Let me give an example of a trick for efficient use of a computer. Suppose we have stored in its memory a great many, say 10^6, eight-digit numbers arranged sequentially and want the computer to decide whether a given number is among those stored. The computer can do this extremely fast by comparing in succession the digits from first to last. Suppose now that we want the computer to decide whether the given number differs from any of the stored numbers by, say, 1 in any of the eight positions. We might program the computer to do this by deciding whether any of the 106 numbers in its memory is that close. That would be a very lengthy operation. There is a much better way to proceed, a way that requires only sixteen times the effort required for the computer to decide whether a single number is among those stored. We first program the computer to produce from the given number the sixteen numbers that do differ by 1 in any of the eight positions and then to decide whether any of the sixteen is among those in its memory.

This example illustrates that a mechanism for producing auxiliary perceptions for comparison with perceptions stored in the memory would be an advantageous acquisition of the nervous system. So also would a mechanism for producing variations of what is stored in the memory for comparison with external stimuli. Perhaps a physiological or anatomical arrangement might serve such functions. Clearly these are merely guesses as to special characteristics the nervous system may have acquired in the course of evolution. ■

An Ulam Distance

by William A. Beyer

Stan had often referred, as he did in this lecture, to a distance between sets based on an encoding of the set points in terms of orthogonal functions. However, he had never explicitly defined such a distance. I do so now to honor the originator of so many seminal ideas.

Let A and B be two-dimensional finite sets enclosed in a square. Let n_A and n_B be the number of points in A and B, respectively. Let $\{f_{i,j}\}$ be a complete set of orthogonal functions on the square, such as two-dimensional Fourier trigonometric functions. Define $\mu^A_{i,j}$ and $\mu^B_{i,j}$, the encodings of A and B mentioned above, as follows:

$$\mu^A_{i,j} = \frac{1}{n_A} \sum_{x \in A} f_{i,j}(x)$$

and

$$\mu^B_{i,j} = \frac{1}{n_B} \sum_{x \in B} f_{i,j}(x).$$

Then $\mu^A_{i,j}$ and $\mu^B_{i,j}$ are functions on the nonnegative lattice points of the plane. Finally, let $\rho(f_1,f_2)$ be some selected distance between such functions. Then $\rho(\mu^A_{i,j}, \mu^B_{i,j})$ is a distance between the sets A and B—an alternative to the Hausdorff distance defined in the lecture. ■