Jet Stability

Numerical simulation is also a powerful tool for studying the effects of instabilities in time-dependent jets. This approach complements the traditional analytic tool of linear perturbation theory, which completely characterizes all modes of instability only in the small-amplitude limit. In contrast to linear theory, our simulations show which modes grow to large amplitudes and how they ultimately affect the flow. We now have some tantalizing hints that Kelvin-Helmholtz instabilities. which disrupt laboratory jets with low Mach numbers, may be reduced or totally absent in the high-Mach-number jets emerging from radio galaxies and quasars. In addition, our simulations confirm recent analytic predictions of the existence in supersonic jets of a new family of modes of the Kelvin-Helmholtz instability and characterize for the first time the nonlinear behavior of the new modes. To motivate our discussion. we review the key results gleaned from analytic studies of jet stability,

Linear Stability Theory

A simple analysis of the growth of axisymmetric, or pinch, modes of instability in supersonic beams is given in Sidebar 4, "Pinch Instabilities and the Bernoulli Effect." This analysis predicts that the relation

$$M = 1 + \sqrt{\eta} \qquad (2)$$

defines a stability boundary in the *(M,n)* plane. For $M < 1 + \sqrt{\eta}$, the pinch modes grow, whereas for $M > 1 + \sqrt{\eta}$, they damp. This analysis applies to a purely longitudinal mode, that is, one that introduces no transverse structure within the perturbed channel. Such a mode corresponds to the fundamental axisymmetric mode of instability}.

In 1981 Attilio Ferrari and coworkers in Bologna determined from perturbation theory that a whole family of unstable (that is. capable of growth) higher order modes exists

Pinch Instabilities

SIDEBAR 4

and the Bernoulli Effect

What happens to a fluid beam when its flow is slightly constricted? Does the flow resist the constriction and remain stable? Or does the constriction grow and pinch off the flow? Here we use a simple argument based on Bernoulli's principle to derive the conditions under which supersonic flow is susceptible to pinch instabilities, or, in other words, is unstable to longitudinal constrictions.

To establish the basic mechanism that drives a pinch instability, we consider first in Fig. Al the steady flow of an incompressible fluid, such as water, in a pipe with a localized constriction. The velocity and pressure of the fluid far from the constriction are v_o and P_o, respectively. At steady state the mass flux, *pvA*, is constant along the length of the pipe, Since the fluid is incompressible (that is, p is constant) the fluid velocity v must increase when the cross-sectional area *A* decreases, The Bernoulli principle states that along a fluid streamline the total specific energy *E* remains constant:

$$E = \frac{1}{2}v^2 + \frac{\gamma}{\gamma - 1}\frac{P}{\rho} = \text{constant}\,. \qquad (1)$$

Consequently, if p remains constant and *v* increases, the pressure *P* in the vicinity of the constriction must decrease. Figure A2 shows what happens if the channel wall is replaced by a static background fluid of uniform pressure P_o. Since the beam pressure *P* near the constriction is less than the background pressure P_o, the channel boundary will move inward, further constricting the flow. By the above reasoning further constriction leads to even higher velocities, lower pressures. and even further constriction.

Precisely this sequence of events is responsible for pinch instabilities in subsonic jets, In principle, an initial constriction of arbitrarily small amplitude will be amplified by the Bernoulli effect until the constriction completely pinches off the flow. In actuality, the ambient fluid following the channel boundary inward at the point of constriction is swept into the channel and carried downstream by the flow as in Fig. A3. As a result, pinch instabilities contribute to the mixing of a subsonic jet with its environment.

When the beam is supersonic, two other effects come into play. First and most important. the compressibility of the fluid can no longer be ignored. Thus the constancy of the mass flux no longer implies an increase in velocity with a decrease in area. Instead the density increases so rapidly as the area decreases that the velocity decreases also. A general mathematical expression relating velocity change to area change in a flow channel of slowly varying cross section is

$$\frac{dv}{v} = \left(\frac{v^2}{c^2} - 1\right)^{-1}\frac{dA}{A}\,, \qquad (2)$$

where c is the speed of sound in the fluid. We see that if the flow is subsonic ($v < c$), its velocity increases ($dv > 0$) at a constriction ($dA < 0$) just as it does in an incompressible fluid, whereas if the flow is supersonic ($v > c$), its velocity decreases ($dv < 0$) at a constriction ($dA < 0$).

A naive application of the Bernoulli principle would predict that all supersonic beams are stable to the pinch instability, since by Eqs. 1 and 2 the pressure now *increases* at a constriction and thus resists further pinching. This prediction is incorrect because we have failed to take into account the effects of the motion of the boundary in the direction of the flow. As shown in Fig. B1, the instability created when one fluid moves past

Fig. A. Incompressible Flow

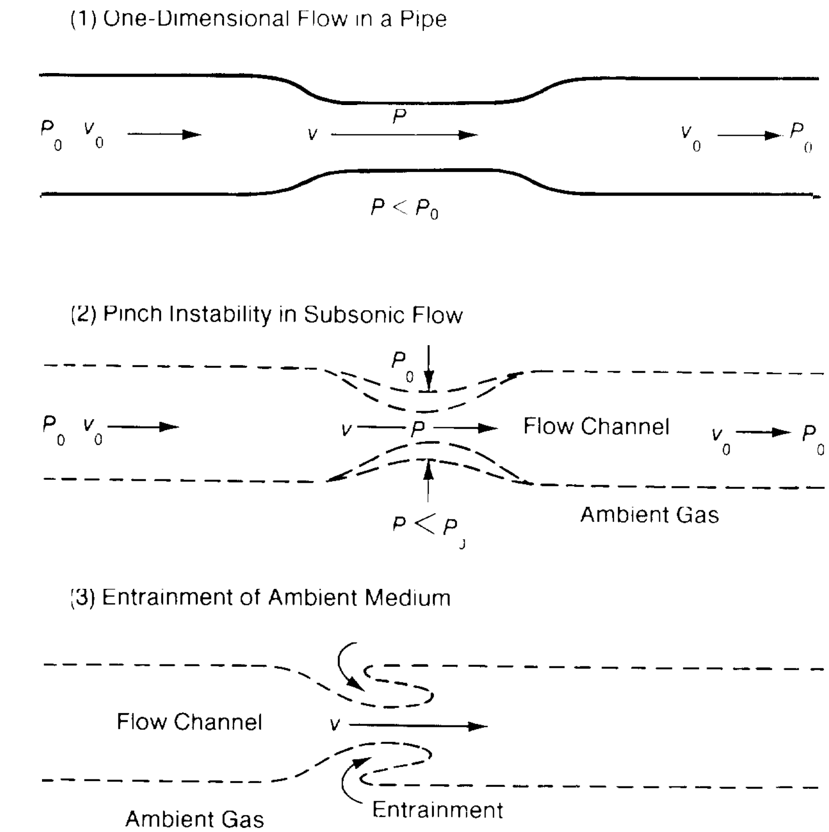
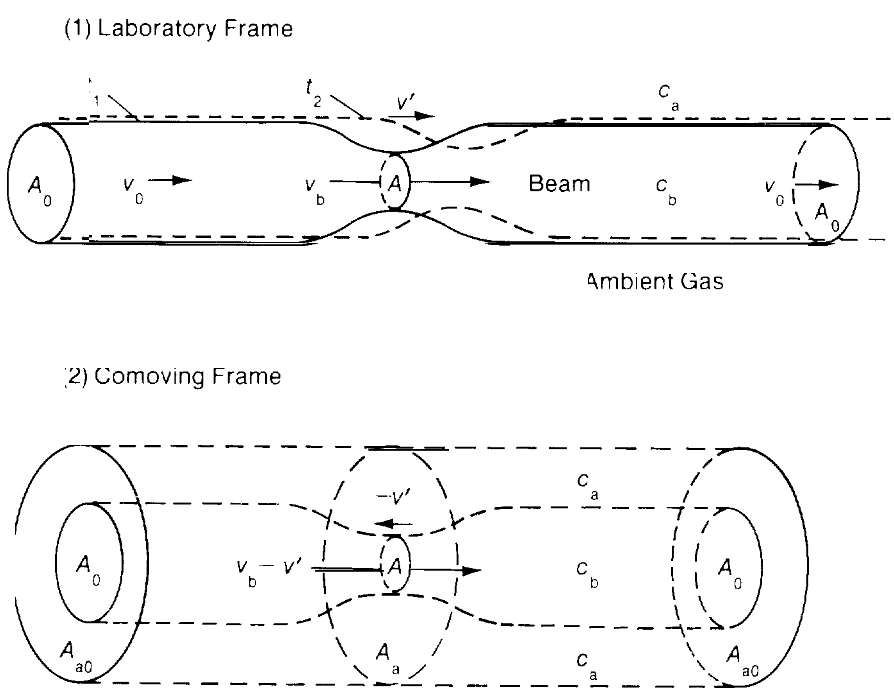


Fig. B. Convective Instability in Compressible Flow



another (the Kelvin Helmholtz instability) is a convective instability; that is, the waveform of the perturbed boundary is carried downstream with some velocity v.

Figure B2 shows the flow in a frame moving at velocity v'. In this frame the beam flows through the constriction with a relative speed of v_b— v', and the ambient gas flows past the constriction with a relative speed of —*v*'. Now consider the Bernoulli effect in the two flow channels shown in Fig. B2—one inside the jet boundary and one surrounding it. To guarantee pinching. we require that at the constriction the pressure decrease inside the jet boundary and increase outside the jet boundary. *This will be the case if both the jet gas and the ambient gas move subsonically with respect to the constriction, since,* from Eqs. 1 and 2, only for relative Mach numbers (ratios of flow velocities in the moving frame to sound speed) less than unity will the channel area and the pressure vary in the same direction. Therefore, a sufficient condition for instability is that $v_b - v' < c_b$ and $v' < c_a$, where c_b and c_a are the speeds of sound in the beam gas and the ambient gas, respectively. Adding the two inequalities eliminates the unknown v' and yields the condition

$$v_b < c_b + c_a \,. \tag{3}$$

Thus, on very simple physical grounds instability requires that the jet speed be subsonic with respect to the sum of the internal and external sound speeds.

To express this condition in terms of the dimensionless parameters Mach number *M* and density ratio n, we eliminate c_a by invoking the pressure balance between the jet and ambient gases ($\rho_a c_a^2 = \rho_b c_b^2$) and obtain

$$M < 1 + \sqrt{\eta} \,. \tag{4}$$

This relationship, which was first deduced by David Payne and Haldan Cohn from a dispersion analysis of the linearized equations of motion, thus follows simply from the fact that the growth of pinch instabilities requires subsonic flow of both beam gas and ambient gas relative to the moving constriction. ■