QCD on a Cray: the masses of elementary particles

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How can we extract answers from QCD at energies below 1 GeV? As noted in the text, the confinement of quarks suggests that weak-coupling perturbative methods are *not* going to be successful at these energies. Nevertheless, if QCD is a valid theory it must explain the multiplicities, masses, and couplings of the experimentally observed strongly interacting particles. These would emerge from the theory as bound states and resonances of quarks and gluons. A valid theory must also account for the apparent absence of isolated quark states and might predict the existence and properties of particles (such as glueballs) that have not yet been seen.

The most promising nonperturbative formulation of QCD exploits the Feynman path integral. Physical quantities are expressed as integrals of the quark and gluon fields over the space-time continuum with the QCD Lagrangian appearing in an exponential as a kind of Gibbs weight factor. This is directly analogous to the partition function formulation of statistical machanics. The path integral prescription for strong interaction dynamics becomes well defined mathematically when the space-time continuum is approximated by a discrete four-dimensional lattice of finite size and the integrals are evaluated by Monte Carlo sampling.

The original Monte Carlo ideas of Metropolis and Ulam have now been applied to QCD by many researchers. These efforts have given credibility, but not confirmation, to the hope that computer simulations might indeed provide critical tests of QCD and significant numerical results. With considerable patience (on the order of many months of computer time) a VAX 11/780 can be used to study universes of about 3000 space-time points. Such a universe is barely large enough to contain a proton and not really adequate for a quantitative calculation. Consequently, with these methods, any result from a computer of VAX power is, at best, only an indication of what a well-done numerical simulation might produce.

We believe that a successful computer simulation must combine the following: (1) physical and mathematical ingenuity to search out the best formulations of problems still unsolved in principle; (2) sophisticated numerical analysis and computer programming; and (3) a computer with the speed, memory, and input/output rate of the Cray XMP with a solid-state disk (or better). We have done calculations of particle masses on a lattice of 55,296 space-time points using the Cray XMP. Using new methods developed with coworkers R. Gupta, J. Mandula, and A. Patel, we are examining glueball masses, renormalization group behavior, and the behavior of the theory on much larger lattices. The results to date support the belief that QCD describes interactions of the elementary particles and that these numerical methods are currently the most powerful means for extracting the predictive content of QCD.

The calculations, which have two input parameters (the pion mass and the long-range quark-quark force constant in units of the lattice spacing), provide estimates of many measurable quantities. The accompanying table shows some of our results on elementary particle masses and certain meson coupling strengths. These results represent several hundred hours of Cray time. The quoted relative errors derive from the statistical analysis of the Monte Carlo calculation itself rather than from a comparison with experimental data. Significantly more computer time would significantly reduce the errors in the calculated masses and couplings.

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Calculated and experimental values for the masses and coupling strengths of some mesons and baryons.

	Calculated Value (MeV/c^2)	Relative Error (%)	Experimental Value (MeV/c^2)
Masses			
ρ meson	767	18	769
Excited ρ	1426	27	1300?
δ meson	1154	15	983
A_1 meson	1413	17	1275
Proton	989	23	940
Δ baryon	1199	17	1210
Couplings			
f_π	121	21	93
f_ρ	211	15	144