CONVERSATIONS *with* ROTA

The cultural affinities and intellectual differences between Stan and Rota were such that they could talk together for hours, though they were worlds apart mathematically and philosophically and never collaborated. I was fascinated by these spontaneous and informal discussions and recorded a number of those that took place in my presence, to transcribe and assemble loosely in a little collection.

Los Alamos Science *selected the following fragments to illustrate the way Stan and Rota interacted and, more generally, the way mathematicians converse about what occupies their minds.*

Francoise Ulam

The Mathematical Dictionary

ULAM: I think it is a very good idea to try to write a mathematical dictionary. First we must settle how many words to think about. Would you say two hundred or less'?

ROTA: Two hundred! No. Ten. maybe!

ULAM: No, no. At least a hundred. They will have to be very diverse. It will be a long project. Logical words like *but* and even have a different character from words which have a topological or kinematical meaning like *mix, find, search.* Then there is another class of words like *involve, intuitive, imaginary.* There are many categories. I think we should have a few in each. We must say what it is for in the introduction,

ROTA May I make an introductory sentence?

ULAM: Yes.

ROTA: Let me make an attempt to give a precise mathematical definition .

ULAM: Not too precise.

ROTA: . a precise mathematical definition of commonplace terms. We will take words like *but, furthermore, nevertheless,* or *crest, degenerate, skid* and describe them in terms of accepted mathematical terminology.

ULAM: And physical. Physics is almost completely mathematized now.

ROTA 1 have *already,* and *perhaps,* and *pending.* They are close.

ULAM: A/ready is difficult mathematically. What about starting with *but?*

ROTA: Logicians claim *but* is the same as *and.*

ULAM: No! Its meaning is entirely different. How would you describe *but* logically? Something that leads us to a conclusion but does not? A disappointment in probability'? A whole essay could be written about it. Someday there will be a tremendous theory devoted to its ramifications, It could be a germ like the word *continuous.* The study of topology is nothing else but the study of the word *continuous.*

ROTA: When I was at Princeton, Alonzo Church gave a two-hour lecture on the meaning of *but* and *and.* It is now written up in his great *Introduction to Mathematical Logic.*

ULAM: So you see! And what does he say? I never read it. I knew he was a logician but did not know he did things like that. Now let's discuss things intelligently, professor.

ROTA: O.K. Let us begin with the word *but,* Stan?

ULAM: I would say that the word *but* suggests to me the following (we'll be more precise later): an element of an algebra whose elements are uttered sentences. I can imagine it as a point in a universe of points interpreted as sentences—physical facts. I see that it won't be easy to avoid circular definitions; we must not use the word *but* in developing a theory of *but,* right? The word *but* means that an element does not belong to a given set of points that was defined *before.* But—I am just saying this on purpose *now—but* expresses that an element belongs to a set which is similar or slightly larger than the already given set. Of course, 1 did not really need to use the word *but* in my explanation. However—Oh! I just used the word however: you see how hard it is to avoid these words? By the way, this poses another interesting philosophical problem, the fact that we cannot explain a mathematical . . .

ROTA: Let's not digress.

ULAM: I just want to see what is in my mind. I do not have a perfect definition right away. Do you agree that *but* is an element which does not belong to a set that was defined before?

ROTA: Yes. Now let me try my definition. We have two sets, *A* and *B,* and a new relation between *A* and *B* which we will call contrast. The word *but is* the contrast between set *A* and set *B.*

ULAM: The set *B,* in every example I know of, is usually given by the speaker. Set *A* is mentioned; set *B* is intended. It is not there at the beginning or maybe it is only in the mind of the second speaker. Would it be a good idea to consider it as a part of *conversation?* One person proposes something, and the other replies, "No, but . . ."?

ROTA: No. I don't think it is a good idea to formalize *conversation.* It would get us too far from our purpose. If we are going to give definitions, they have to be objective.

ULAM: Whatever is done, you always stick to tempus acti, and you do not want to do something unorthodox. Why reduce it to the existing formalism? It is good to try, but it is not necessary.

ROTA: If possible, do it. Only when you have to, give up.

ULAM: O.K. I agree. Continue.

ROTA: So you have two sets and the contrast between two sets, and the word *but is* an expression of this contrast. And now I would say the word *but is* used when this contrast has to be brought out.

ULAM: Very good. But is that really always true?

ROTA: That is my story.

ULAM: We should have examples, like in dictionaries. They always give quotes from Shakespeare.

ROTA: Let me give an example: "We were going to go out touring today, but it is raining and we didn't go." Analysis: There are two situations, or sets, if you wish. One, going touring; two, assumption that the weather is fair. Then the weather turns out not to be fair, so there is a contrast between fair and unfair and the word *but* arises.

ULAM: I agree. Let me give another example: "The snail is not an insect, but it is still an animal."

ROTA: Here again you have a set. You presume the snail to belong to this set. The contrast arises because you see a further subdistinction inside this set.

ULAM: Simply, these sets are not equal; one set contains the other.

ROTA: Be that as it may, either not equal or partitioned, one contains the other.

ULAM: This is part of it but perhaps not all. We will have to have detailed discussions like that about every word, as they do at l'Academie francaise!

ROTA: Let me say that any definition is necessarily incomplete. It is a property of definitions to be incomplete.

ULAM: Incomplete perhaps, but still it should try to be as broad as possible.

ROTA: Then it will never end. There is a point where one says fine, adequate, even though it is not the whole story.

ULAM: Yes, I agree.

ROTA: Let's take another example. One says of a person: "He is good, but he is also careless." How would you analyze that?

ULAM: A point belongs to two sets. If you say *good,* the presumption is that everything is good about him, so you add another set.

ROTA: Suppose I replaced the word ***but*** by the word *and.* In your opinion how would the meaning of the sentence change?

ULAM: It would be an entirely different meaning.

ROTA: Why?

ULAM: Because the set of *carelessness is* not a set which normally is associated with the set *being good.*

ROTA: It is not a complete explanation. *But* always requires a contrast.

ULAM: True. What about a distinction?

ROTA: Distinction is too weak. *But* requires contrast and unexpectedness.

ULAM: Unexpectedness. Exactly. This is the essential thing to my mind. Namely the first set suggests something, and the second implies the suggestion does not hold. A set of properties implies a lot of others, but an exception is made. *But* suggests exception.

ROTA: No exception is involved. For example, "I was going to go out but the phone rang." That is no exception.

ULAM: That is yet a different meaning of *but.* It says that the normal pattern is being abruptly changed.

ROTA: Let me say this. A lot of examples have the following structure. You have two sets, *A* and *B,* and you have an element c. You expect *c* to belong to *A,* but then it turns out to belong to *B.* That is the typical use of *but.*

ULAM: Right. So it is not a relation of the contrast but of difference.

ROTA: We have abstracted a set-theoretic relationship for the word *but;* namely we have two sets and an element. The element may belong to *A* but instead it belongs to *B.*

ULAM: Very good! However the two sets are somehow close. They are not too different or one contains the other because you could not say, "The pencil is long but it is black."

ROTA: Right.

ULAM: Why?

ROTA: There must be a similarity between the sets.

ULAM: Ah! Now we have caught one essential point.

ROTA: So there are these properties of sets which somehow are similar, and then there is the confusion of one element belonging to one instead of the other.

ULAM: In general the two sets are in some relation of similarity, close in the sense of a Hausdorff distance or whatnot, and not completely separate.

ROTA: Two or more sets are in turn subsets of a set of sets which is predetermined. They are members of the same family of sets.

ULAM: What does it mean, the same family?

ROTA: The family is the similarity class.

ULAM: Right, that is what one could say. Very good. We are getting somewhere.

ROTA: You see, I am becoming Ulamian. Set of sets!

ULAM: My example about the pencil was crucial. It did not make sense. Let us take something else. For example, *however.* It is not quite the same as *but.*

ROTA: Later. Let us finish with *but.*

ULAM: We have to warn our readers and ourselves that there are words that mean almost the same, with subtle shades of difference. In French there is *mais* and *cependant.* We ought to analyze that.

ROTA: What about *nevertheless* and *yet?*

ULAM: *Nevertheless* has a greater degree of something. We should analyze all these. They are all coming together.

ROTA: *In spite cf...*

ULAM: I would very much like to define the word *key* or *lock,* because there is a sort of labyrinth, a maze. You have to enter a lock a certain way, which at random is difficult, and perform a sequence of operations.

ROTA: Key is absolutely one of the best.

ULAM: *Key, lock, labyrinth-* there is a whole topological, combinatorial meaning there. Logical too. *Key* also has an abstract meaning, a *key* to something. We are just beginning. This is a project for several months.

ROTA: We could get a grant!

ULAM: From some cultural whatnot— there are such. Philosophers do not give grants, but we are rich old men, as Erdos says. If we could meet an hour a day, we could get somewhere in one month.

FRANCOISE: Next summer in Santa Fe.

Gainesville
January 1974

On Teaching And Learning

ULAM: Most of what I've learned was subconscious, by osmosis. When I read, 1 am not aware that I am learning. I learn mainly from conversations, from people rather than from lectures, and I did not realize until a few years ago that I have a good memory.

I could start teaching mathematics with courses for college freshmen and go to junior or senior courses without any preparation, because in mathematics one thing leads to another.

Let us discuss whether teaching mathematics really makes any sense, Either the student is so good that he does not need a teacher, or else, if he needs help, he is not cut out to become a mathematician. At Harvard I had some good students with whom I could talk and feel that teaching was not merely an empty gesture.

I think I influence people more than I teach them. 1 influence their taste or their choices.

ROTA: I learned from you to argue in a short way, to give only ideas followed by simple examples. That is the Ulamian influence.

ULAM: I don't mind teaching, but I don't like to do it regularly. When I have to do something at a fixed hour, even if it is a pleasant dinner or cocktail, I fret. I hate not feeling completely free. But of course, being completely free immediately brings on a feeling of restlessness, of not knowing what to do!

Each of us has taught several thousand hours. If you think that a normal working year in America has about 2000 hours—an 8-hour day for about 250 days—that is quite a bit of your waking time, isn't it? But maybe it is not entirely waking time. One does it in a trance, partly asleep sometimes!

I am told I teach calculus well. It is possible, for I believe one should concentrate on the essence. One should not teach everything at a uniform level either. One should stress some important as well as some unimportant details on purpose—in a sense to follow the way I think the memory works.

When you remember a proof you remember a sequence of pleasant, unpleasant points, zeros and ones. Here comes a difficulty you try to remember, and you make an effort. Then you come to something that goes automatically and it is zero, zero, zero. Then again a special trick that has to be remembered. It is like going through a labyrinth.

ROTA: I am amazed at your labyrinth!

ULAM: I learn best from conversations. I love them, and that is how I learned physics in Los Alamos.

Some people are different in this respect. They prefer to learn slowly and methodically. How about you?

ROTA: I learn best when I am forced to do it.

ULAM: Speaking of being forced to learn, in Poland it happened several times that I announced that I would speak on a certain subject at a meeting of the mathematical society before I had a proof. I felt absolutely confident that once I had agreed to speak, I would get a proof. It could have been an embarrassment otherwise.

On the other hand, when I look at a paper of mine which has been published, I discard it after one glance, from fear that I will discover that it is wrong. There is also this tiny gnawing doubt about whether the result is new or not. Yet even in a field about which I know nothing, I can always tell whether a theorem or a point of view is good or not. This feeling comes somehow from the way the quantifiers are arranged, from the tone or music of the piece.

Do you remember what Galois wrote in his final letter before his fatal duel? He wrote that in their publications mathematicians really conceal the way they obtain their results because the process of discovery is different from what appears in print. It is important to repeat this again and again.

Gainesville
February 1974

John von Neumann

ULAM: Hot! What is the temperature?

ROTA: 80 or so.

ULAM: Pas possible! It must be the hottest day in thirty years. Which reminds me, once flying back to Los Alamos on Carco on a hot summer day, I opened the little window and my handkerchief flew out of the plane. Behind us there was a second plane carrying Johnny and others. What do you think the probability is that my handkerchief could have gotten enmeshed in the propeller of the other plane?

ROTA: Von Neumann was older than you.

ULAM: Six, seven years.

ROTA: An older man!

ULAM: Yes. You know how it is. In the beginning the percentage was twenty or so; later it went down to ten.

ROTA: So you considered him a senior, and yet you made fun of him?

ULAM: Oh always! Of Banach too. I was always impudent.

ROTA: He did not treat you as someone younger?

ULAM: No. I don't think he knew anybody more intimately and vice versa, despite our difference in age. For a man of his stature he was curiously insecure, but his understanding, intelligence, mathematical breadth, and appreciation of what mathematics is for, historically and in the future, was unsurpassed. His immense work stands at the crossroads of the development of exact sciences. The rationalization of the idea of infinity-the life blood of its history—with its mysterious power to encode succintly and generally the properties of numbers and the patterns of geometry, received some of its definite formulations from his work. His ideas also advanced immeasurably the attempts to formalize the new, strange world of physics in the philosophically strange work of quantum theory. Fundamental ideas of how to start and proceed with the formal modes of operations and the scope of computing machines owe an immense debt to his work, though they still today give hints that are only dimly perceived about the workings of the nervous system and of the human brain itself.

Other mathematicians strike me as virtuosi who play their own special instruments. None are comparable to Johnny.

By the way, you were supposed to ask about the foods von Neumann liked.

ROTA: List the foods von Neumann liked and those he did not like!

ULAM: He was not a gourmet, but he liked to eat. He liked to go to restaurants, mainly, I think, to escape from the usual scene or routine. It was an excuse for not working, because he was a very hard worker. At home he worked at a desk, writing, a thing which irritated me a bit. When I stayed at his house and saw him suddenly leave to go upstairs and write, I, cruelly and foolishly I must say, would make fun of it. So for relaxation he liked to drive out for dinner. In Princeton we often went to a restaurant called Marot, on the highway to Trenton.

He never smoked, but he ate voluminously, which accounted for his increasing rotundity and portliness as the years went by. Sometimes when Klari, his second wife, could not finish what was on her plate, she would give it to Johnny or to me and say, "Both of you are human garbage cans!" Klari, by the way, was a very intelligent, very nervous woman who had a deep complex that people paid attention to **her** only because she was the wife of the great von Neumann, which was not true of course.

Johnny liked Mexican food, hot peppery stuff. I suspect it was because if he had a stomachache later, he **would** know what to blame it on! I always have such Machiavellian suspicions. It is probably just that he was used to Hungarian goulashes and hot paprika. He liked sweets too, but on the whole what he wanted was volume, like me, like you too. You like the volume of pasta.

He had this nervous trait, an almost automatic response. For example, whenever he saw the words *chicken mole* on a menu, he would automatically intone Moles *Hadriani*, and I would respond *Jacques de Molay—you* know, the Grand Master of the Knights Templar. It was a game of association, just like you always add *Pal* [Hungarian for Paul] when you hear the word *Erdos!*

He also had occasionally an infrequent but noticeable stutter. He would say a word and repeat it two or three times in quick succession. I wonder whether it could have been an incipient physical lesion, for he died of things affecting his brain. Actually, on second thought it couId not, because his cancer started somewhere else. Sometimes I suspect that his stutter was in order to gain time while thinking over a riposte or considering quickly some other angle for a statement, like a person lighting a pipe to gain time.

ROTA: How long did you know von Neumann?

ULAM: I first met him in Warsaw in 1935, but I had already started corresponding with him the year before, and that is when he invited me to visit him at the Institute in Princeton.

ROTA: What was he working on at Los Alamos during the war?

ULAM: On everything. He was one of the originators, one of the "influencers" of implosion. By the way, you are my most eminent "influencee"; it is a relationship different from teacher-student.

He worked on the whole project, scientifically and politically, especially with the hydrogen work.

ROTA: But actual *work?*

ULAM: Of course, mostly hydrodynamics.

ROTA: Did he know much physics?

ULAM: To some extent, but he did not have the physicist's feeling for experiment. His interest was more modem than Hilbert's. His interest was in the foundations of quantum mechanics, which were mathematical. And that could be taken as an example of mathematics not really useful for real physics.

But there was no bullshit in him. That is an expression he used about certain people. He would say, "It is very rare, but there is no bullshit in so-and-so."

Of course he worked, in answer to your question. In fact he was unable to play the role of senior scientist or advisor without being actively engaged, like with computing. Even towards the end of his life, when he was chairman of the ICBM Committee, a committee established by the President after Sputnik.

ROTA: 1 still don't have a picture of von Neumann's personality.

ULAM: He loved jokes, though I don't think he invented many, but he remembered and repeated them, and occasionally he made original and very witty remarks or saw comparisons which were comical. Most are unprintable.

A propos of the church knowing about the atom bomb, he said, "Priests will bless the active cores." And when he noticed all the churches of Los Alamos, he was much amused when I pointed to one church and called it "San Giovanni delle Bombe"! One of the first solid non-wood buildings in Los Alamos was built for the offices of the AEC. He called it "El Palacio de la Seguridad"!

Oh! One thing about Johnny, he tended to tell people what they wanted to hear. He also used to tell me his little tactical discoveries. Once he said, "In Los Alamos it is very difficult to introduce novelty, but once introduced, it is impossible to get rid of it!"

After the war he was for a Pax Americana, and one could probably have established it, but the historical perspective, the desire to do it were not present in the country. The general population was not thinking in those terms. Although, when World War 11 ended, Americans were like Roman citizens during the Roman Empire. By commuting through the American bases one could go anywhere in Europe without encountering the native populations. This was really a beginning of that sort of thing, but for good or for bad—who knows—it quickly dissipated.

What else would you like to know about von Neumann?

ROTA: Always well dressed, wasn't he?

ULAM: Not really well dressed, but simple, decent, well-cut, classic city dress.

ROTA: I still don't have a picture of the man.

ULAM: He became an important government figure and very influential in ballistic missile development.

ROTA: It is strange how you like every thing about him except his work in mathematics.

ULAM: Really? No, not quite so. But he was not a mathematician's mathematician. He did little in number theory, some in continuous geometry and operators and Hilbert space, and some in measure theory and group theory.

To my mind and to my taste, the most important work he did is what he did when he was getting older, which mathematicians don't appreciate, namely his speculations on automata, on the brain, and his contributions to computing and to problems in hydrodynamics.

He knew about quantum theory and some parts of theoretical physics, which few mathematicians did. He contributed to the grammar of physics, so to say. One must also mention the theory of games. What interested me less was his work in the almost-periodic functions of groups.

ROTA: Can you tell me something about how his mind worked?

ULAM: It is curious tome that in our many mathematical conversations on topics belonging to set theory and allied fields, he always seemed to think formally. Most mathematicians, when discussing problems in these fields, seem to have an intuitive framework based on geometrical or almost tactile pictures of abstract sets, transformations, and such. Johnny gave the impression of operating sequentially by formal deductions. His intuitions seemed very abstract; they involved a complementarily between the formal appearance of a collection of symbols, the games played with them, and the interpretation of their meanings. Something like the distinction between a mental picture of the physical chess board and a mental picture of a sequence of moves on it written down in algebraic notation!

The quickness of his thinking was quite remarkable. He saw immediately the possibilities of Monte Carlo. To my mind this was much more important than one hundred papers in partial differential equations! It is at least a general procedure—I would not quite call it a method—and he invented many tricks for it and specific ways to get random distributions. It was very pleasant to discuss it with him.

Too bad he did not live to see how computers have revolutionized everything and what influence they will have on science in general and even on pure mathematics. His role in their development was tremendous, and if I may say so I would say I too played a modest role in showing how to use computers!

ROTA: How would you characterize his influence?

ULAM: There used to be a time when there were mathematicians who gave specific ideas and choice of topics and directions either explicitly or by implication to the work of other mathematicians. Not to go back centuries but less than a hundred years, let us say Poincare, Hilbert, in more recent times Herman Weyl. Hilbert had laid what was hoped would be a foundation for the final axiomatization of mathematics and beyond, of all science. Little did he know that in the thirties the unavoidable limitations of this approach would be revealed.

Von Neumann was one of these giants too in the breadth of his knowledge, especially when one remembers that now the diversity and complexity of contemporary problems enormously surpass the situation confronting Poincare and Hilbert. Yet, he admitted to me that he felt he did not know even a third of mathematics, that he did not think it was possible nowadays for any one brain to have more than a passing knowledge of more than one-third of pure mathematics.

So, at his suggestion and for his amusement I concocted an oral doctoral examination in various fields in such a way that he would not be able to pass it. And indeed, when I thought about what problems to give him in each domain, I found one in differential geometry, one in number theory, one in algebra and a couple of others. And he agreed that he could not have answered any of the questions and the exam would have been a complete failure. Which goes to show that doctoral exams are to some extent meaningless. Of course, if one prepares for some specific topics, that is something else.

ROTA: Who was von Neumann a student of?

ULAM: He considered himself a student of Ehrardt Schmidt. It was not easy for me to get to the bottom of this. One reason, I suspect, is that Schmidt did some work in combinatorics which always interested Johnny very much.

ROTA: It was the Hilbert space. Schmidt was the only person at the time who studied nonlinear operators.

ULAM: But Johnny did not.

ROTA: That is why he admired Schmidt!

ULAM: Also I remember that he told me that Schmidt did not like to write. That surprised Johnny. I also think he secretly admired it. He said that Schmidt had told him that he felt faint whenever he saw a blank sheet of paper. Johnny was not at all like that. On the contrary, whenever he had a mathematical thought, he immediately wanted to write it down and elaborate.

ROTA: Did he have any students?

ULAM: Not really, even though at the Institute he gave several courses every year. Murray and Halperin may be considered his students.

ROTA: What about Godel and von Neumann?

ULAM: One summer before the war when I was returning to the States, Johnny was waiting for me at the pier. His first words were that Godel had shown that the continuum hypothesis was undecidable. This was how I heard for the first time about the existence of undecidable propositions in any formal system. So I said to him, "oh! That is because he defines what is meant by a set. " Johnny opened his eyes wide and expressed surprise that I had seen right away what was indeed the essential point. He thought I had some supernatural intuition.

I asked him whether Godel was not a little afraid that his result was nothing but a sort of super paradox of the existing set theory, merely a diagonal method. In a sense it *is* a diagonalization. He agreed that this was probably right and that Godel did not quite realize the importance of his discovery because of the fear that it would turn out to be merely another version of the whole series of set-theoretical paradoxes. Of course it was much more than that because he had made it all formal. The other paradoxes were special and dependent on metamathematical considerations that were not truly part of mathematics, whereas his results were. Curious how nervous people can be about their own work when it is *the* work!

ROTA: You have a higher opinion of Godel than I have.

ULAM: Yes, I know. It was so unexpected at the time, and poor Hilbert was . . .

ROTA: Not to speak of poor von Neumann.

ULAM: Johnny told me that Godel's results made him very downcast, not quite despairing but disappointed. You must remember that his work on the axiomatization of set theory, which was way back in the twenties, constitutes to this day one of the best foundations for set-theoretical mathematics. Basically he believed in Hilbert's goal of a final and conclusive axiomatization of mathematics, and yet, in a 1925 paper, in a mysterious flash of intuition, he pointed out the limits of any axiomatic formulation of set theory. That was perhaps a vague forecast of Godel result. But it was left to Godel to follow it through, and it has changed the direction of all science.

Gainesville
January 1974

On Ethnic Minds

ROTA: What is the difference between the Slavic mind and the German mind?

ULAM: The German mind is systematic; the Slavic is not. Slavs tend to be soulful, expansive, pensive, but they are not as nebulous or as much carried away by the sound of words as Germans are. In the German language syllables and words concatenate, and they concatenate thoughts which sometimes don't go very well together.

ROTA: Whereas the Slavic?

ULAM: Slavs tend, I think, to be self-analytic, more psychological than philosophical, full of regrets, feelings of guilt, but more fundamentally optimistic than the German, and with humor, which if it is not showing, is at least not far away. German humor is based on ridicule, I don't know why. Latins are something else.

ROTA: Describe the Latin mind.

ULAM: Order. Clarity is always there. Words are separated, they don't stick together. It is like well-cooked rice compared to the sticky overcooked stuff that comes out of a German mess.

What would you say about the Jews? Would you say there is a Jewish mind?

ROTA: I don't think so. Italian Jews are Italian, German Jews are German, and so on.

ULAM: Don't you think that the Jewish mind is a little truculent, that there is a bent for contradiction? I feel I have myself this Jewish characteristic of always wanting to change what exists. It is a sort of rebelliousness, an inability to kowtow to authority. Think of the great revolutionaries—Jesus, Marx, Freud, Einstein, Cantor. Cantor by the way was only half Jewish. Most Jews are only part Jewish, you know, but the Jewishness comes through all the same.

This rebellious spirit of the Jews does not show in music, where the Jews are much less creators than performers, interpreters.

Gainesville
January 1974

Gamow And Teller

ULAM: It is Gamow who brought Teller to George Washington University originally.

ROTA: From zero'?

ULAM: From Europe, in Hitler time, when he had no job.

ROTA: How did Gamow and Teller get along?

ULAM: Gamow ruined Teller a bit. Gamow had this fantastic talent—an intuition, a lightness of touch for what is important, without doing too much work, without much mathematics, without any laborious Grundlichkeit. Teller wanted to be like that. He had other talents. complementary perhaps. Comedians always want to be tragedians and vice versa. Under Gamow's influence Teller wanted to have "ideas" at any cost.

Gainesville
January 1974

Paul Erdös

ULAM: Plutarch compared lives, and it may have a certain sense, a certain value, to compare pairs of mathematicians. Take Erdos. Erdos and I have something in common, a tremendous facility for finding difficult combinatorial problems out of thin air.

I'd like to take Erdos, Rota, and Everett and, like in the theory of colors, see whether by mixing them one could produce all other colors!

ROTA: Your style is completely different from Erdos's. He is interested in proofs; you are not interested in proofs. You are interested in problems interesting to state and don't care very much how they are solved. Erdos cares about techniques that he uses all the time.

ULAM: Really? He likes to think from the beginning; he does not quote somebody's theorem to prove something else,

ROTA: Your typical problem can almost always be restated as follows: Develop a theory of . . along the following lines. That is what your problems are about, whereas Erdos are never this way.

ULAM: Maybe he exaggerates by trying to put everything on paper immediately.

ROTA: There is a primitivism to Erdos.

ULAM: Yes. I have that feeling too, very much. Once you said something which if true is very flattering, namely that things I mention are germs of whole theories, whereas his on the whole are more special.

Erdos is not really narrow, but it is hard to get things out of him. I think he knows a great deal, though I don't think he has read much belles-lettres.

ROTA: He has no outside interests.

ULAM: I think he reads quickly and efficiently and gets the gist of things. 1 don't know how much he knows, say, of French literature, the classics, history. He does know some history because he is interested in politics. He reads about current things. progress in medicine, a little about physics. He forms impressions.

He is really very nice, never diminishes people, does not make fun of anybody, and is very much interested in young people in the sense that he is always searching for young geniuses. Wouldn't you say that in a sense he is more human than von Neumann or Fermi? Fermi was enormously aware of but not warmly interested in others.

ROTA: I really don't understand Erdos as a person. 1 understand him mathematically.

ULAM: He wants to be famous. He is very well known. Every mathematician knows him. He has written over 800 papers. You know the "Erdos number"- who wrote papers with him. People have a weakness for him. He has some sense of humor. Politically he is not naive at all. He is very well wishing, and really I have never heard him speak badly of anyone. Very few people are like that, You speak badly of people. 1 speak badly of people.

The death of his mother was a terrible blow; he still has not recovered. She was ninety-one, and he says she still could have lived another three or four years.

Erdos is interested in human destiny, in sickness, in death. He has no home. Now he refuses to go to Hungary because of their attitude towards two Israelis. Last summer, at the time of a meeting in his honor. Hungary did not let two Israeli mathematicians in. This infuriated him, and he said he would not return for several years. He is a true man of principles and in a way very courageous.

Gainesville
March 1974

Erdos

Paul Erdos is the most prolific mathematician of modern times and is second only to Euler in the volume of work produced. He was a long-time friend and collaborator of Ulam. In Ulam's files are 191 letters from Erdos, mostly in longhand. Erdos collaborates the world over and has done more for collaboration in mathematics than anyone else.

Teaching Physics To Rota

ROTA: What are your views on classical physics versus quantum mechanics?

ULAM: Quantum mechanics uses variables of higher type. Instead of idealized points, or groups of points or little spheres or atoms or bodies, the primitive notion is a probability measure. Quite a logical leap from the classical point of view.

Nevertheless you find in quantum mechanics the strange phenomenon that a theory dealing with variables of higher type has to be imaged on variables of lower type. It is the complementarily between electron and wave.

In our minds, because of habit or historical conditions, an electron is a localized small object, whereas a wave is something diffuse. But some phenomena show a dual nature; they share properties of one and the other. I don't think there is yet a satisfactory logical or mathematical discussion of this duality. In my opinion it does not do any good to write down axioms which sanctify the usual dicta. People accept what works. Quantum theory is very successful at describing atomic phenomena, and some of its general features seem to be valid even in the subatomic nuclear and elementary-particle phenomena. But the overall success is not too striking, except perhaps in quantum electrodynamics.

To me the situation in theoretical physics seems to be the following. There are about one hundred bright young physicists in the country, all mathematically very skillful and learned-too much so for my taste! To predict or explain some of their observations, they fudge a little, which is only natural. However the next experiments at CERN or Fermilab always seem to invalidate their calculations. You would think that among so many guys making so many different predictions, at least a few would get some correct answers, but no! Whatever the prevailing beliefs or attempts, the new experiments show something else. How can this be? Nature is not that malicious. Maybe today's physicists are technically very skilled but not really imaginative or innovative enough.

ROTA: What is to your mind problem number one in physics?

ULAM: Is there a true infinity of structures going down into smaller and smaller dimensions? Is it not a precise problem, or recognized as such.

In physics there has always been an atomistic or a field point of view. If there is a field, then points are mathematical points and they are all the same. But another possibility is a very strange structure of successive stages, each stage different. The topology or the scene on which they exist, that is, space and time themselves, need not be the uniform, smooth Euclidean topology. The miracle is that physics would not be possible if protons and electrons were not very much the same. If this similarity or identity of subsets of the universe did not exist, there would be no physics. The role of physics to some extent is to divide the existing groupings+ all them particles-into entities isomorphic or almost isomorphic to each other.

The great hope of physics lies in the fact that one can almost repeat the same situations. Having twenty or twenty-two bodies does not radically change a physical law. In mathematics too there are similar analogies. In physics such analogies are essential.

It may be that in reality for phenomena in the small and involving high energy, there may be an underlying true infinity that does not allow for similarities. It may be that at the present stage of evolution of the universe a sufficient number of identical situations has not yet been produced. If this is so, then physics will become fundamentally more complicated.

Who knows whether there are not fundamental complications in the nature of subparticles? Are the billions of protons that compose our bodies or this table really the same? This stability is far from guaranteed. There might be critical numbers, critical crises not only in technology but in fundamental physics itself.

Since Godel, even in mathematics it is not simple anymore. Have I told you that van Hove asked me to give a talk on infinities in physics at CERN?

ROTA: What did you say?

ULAM: I intend to write it up in my future *Physics for Mathematicians.*

In recent years you seem to have lost your feeling of horror towards physics!

ROTA: I did not understand. I like to understand.

ULAM: Do you understand mathematics? It is easier to get accustomed quickly to a fixed symbolism, like that of mathematics. But this is largely an illusion. Mathematics has a restricted range; it has not changed since Archimedes. There are axioms, proofs, lemmas, theorems. In physics it is not clear what one really does and at what point one becomes satisfied that the formulation is correct.

Santa Fe
July and August 1974

Miscellaneous Comments About Mathematics

Mathematics and Games

ULAM: A French philosopher whose name I forget said that nowhere has the human mind shown itself so inventive as in devising new games.

ROTA: Inventors of games are always anonymous, Why'? What is your philosophy of the anonymity of games?

ULAM: Probably other people quickly perfected the original invention, and it is difficult to find out who thought of it first.

Are games part of combinatorics or the other way around? I claim that much of mathematics can be "paisaised," a Greek word which means to play.

Here is an example of a problem inspired by a game. Suppose n is a given integer and we are to build, you and I, two permutations of n letters. We construct them in turn as follows: For the first permutation I take n_1, you take n_2, I take n_3, and so on. Finally we get a permutation. Then we play for the second permutation. If the two permutations generate the group of all permutations, I win; if not you win. Who has a winning strategy in this game'? I don't know,

If we do it at random, what is the chance [that there is a winner]'? This then becomes a combination of measure, probability, and combinatorics. I talk about this racket in my book of problems. It is amusing, isn't it'? It can be done in any branch of mathematics.

Paris
April 1972

Combinatorics

ULAM: Combinatorics is devoid of general methods, It is full of nice individual curiosities, it is Erdosian. I have nothing against it, it is amusing. But it throws no light on anything else.

ROTA: You are not being fair.

ULAM: Complex functions, the idea of entropy are broader, Ramsey's theorem, interesting as it is, is like progress in zoology when a new species of insects with one red eye and one green eye has been discovered !

ROTA: Ramsey's theorem tells more about the nature of sets than all the axioms of set theory!

ULAM: It is one of numerous properties of infinity. Why take two sets of pairs and divide them into two classes'? My master's thesis already contains that sort of thing,

Some problems, big or small, are solved with a bang; they open new vistas. Others are solved with a whimper, in a way which is very specific and leaves nothing to be said or asked, regardless of whether the problems are important or interesting.

Paris
April 1972

Ramsey's Theorem

One consequence of Ramsey's theorem is the following: Among a gathering of 6 people, there will be at least 3 all of whom know one another or else there will be 3 none of whom know one another. This is not true if only 5 are gathered together. In general, for each positive integer k there is a positive integer n = n(k) such that if n people are gathered together, then there will be k all of whom know one another or else none of whom know each other. To this date we know only that n(k) exists but not its value for arbitrary k. It is known, however, that n(2) = 2, n(3) = 6, and n(4) = 18.

Cantor

ULAM: Set theory revolutionized mathematics. It is largely the work of Cantor. What made set theory is the fact that Cantor proved that the continuum is not countable. It is hard to imagine that a field that arose from trigonometric series quickly transformed the shape and flavor of math.

Paris
April 1972

Godel

ULAM: A second landmark on the scale of centuries was Godel's undecidability theorem. Now there is a flood of results that show that our intuition of infinity is not complete. Cohen's results opened the flood gate.

Mathematics is not a finished object based on some axioms. It evolves genetically. This has not yet quite come to conscious realization.

Paris
May 1972

A Few Unsolved Problems

The Continuum Hypothesis
$$2^{\aleph_0} = \aleph_1$$
or
if E is an uncountable subset of the interval [0,1], then there is a one-to-one correspondence between the elements of E and all the numbers between O and 1.

Twin Primes Conjecture
There are infinitely many primes p such that p + 2 is also a prime.

Goldbach Conjecture
Every even integer equal to or greater than 6 can be expressed as the sum of two odd primes in at least one way . For example, 12= 5+7 and 16= 3 +13 = 11+5

ROTA: Can you list ten unsolved problems in mathematics which you consider important?

ULAM: First, the continuum hypothesis. If you take the existing axioms for set theory, then it is independent.

ROTA: One!

ULAM: But the existing axioms are probably not enough to give expression to our intuitions about sets. In that sense the continuum hypothesis is not a closed story.

Two. In number theory, any problem is as good as any other. I don't know which to choose, the infiniteness of twin primes or the Goldbach conjecture. The fact that they are very difficult and so simple makes them in my opinion very important. I have to list the Riemann hypothesis because it has so many consequences, although it is not one of my favorite problems, for a reason which I cannot express.

ROTA: Would you list the Riemann hypothesis as third?

ULAM: I don't like to order them. Snobbism plays a role in the ranking of mathematical problems. By chance some so-called great mathematician mentions something. For example, out of Hilbert's marvelous twenty-three problems, several would not be considered important if it were not for the fact that it was Hilbert who proposed them! Now what would you say besides these?

It is like asking someone to please mention ten best dishes or paintings! I don't know whether any single problem is really important, except in foundations of set theory. They are mainly important for what they suggest or allude to, Think of Fermat's conjecture. It is important because it is difficult but probably also because whoever will solve it will have found some new trick or method. The important thing is that the break is simple and difficult. I came to this conclusion sort of gradually. I am being honest, which most people are not.

A great problem is: Why are some problems sometimes difficult to solve? That is metamathematical, but it may some day be mathematized. The notion of complexity is beginning to be made precise, and what I just said will become a super problem.

ROTA: Why should Goldbach's conjecture be more interesting than a Chinese puzzle'?

ULAM: Because it is simple. Any child can understand it. Isn't it curious that a child can ask questions about numbers that no mathematician can answer?

Gainesville
January 1974

Riemann Hypothesis
Let $\xi(z) = 1 + 1/2^z + 1/3^z + \cdots$. ***If*** $\xi(z) = 0$, ***then*** $Re\ z = 1/2$.

Fermat's "Last Theorem"
If $n > 2$, *there do not exist positive integers* x,y, *and* z *such that* $x^n + y^n = z^n$.

Infinity

ULAM: Why is it that calculus, which deals with limits, is so effective? Or why are asymptotic theorems so much simpler than finite approximations? Infinity does not correspond to the popular image. It is a guiding light, a star that draws us to finite ways of thinking, God knows why.

Santa Fe
July 1974

The Value of Mathematics

ROTA: What is the value of mathematics?

ULAM: Value? In what sense? In what market?

It has value because it trains the brain. Just like in any other game, practice sharpens the organ. I don't know if today mathematicians' brains are any sharper than in the time of the Greeks. Yet I think mathematics plays a genetic role. It is one of the few ways to perfect the brain, to perhaps develop new connections in the brain. It has a peculiar sharpening value. Nothing could be more important. I don't know if' any other science plays the same role. Another value is the aesthetic one, which is for the practitioners.

ROTA: What is its ugliness'? Could you state an ugly theorem?

ULAM: Ugliness lies in the fact that one has to be punctilious, make sure of every step. In mathematics, one cannot paint with a wide brush, one has to fill in all the details.

The same is true in chess. There are chess games which have flaws. In fact most do. Otherwise there would not be a loser.

ROTA: Compare mathematics to the classics as an educational technique.

ULAM: I would say they are complementary. Latin grammar is good training in logic, not Boolean logic, but relational logic.

Santa Fe
July 1974

Foundations of Mathematics

ULAM: Mycielski disagrees with me when I say there will be systems of axioms for set theory other than the Zermelo-Fraenkel point of view. He claims that everything that we can think of can be expressed in those terms. This may be true but there might someday be entirely new points of view, even about sets or classes of sets. Sets may some day be considered as "imaginary." I think that will come to pass, though at present it is not admissible.

Everything that is conceivable somehow eventually comes into existence, in what form we cannot say. Ideas which begin in a prosaic way, like the study of complexity, are the ones that go very far.

ROTA: As a phenomenologist I agree.

Santa Fe
July 1974

The Future of Mathematics

ROTA: What about *l'avenir des mathematiques* today?

ULAM: Mathematics will change. Instead of precise theorems, of which there are now millions, we will have, fifty years from now, general theories and vague guidelines, and the individual proofs will be worked out by graduate students or by computers.

Mathematicians fool themselves when they think that the purpose of mathematics is to prove theorems, without regard to the broader impact of mathematical results. Isn't it strange?

In the next fifty years there will be, if not axioms, at least agreements among mathematicians about assumptions of new freedoms of constructions, of thoughts. Given an undecidable proposition, there will be a preference as to whether one should assume it to be true or false. Iterated this becomes: Some statements may be undecidable undecidable. This has great philosophical interest.

ROTA: I disagree. I don't think the current work in set theory is going anywhere, and I deny that it has philosophical import. It is a bunch of technicians doing Talmudic, irrelevant exercises.

ULAM: You may not like it, but it is as relevant as Heidegger!

Set theoreticians are workers, not generals, discovering interesting facts on the behavior of axioms and how incomplete they are. To me this is of great interest. One used to assume certain ideas of infinity and suddenly, 10 and behold, they are incomplete.

Santa Fe
August 1976