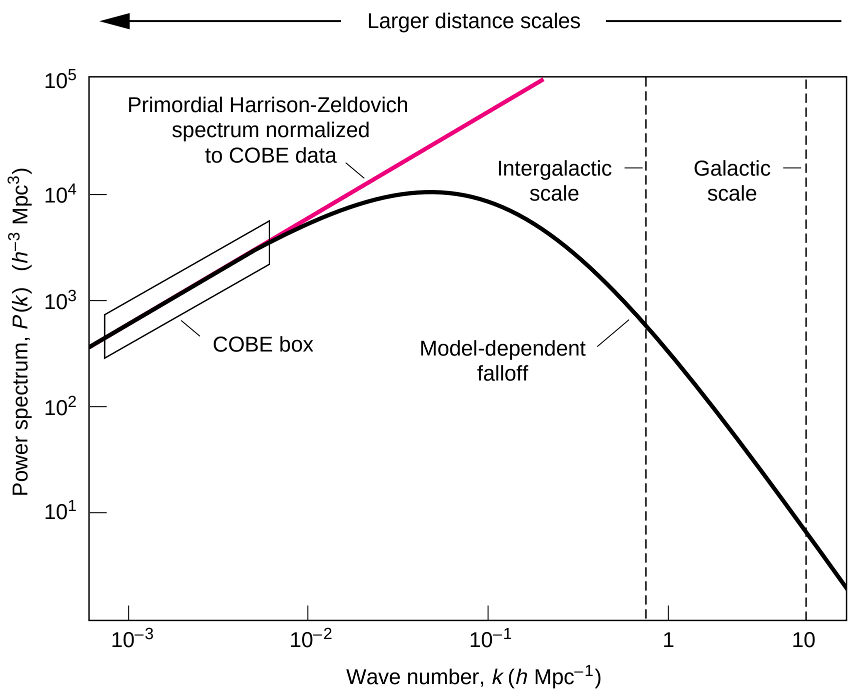


Figure 3. Temperature Fluctuations across the Microwave Sky

(a) The DMR experiment on the COBE satellite measured root-mean-squared (rms) temperature variations, $\Delta T/T$, in the CMBR to be on the order of 1 part in 10^5. (The average temperature of the background is 2.728 ± 0.004 kelvins.) The variations can be related to density fluctuations at the time of recombination that seeded the current large-scale structures seen in Figure 2. (The scale of the map shown in (a) is enormous. The largest structure of Figure 2 would easily fit within the smallest feature of the map.) (b) Data from 16 experiments that have measured the CMBR with varying degrees of angular resolution are shown in this figure of ΔT_l (the rms temperature fluctuations per logarithmic interval $\Delta l/l$) versus the angular multipole l (plotted on a log scale). The black curve is a theoretical prediction for the CMBR, based on a cold dark matter model (discussed later in the text.) At very large distance scales (small l), the anisotropy is determined by the primordial power spectrum and is predicted to be a flat line for the Harrison-Zeldovich spectrum. Values of $l < 80$ correspond to distance scales that were not causally connected at the time of recombination.

Figure (a) provided by the National Space Science Data Center (http://www.gsfc.nasa.gov/astro/cobe/cobe_home.html). The COBE data sets were developed by the the NASA Goddard Space Flight Center under the guidance of the COBE Science Working Group. (b) Compilation of data and the theoretical curve are courtesy of E. Gawiser and J. Silk, CMB Theory group, UC Berkeley.



igure 4. The Harrison-Zeldovich Spectrum and a Processed Spectrum
he primordial Harrison-Zeldovich power spectrum (shown in red) is plotted against ave number, *k*, on a log-log scale. (Notice that *k* varies inversely with distance.) Data om COBE (rectangular box, derived from the CMBR data at large angular multipoles) fixes the normalization, that is, the position of the spectrum in the vertical direction. The ower spectrum that one compares with observations is a processed spectrum like the ne shown in black. At low *k*, the spectrum is Harrison-Zeldovich, while the falloff at rge *k* is model dependent. For any given model, the processed power spectrum is the arrison-Zeldovich spectrum multiplied by a transfer function, *T(k)*, which incorporates he contribution of all relevant physical processes.

Nonbaryonic dark matter, however, oes not couple to photons and thus oes not suffer from collisional damping. Density perturbations can begin o grow well before recombination, as oon as matter-radiation equality is chieved. This allows the development f large density perturbations without iolating the density constraints implied y the small anisotropy in the microwave background. Some form of nonaryonic dark matter, therefore, appears o be necessary to explain the formation f structure in the Universe.

Despite the complete absence of irect experimental evidence for a nonaryonic, dark matter particle, theorists ave had no trouble in suggesting a lethora of possible candidates. Although their specific properties vary, all candidates are bound by the common constraints that they have mass, not be made of quarks, and have neither strong nor electromagnetic interactions.

Structure Formation and Dark Matter

Of all the proposed dark matter candidates, massive neutrinos have always been the most natural: neutrinos are known to exist, and they were produced in very large numbers during the Big Bang. (Roughly a billion neutrinos were created for every baryon.) Since the mean density of the Universe has to satisfy observational constraints, there exists a calculable range of allowed neutrino masses. Assuming a single, two-component neutrino species, the relevant formula is

$$m_\nu = 92\ \Omega_\nu\, h^2 \ ,$$

where the neutrino mass m_ν is measured in electron volts. If we assume that the Universe is at critical density, this yields a neutrino mass of roughly 30 eV (assuming $h \approx 0.6$). With $\Omega_\nu = 0.2$, the mass range falls to several electron volts.

However, structure formation must also be considered. We would like to know in what way the evolution of structure depends on the type of nonbaryonic dark matter. This question leads to a simple hot or cold classification of dark matter, a classification based on the random velocities of dark matter particles at the moment they fall out of thermal equilibrium with the photon heat bath in the expanding Universe. Relativistic particle velocities are characteristic of hot dark matter, while cold dark matter particles are either very heavy, and hence nonrelativistic early on, or are created with essentially no random velocity.

In the case of massive neutrinos, which decouple from the rest of the Universe at a temperature of 1 MeV, masses on the order of tens of electron volts or less make them highly relativistic. They are therefore an excellent (and currently the only) candidate for hot dark matter. In a massive neutrino, hot dark matter model with adiabatic initial perturbations (radiation strongly coupled to matter), typically very large scale structures form first. The large bodies then break up to form objects at smaller scales: this is "top-down" structure formation.

Such a growth pattern results because small-scale structure in hot dark matter models is washed away by neutrino free-streaming, a collisionless effect analogous to Landau damping in plasmas. As long as density fluctuations are stable against the Jeans instability, collisionless particles such as neutrinos can stream out of the higher-density regions into the lower-density regions, thereby smoothing inhomogeneities. Regions separated by distances larger than the neutrino free-streaming length survive this smoothing, so that on larger scales, differences in density are maintained and can grow.

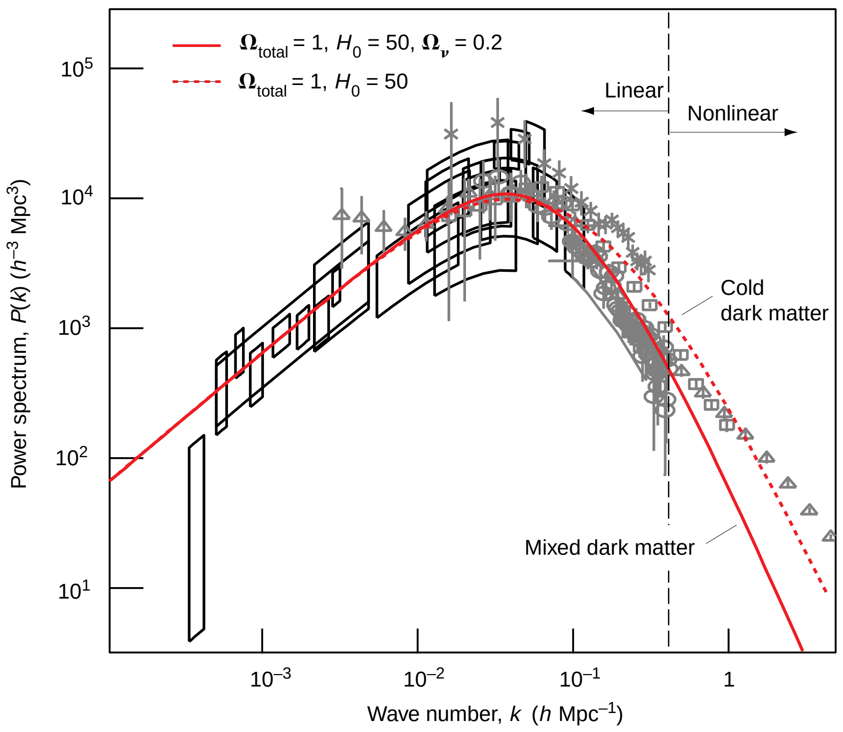


Figure 5. Large-Scale Structure Data and Dark Matter Power Spectra
Two theoretical, linear power spectra (best-fit mixed dark matter and standard cold dark matter) are shown superimposed on observational data. The black boxes are reconstructed in a model-dependent way from the measurements of the CMBR data shown in Figure 4 and are given here for mixed dark matter. (The box height reflects a 1-sigma confidence level. The boxes differ slightly near the peak of *P(k)* if a cold dark matter model is assumed.) Observations from matter surveys are shown in light grey. For $k > 0.3$ *h* Mpc^{-1}, the data is measuring nonlinear structure, beyond which point it cannot be directly compared with the linear theoretical power spectra. The overproduction of small-scale structure by cold dark matter models is best seen in the region around *k* of 10^{-1} *h* Mpc^{-1}, where mixed dark matter is very successful.

Figure courtesy of E. Gawiser and J. Silk, CMB Theory group, UC Berkeley.

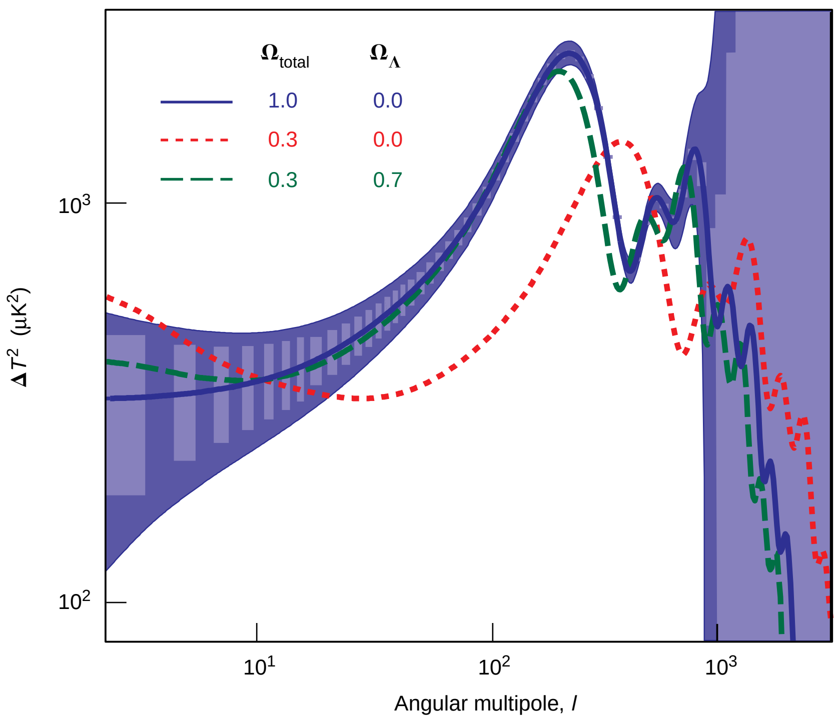
Unfortunately, hot dark matter models have been shown to be incompatible with observations. Assuming the Harrison-Zeldovich spectrum predicted by inflation and normalized to COBE, numerical simulations have shown that, owing to suppression of small-scale fluctuations by neutrino free-streaming, cluster and galaxy formation occurs far too close to the present time. Another difficulty is that the large coherence length of neutrinos makes it difficult for them to explain the localization of dark matter in individual galaxies. Thus, any model containing only adiabatic fluctuations and massive neutrinos has been ruled out.

Cold dark matter is composed of particles that are massive enough to become nonrelativistic shortly after their birth (or that are born with no random velocity). Although there is no experimental evidence for such particles, the supersymmetric candidates and the axion (Peccei-Quinn symmetry)—particles that are associated with central ideas in high-energy physics—have been suggested as possible cold dark matter candidates. Because it is nonrelativistic, cold dark matter readily clumps together, and structure formation typically proceeds in a "bottom-up" manner. Galaxies form first, which then get grouped into clusters and superclusters, possibly in a hierarchical way.

However, predictions from cold dark matter models also disagree with observations. With the Harrison-Zeldovich assumption for the primordial density perturbations plus the COBE anisotropy constraints, we find that cold dark matter tends to produce too much structure at small scales.

The complementary nature of structure formation theories—massive neutrinos generate too little structure at small scales, while cold dark matter overproduces it—has naturally led to a model enjoying some popularity at present: mixed dark matter. Both kinds of dark matter are assumed to exist. Adding some massive neutrinos to a predominantly cold dark matter model tends to reduce the overproduction of small-scale structure characteristic of pure cold dark matter because of the neutrino free-streaming. It has been shown that in a critical-density Universe, if we choose a cold to hot dark matter ratio of 5 to 1 in mean mass density, then a mixed dark matter model might be viable and escape the serious problems of both the hot and cold dark matter models (see Figure 5). (There are indications of problems in forming structure at early enough times, but it is difficult to tell how serious these problems really are.) For such a mixed dark matter model to work, the neutrino mass has to be in the range of several electron volts, and this mass range is compatible with results from the liquid scintillator neutrino detector (LSND) experiment.

But mixed dark matter models also face tight constraints. One problem is that even a small admixture of hot dark matter reduces structure formation on small scales to the extent that a fairly large amount of cold dark matter is needed to compensate. It turns out that even for $m_\nu \sim 2$ eV, corresponding to



igure 6. MAP and the Future

emperature fluctuation squared is plotted to show how the MAP mission can distinuish between different theoretical models, here cold dark matter (dark blue curve), an pen model with $\Omega = 0.3$ (dashed red curve), and a model with a nonzero cosmological onstant Ω_Λ (dashed green curve). The rectangles that follow the blue curve are procted error bars, while the solid-blue envelope represents the so-called "cosmic varince," a finite sampling effect due to the fact that the observation is restricted to only fraction of the Universe. Thus MAP will provide useful information for multipoles up o $l \sim 1000$ and is dominated by cosmic variance up to $l \sim 600$. If this figure correponded to real data, only the cold dark matter model would be viable.

gure courtesy of Wayne Hu; available at http://www.sns.ias.edu/~whu/physics/physics.html.

$\Omega_\nu \sim 0.1$, a large value of Ω_{total} is avored. The contribution from the old dark matter (Ω_{CDM}) must be at east greater than 0.3, which is already t the upper range of observational mits. If Ω_{total} turns out to be low, mixed dark matter models would be rongly disfavored.

Another alternative model for tructure formation is based on decayng heavy neutrinos. The overproducon of small-scale structure in cold ark matter models can be mitigated with either of two strategies, either y reducing the density of cold dark matter particles as in mixed dark matter models or by increasing the nergy in radiation. The latter can e accomplished with unstable cold dark matter particles that decay.

Decaying-neutrino models are characterized by two parameters: the mass of the neutrino, m_ν, and its lifetime, t_d. In the early Universe, when the thermal energy of the neutrinos is much greater than m_ν, they behave as essentially massless particles. At later times, when the temperature falls, the energy density in the Universe can be dominated by these species, after which time they decay and release their rest energy into relativistic particles. After this point, the evolution is similar to that of a cold dark matter model except for the additional energy density due to relativistic particles. This is what allows for the tailoring of the perturbation power spectrum in order for the model to be observationally viable. The acceptable mass range is $m_\nu \geq 10$ keV, while decaying neutrinos with $m_\nu \leq 50$ eV are ruled out (for all values of t_d).

So far, our discussion of both hot and cold dark matter models has assumed the Harrison-Zeldovich form for the primordial spectrum with adiabatic perturbations. A natural question is whether the situation is any different when other types of perturbations are considered. An alternative to inflation in this respect comes from theories in which the initial density perturbations are seeded through the formation of topological defects in early Universe phase transitions.

In these theories, topological defects such as cosmic strings give rise to perturbations either through the formation of wakes of overdensity as they move through the Universe or through the accretion of matter onto string loops. Calculations with these models are much more difficult than with inflationary models. Until recently, cosmic string/hot dark matter models were viable candidates for large-scale structure formation; unfortunately, topological defect models have now been ruled out. For a given anisotropy in the CMBR temperature, the corresponding amplitude in density fluctuations is several times too low to explain structure formation. In addition, the predicted small-angular-scale CMBR anisotropies are in conflict with present ground-based and balloon-borne observations.

Outlook

Although we have presented an up-to-date summary of dark matter and its relationship to structure formation, it should be noted that the outcome of models of structure formation and CMBR anisotropy depends critically on the values of cosmological parameters such as Ω_{total} and the Hubble constant H_0. It is important to pin down their values to within a few percent. At present, the constraints on these parameters from CMBR anisotropy measurements are quite weak.

But the future is full of promise. The constraints on Ω_{total} and H_0 are expected to improve dramatically with the next generation of satellite observations. The Microwave Anisotropy Probe (MAP) is scheduled to fly in 2001, followed several years later by the PLANCK satellite. In addition, deep, high-statistics redshift surveys of galaxies are expected to yield data within the next several years. The 2 Degree Field (2dF) and the Sloan Digital Sky Survey (SDSS) will go out about the same distance as Las Campanas (roughly 600 h^{-1} Mpc), but they will survey many more galaxies: a quarter of a million for 2dF and a million for SDSS, compared with roughly 25,000 for Las Campanas.

Analysis of the new CMBR data combined with the large-scale structure information from the redshift surveys will provide a very powerful discriminator between competing models of structure formation (see Figure 6). A value of $\Omega_{total} \ll 1$ would be unfavorable for models incorporating light neutrinos and would be difficult to reconcile with inflation. (If the matter density is less than critical, that is, $\Omega_{matter} < 1$, it is still possible to save $\Omega_{total} = 1$ by introducing a large cosmological constant, an alternative espoused by some theorists.) On the other hand, if standard inflation is vindicated and $\Omega_{matter} = \Omega_{total} = 1$, a light neutrino might be just what theory needs to satisfy the constraints imposed by structure formation. Even if this were the case, however, the neutrino would still not play a major role in dictating the dynamics of the Universe.

It is unlikely that the last word has been spoken on the cosmological consequences of a massive neutrino. Today, such a particle is not the favored dark matter candidate given our theories of initial conditions and structure formation. Just how good or bad these theories are will not be known until the next generation of CMBR observations yield results. Can the massive neutrino regain the dark matter center stage? The turn of the millennium may bring us the answer to that question. ■

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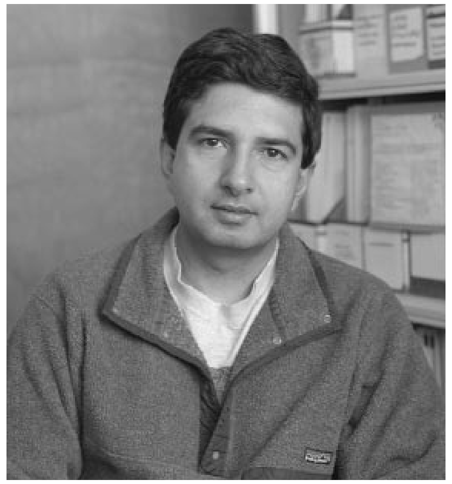

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