Steady-State Jets

Supersonic gas jets are created in the laboratory by allowing a highly compressed gas to escape through a nozzle into the atmosphere or some other ambient gas. In contrast to astrophysical jets, laboratory jets usually have high density ratios ($\eta \equiv \rho_{jet}/\rho_{ambient} > 1$), moderate Mach numbers ($M \equiv v_{jet}/c_{jet} \leq 3$), and large pressure mismatches at the source. Investigations of laboratory jets have, until recently, focused on their steady-state behavior rather than the initial time-dependent behavior of interest in astrophysical contexts. Nevertheless, our understanding of the steady-state structures observed in laboratory jets provides a point of departure for interpreting our numerical simulations of time-dependent jet flow.

An Idealized Supersonic Jet

Figure,4 shows the characteristic structure taken on by a slightly underexpanded supersonic jet, that is, one for which the pressure of the gas at the nozzle orifice. P.. is slightly greater than the ambient gas pressure, P_a. (A jet is referred to as underexpanded if $K \equiv P_n/P_a > 1$, overexpanded if $K < 1$, and pressure matched if $K = 1$.)

This complicated axisymmetric structure has several remarkable features. First, the jet boundary oscillates as the jet gas periodically overexpands and reconverges in its attempt to match the ambient pressure. The gas continually overshoots the equilibrium position because the effects of the boundary arc communicated to the interior of the jet by sound waves, which, by definition, travel more slowly than the bulk supersonic flow. The characteristic paths of the sound waves converge to form the second remarkable feature of the jet, the network of crisscrossed shock waves, or shock diamonds (red lines). These standing shocks alternate with rarefaction fans (blue lines). The gas in the jet interior expands and cools (shades of blue) as it flows through the rarefaction fans and is compressed and heats (shades of red) as it passes through the shock diamonds. The figure clearly illustrates that the jet interior is always out of step with the jet boundary. For example, the positions of greatest gas compression (dark red) do not coincide with the

SIDEBAR 3

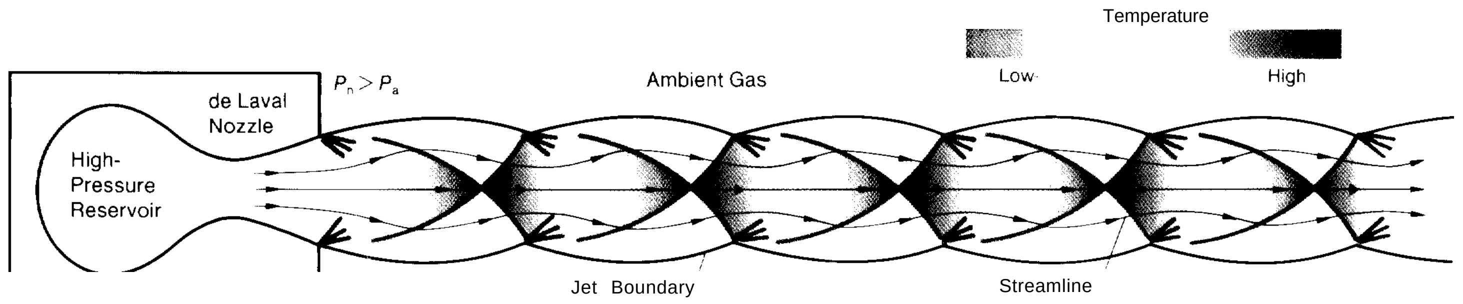

Fig. A. Idealized steady-state structure of a slightly underexpanded (P_n slightly greater than P_a) supersonic jet. The red lines represent *incident and reflected shocks (see Fig. B), and the blue lines indicate the beginnings of rarefaction fans. The gas temperature varies according to the key. Black streamlines follow the oscillating flow path of the jet gas.*

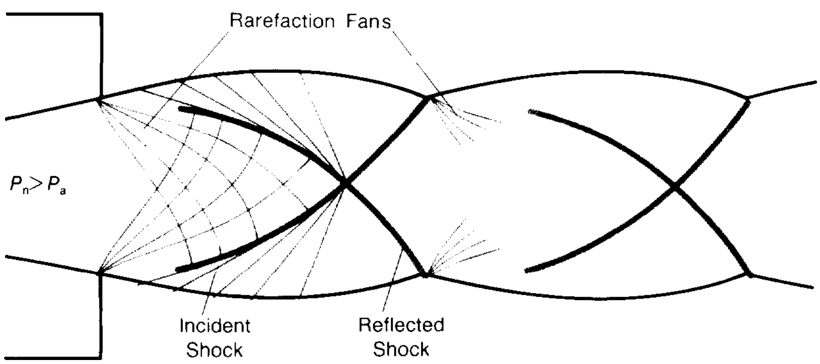

Fig. B. Regular reflection in a slightly underexpanded jet. The pattern of crisscrossed shocks in Fig. A can be understood in terms of characteristics. Diverging characteristics (blue) form rarefaction zones. Converging characteristics (red) from the boundary form the incident cortical shocks, which reflect off the jet axis to form the reflected shocks.

positions of minimum jet diameter. The black streamlines in the figure indicate the flow paths of the gas. The gas bends out toward the boundary as it passes through rare faction fans and bends back toward the axis as it passes through shock fronts.

Regular Reflections

Figure B shows how the shock structure in Fig. A can be understood in terms of characteristics. As the gas leaves the nozzle, it expands and a rarefaction fan (diverging blue characteristics) emanates from the nozzle orifice. The gas overexpands, and the pressure of the ambient gas at the boundary, acting like the piston in Sidebar 1, pushes the jet gas back toward the axis, creating the red characteristics. These characteristics form a converging conical shock. When this so-called incident shock reaches the jet axis it undergoes a regular reflection; that is, it forms a diverging shock. At the point where this reflected shock reaches the jet boundary, it knocks the boundary outward, creating a new rarefaction fan, and the process begins all over again. The geometric pattern of the (x, y) characteristics that form the shocks and rarefaction fans in this steady-state two. dimensional flow is similar to that of the

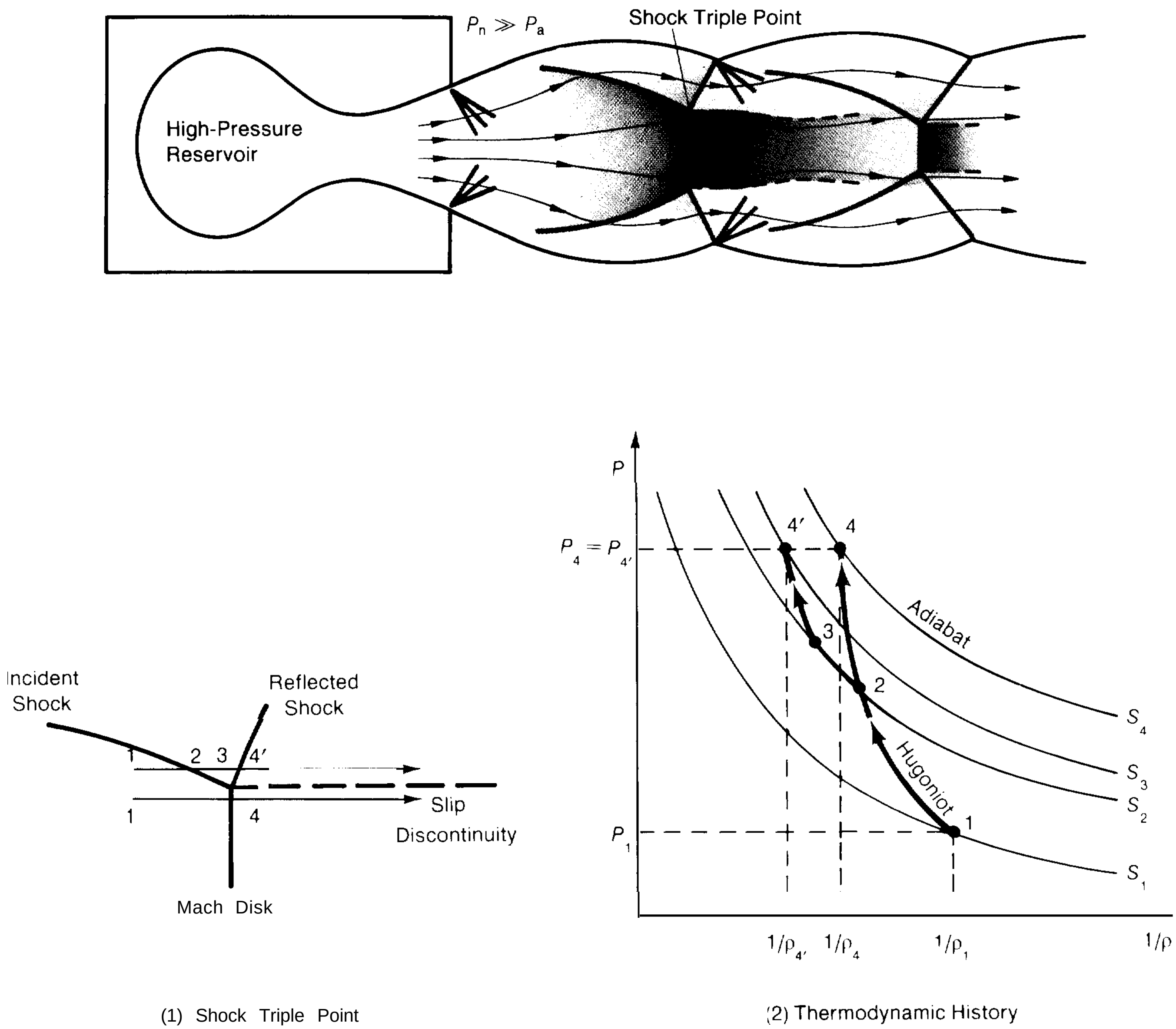

Fig. C. Mach reflection in an underexpanded ($P_n \gg P_a$) supersonic jet. A Mach reflection creates a shock triple point where three shocks meet. Figures C1 and C2 show how passage through the Mach disk {from point 1 to point 4) slows down the beam much more than passage through the incident and reflected shocks (from point 1 through points 2 and 3 to point 4'. As a result, a slip discontinuity is formed {dashed line).

(x, t) characteristics for one-dimensional time-dependent flow described in Sidebar 2.

Mach Reflections

A striking change in flow structure occurs when the pressure mismatch at the orifice is large ($K \gg 1$ or $K \ll 1$). As shown in Fig. C for an underexpanded jet, the incident shock, rather than converging to a point on the axis, reflects at the perimeter of a Mach disk—a strong shock normal to the flow direction (so named because Ernst Mach was the first to record its existence). The angle between the incident shock and the jet axis determines the type of reflection: small angles of incidence yield the regular reflections shown in Figs. A and B, and large angles of incidence yield Mach reflections. When gas passes through a shock, its velocity component normal to the shock is greatly reduced but its parallel component remains unchanged. Thus shocks with large angles of incidence relative to the flow axis are much more effective at slowing down the flow than shocks with small angles of incidence. The critical angle for transition from regular reflections to Mach reflections is approximately the angle that yields a sonic relative velocity for the gas downstream of the shock.

SIDE BAR 3

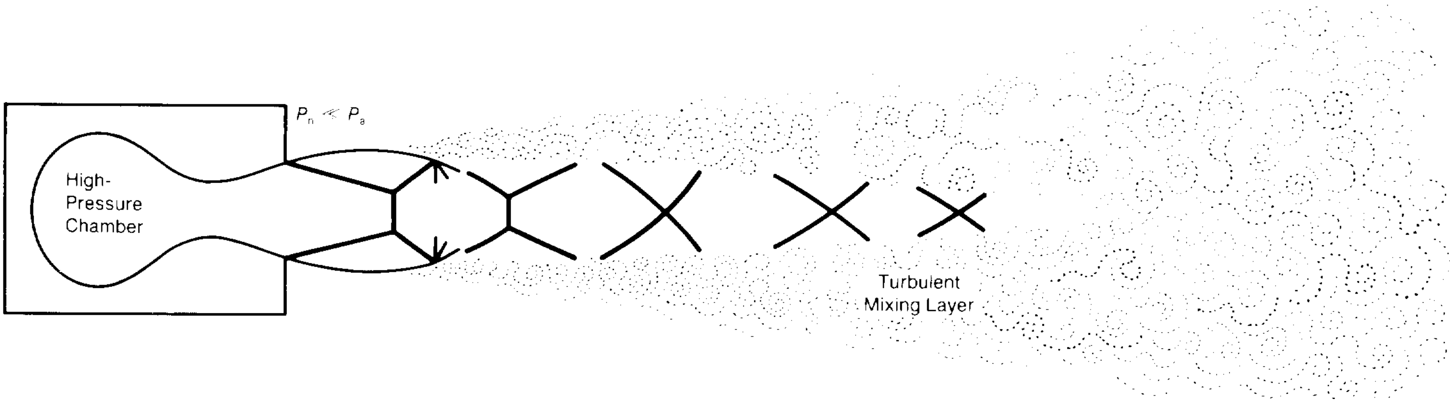

Fig. D, Realistic steady-state structure of an overexpanded ($P_n \ll P_a$) supersonic jet, showing the presence of both Mach reflections and regular reflections. Since this is an overexpanded jet, shocks rather than rarefactions emanate from the nozzle orifice. Shear instabilities create a mixing layer that grows until it reaches the beam axis.

A prominent feature of Mach reflections is the emergence of a slip discontinuity (dashed line) from the shock triple point where the incident shock, reflected shock, and Mach disk meet, The flow velocity, density, and temperature are discontinuous across this contact surface. This slip discontinuity arises because. the thermodynamic pathway through the incident and reflected shocks *does not equal* the pathway through the Mach disk.

In Fig. Cl we display two adjacent streamlines, one on each side of the shock triple point, and in Fig. C2 we display the corresponding thermodynamic pathways.

Adjacent fluid elements at some initial point on either streamline have the same state variables and the same total (kinetic plus internal) specific energy, that is, energy per unit mass. This quantity is given by

$$\frac{1}{2}v^2 + \frac{\gamma}{\gamma - 1}\frac{P}{\rho},$$

where *y* is the ratio of the specific heat of the fluid at constant pressure to the specific heat at constant volume.

By Bernoulli's principle the total specific energy remains constant along a streamline, Therefore, when the two adjacent fluid elements arrive at points 4 and 4' in Fig. Cl they must still have the same total specific energy. They must also have the same pressure because they are still adjacent. However, we see from Fig. C2 that their densities and entropies are different. The element at point 4 has been shock-heated along a Hugoniot to a higher entropy and lower density than the element that passed through the incident and reflected shocks along points 2 and 3 to point 4'. The lower density of the fluid element at point 4 implies that its internal energy (which is proportional to *P/p)* is greater than that of the fluid element at point 4'. Bernoulli's principle then implies that the fluid element at point 4 must have a correspondingly lower kinetic energy, and hence flow velocity, than the adjacent element at point 4'. A slip discontinuity results from this difference in flow velocities,

Laboratory Supersonic Jets

The jet structures shown in Figs. A, B, and Care idealizations in that real supersonic jets do not have sharp, stable boundaries but turbulent boundaries where jet and ambient gases mix. Figure D shows a more realistic steady-state structure for an overexpanded laboratory jet. Near the orifice, where the pressure mismatch is large, Mach reflections occur, but farther downstream the reflections are regular. The mixing layer, which grows as a result of Kelvin-Helmholtz (shear) instabilities, progressively eats its way into the supersonic core of the jet. When the mixing layer reaches the axis of the jet. the flow is subsonic and fully turbulent. It is then susceptible to twisting and bending motions, like a smokestack plume in a crosswind,

Strictly speaking the wave structures within the supersonic core are not steady since they are buffeted by the turbulent boundary layer. However, their average positions, those in Fig, D, are well defined. ■