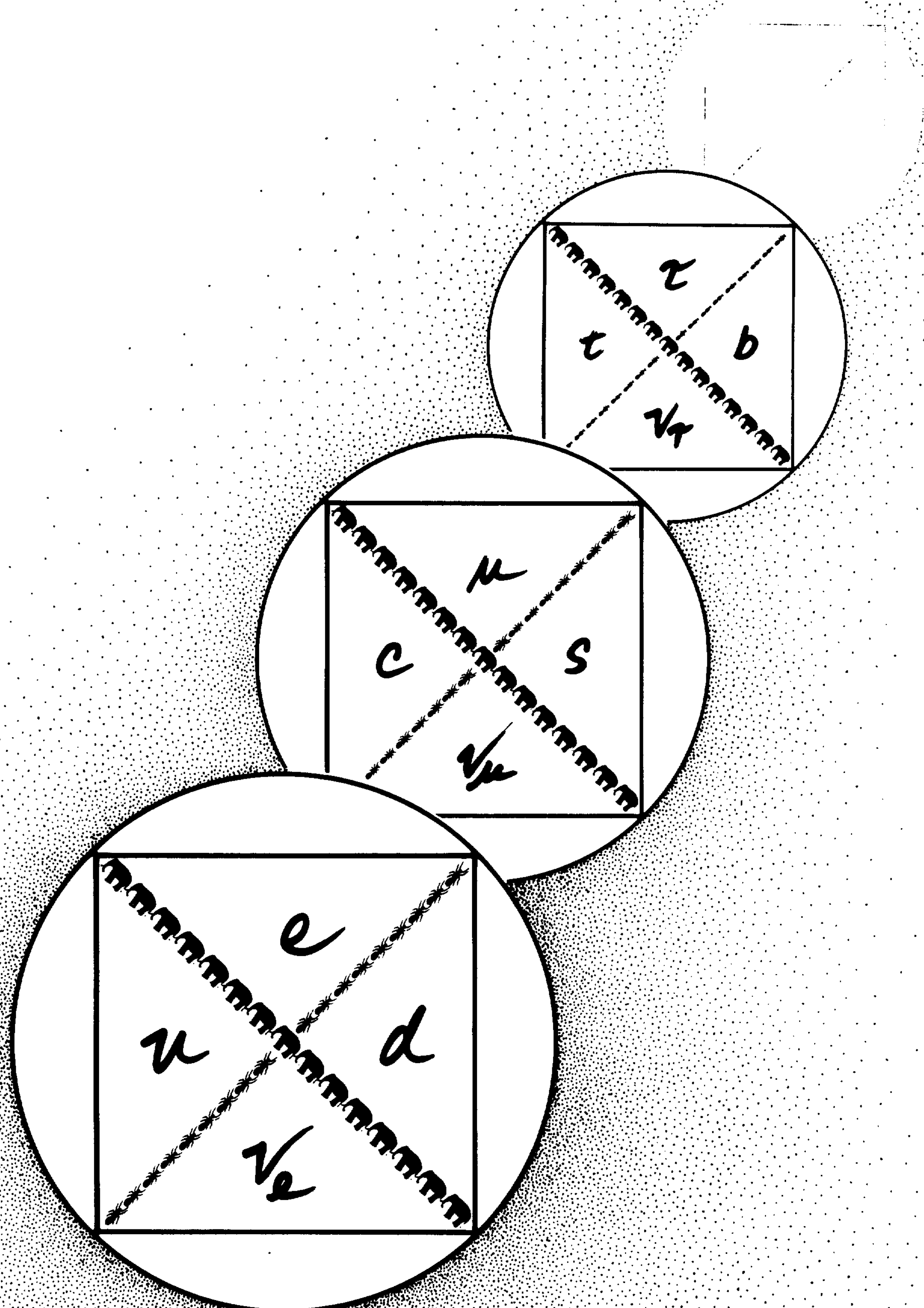
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The Family Problem

by T. Goldman and Michael Martin Nieto

The roster of elementary particles includes replicas, exact in every detail but mass, of those that make up ordinary matter. More facts are needed to explain this seemingly unnecessary extravagance.

The currently "standard" model of particle physics phenomenologically describes virtually all of our observations of the world at the level of elementary particles (see "Particle Physics and the Standard Model"). However, it does not explain them with any depth. Why is $SU(3)_C$ the gauge group of the strong force? Why is the symmetry of the electroweak force broken? Where does gravity fit in? How can all of these forces be unified? That is, from what viewpoint will they appear as aspects of a common, underlying principle? These questions lead us in the directions of supersymmetry and of grand unification, topics discussed in "Toward a Unified Theory."

Yet another feature of the standard model leaves particle physicists dissatisfied: the multiple repetitions of the representations* of the particles involved in the gauge interactions. By definition the adjoint representation† of the gauge fields must occur precisely once in a gauge theory. However, quantum chromodynamics includes no less than six occurrences of the color triplet representation of quarks: one for each of the *u*, *c*, *t*, *d*, *s*, and *b* quarks. The *u*, *c*, and *t* quarks have a common electric charge of $\frac{2}{3}$ and so are distinguished from the *d*, *s*, and *b* quarks, which have a common electric charge of $-\frac{1}{3}$. But the quarks with a common charge are distinguished only by their different masses, as far as is now known. The electroweak theory presents an even worse situation, being burdened with nine left-chiral‡ quark doublets, three left-chiral lepton doublets, eighteen right-chiral quark singlets, and three right-chiral lepton singlets (Fig. 1).

Nonetheless, some organization can be discerned. The exact symmetry of the strong and electromagnetic gauge interactions, together with the nonzero masses of the quarks and charged leptons, implies that the right-chiral quarks and charged leptons and their left-chiral partners can be treated as single objects under these interactions. In addition, each neutral lepton is associated with a particular charged lepton, courtesy of the transformations induced by the weak interaction. Thus, it is natural to think in terms of three quark sets (*u* and *d*, *c* and *s*, and *t* and *b*) and three lepton sets (e^- and ν_e, μ^- and ν_μ, and τ^- and ν_τ) rather than thirty-three quite repetitive representations. Furthermore, the relative lightness of the *u* and *d* quark set and of the e^- and ν_e lepton set long ago suggested to some that the quarks and leptons are also related (quark-lepton symmetry). Subtle mathematical properties of modern gauge field theories have provided new backing for this notion of three "quark-lepton families," each consisting of successively heavier quark and lepton sets (Table 1).

****We give a geometric definition of "representation," using as an example the $SU(3)_C$ triplet representation of, say, the up quark. (This triplet, the smallest non-singlet representation of $SU(3)_C$, is called the fundamental representation.) The members of this representation (u_{red}, u_{blue}, and u_{green}) correspond to the set of three vectors directed from the origin of a two-dimensional coordinate system to the vertices of an equilateral triangle centered at the origin. (The triangle is usually depicted as standing on a vertex.) The "conjugate" of the triplet representation, which contains the three anticolor varieties of the up quark with charge $-\frac{2}{3}$, can be defined similarly: it corresponds to the set of three vectors obtained by reflecting the vectors of the triplet representation through the origin. (The vectors of the conjugate representation are directed toward the vertices of an equilateral triangle standing on its side, like a pyramid.) The "group transformations" correspond to the set of operations by which any one of the quark or antiquark vectors is transformed into any other.***

†The "adjoint" representation of $SU(3)_C$, which contains the eight vector bosons (the gluons), is found in the "product" of the triplet representation and its conjugate. This product corresponds to the set of nine vectors obtained by forming the vector sums of each member of the triplet representation with each member of its conjugate. This set can be decomposed into a singlet containing a null vector (a point at the origin) and an octet, the adjoint representation, containing two null vectors and six vectors directed from the origin to the vertices of a regular hexagon centered at the origin. Note that the adjoint representation is symmetric under reflection through the origin.

‡A massless particle is said to be left-handed (right-handed) if the direction of its spin vector is opposite (the same as) that of its momentum. Chirality is the Lorentz-invariant generalization of this handedness to massive particles and is equivalent to handedness for massless particles.

If the underlying significance of this grouping by mass is not apparent to the reader, neither is it to particle physicists. No one has put forth any compelling reason for deciding which charge $^2/_3$ quark and which charge $-^1/_3$ quark to combine into a quark set or for deciding which quark set and which charged and neutral lepton set should be combined in a quark-lepton family. Like Mendeleev, we are in possession of what appears to be an orderly grouping but without a clue as to its dynamical basis. This is one theme of "the family problem."

Still, we do refer to each quark and lepton set together as a family and thus reduce the problem to that of understanding only three families—unless, of course, there are more families as yet unobserved. This last is another question that a successful "theory of families" must answer. Grand unified theories, supersymmetry theories, and theories wherein quarks and leptons have a common substructure can all accommodate quark-lepton symmetry but as yet have not provided convincing predictions as to the number of families. (These predictions range from any even number to an infinite spectrum.)

Such concatenations of wild ideas (however intriguing) may not be the best approach to solving the family problem. A more conservative approach, emulating that leading to the standard model, is to attack the family problem as a separate question and to ask directly if the different families are dynamically related.

Here we face a formidable obstacle—a paucity of information. A fermion from one family has never been observed to change into a fermion from another family. Table 2 lists some family-changing decays that have been sought and the experimental limits on their occurrence. True, a μ^- may appear to decay into an e^-, but, as has been experimentally confirmed, it actually is transformed into a ν_μ, and simultaneously the e^- and a $\bar{\nu}_e$ appear. Being an antiparticle, the $\bar{\nu}_e$ carries the opposite of whatever family quantum numbers distinguish an e^- from any other charged lepton. Thus, no net "first-familiness" is created, and the "second-familiness"

Quark Representations

$$\left(\begin{bmatrix} u_r \\ d'_r \end{bmatrix}_L, \begin{bmatrix} u_b \\ d'_b \end{bmatrix}_L, \begin{bmatrix} u_g \\ d'_g \end{bmatrix}_L\right) \quad ([u_r]_R, [u_b]_R, [u_g]_R) \quad ([d_r]_R, [d_b]_R, [d_g]_R)$$

$$\left(\begin{bmatrix} c_r \\ s'_r \end{bmatrix}_L, \begin{bmatrix} c_b \\ s'_b \end{bmatrix}_L, \begin{bmatrix} c_g \\ s'_g \end{bmatrix}_L\right) \quad ([c_r]_R, [c_b]_R, [c_g]_R) \quad ([s_r]_R, [s_b]_R, [s_g]_R)$$

$$\left(\begin{bmatrix} t_r \\ b'_r \end{bmatrix}_L, \begin{bmatrix} t_b \\ b'_b \end{bmatrix}_L, \begin{bmatrix} t_g \\ b'_g \end{bmatrix}_L\right) \quad ([t_r]_R, [t_b]_R, [t_g]_R) \quad ([b_r]_R, [b_b]_R, [b_g]_R)$$

Lepton Representations

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}_L \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}_L \quad (e^-)_R \quad (\mu^-)_R \quad (\tau^-)_R$$

Fig. 1. The electroweak representations of the fermions of the standard model, which comprise nine left-chiral quark doublets, eighteen right-chiral quark singlets, three left-chiral lepton doublets, and three right-chiral lepton singlets. The subscripts* r, b, *and* g *denote the three color charges of the quarks, and the subscripts* R *and* L *denote right- and left-chiral projections. The symbols* d′, s′, *and* b′ *indicate weak-interaction mass eigenstates, which, as discussed in the text, are mixtures of the strong-interaction mass eigenstates* d, s, *and* b. *Since quantum chromodynamics does not include the weak interaction, and hence is not concerned with chirality, the $SU(3)_c$ representations of the fermions are fewer in number: six triplets, each containing the three color-charge varieties of one of the quarks, and three singlets, each containing a charged lepton and its associated neutral lepton.

of the original μ^- is preserved in the ν_μ.

In spite of the lack of positive experimental results, current fashions (which are based on the successes of the standard model) make irresistible the temptation to assign a family symmetry group to the three known families. Some that have been considered include SU(2), SU(2) × U(1), SU(3), and U(1) × U(1) × U(1). The impoverished level of our understanding is apparent from the SU(2) case, in which we cannot even determine whether the three families fall into a doublet and a singlet or simply form a triplet.

The clearest possible prediction from a family symmetry group, analogous to Mendeleev's prediction of new elements and their properties, would be the existence of one or more additional families necessary to complete a representation. Such a prediction can be obtained most naturally from either of two possibilities for the family symmetry: a spontaneously broken local gauge symmetry

Table 1

Members of the three known quark-lepton families and their masses. Each family contains one particle from each of the four types of fermions: leptons with an electric charge of −1 (the electron, the muon, and the tau); neutral leptons (the electron neutrino, the muon neutrino, and the tau neutrino); quarks with an electric charge of ⅔ (the up, charmed, and top quarks); and quarks with an electric charge of −⅓ (the down, strange, and bottom quarks). Each family also contains the antiparticles of its members. (The antiparticles of the charged leptons are distinguished by opposite electric charge, those of the neutral leptons by opposite chirality, and those of the quarks by opposite electric and color charges. For historical reasons only the antielectron has a distinctive appellation, the positron.) Family membership is determined by mass, with the first family containing the least massive example of each type of fermion, the second containing the next most massive, and so on. What, if any, dynamical basis underlies this grouping by mass is not known, nor is it known whether other heavier families exist. The members of the first family dominate the ordinary world, whereas those of the second and third families are unstable and are found only among the debris of collisions between members of the first family.

First Family	Second Family	Third Family
electron, e^- 0.511 MeV/c^2	muon, μ^- 105.6 MeV/c^2	tau, τ^- 1782 MeV/c^2
electron neutrino, ν_e 0.00002 MeV/c^2 (?)	muon neutrino, ν_μ $\leq$0.5 MeV/c^2	tau neutrino, ν_τ $\leq$147 MeV/c^2
up quark, u $\simeq$5 MeV/c^2	charmed quark, c $\simeq$1500 MeV/c^2	top quark, t $\geq$40,000 MeV/c^2 (?)
down quark, d $\simeq$10 MeV/c^2	strange quark, s $\simeq$170 MeV/c^2	bottom quark, b $\simeq$4500 MeV/c^2

Table 2

Experimental limits on the branching ratios for some family-changing decays. The branching ratio for a particular decay mode is defined as the ratio of the number of decays by that mode to the total number of decays by all modes. An experiment capable of determining a branching ratio for $\mu^+ \to e^+\gamma$ as low as 10^{-12} is currently in progress at Los Alamos (see "Experiments To Test Unification Schemes").

Decay Mode	Branching Ratio (upper bound)	Dominant Decay Mode(s)
$\mu^+ \to e^+\gamma$	10^{-10}	$\mu^+ \to e^+\nu_e\bar{\nu}_\mu$
$\mu^+ \to e^+e^+e^-$	10^{-12}	$\mu^+ \to e^+\nu_e\bar{\nu}_\mu$
$\pi^0 \to \mu^\pm e^\mp$	10^{-7}	$\pi^0 \to \gamma\gamma$
$K^+ \to \pi^+\mu^\pm e^\mp$	10^{-8}	$K^+ \to \pi^+\pi^0$ or $\mu^+\nu_\mu$
$K_L \to \mu^\pm e^\mp$	10^{-8}	$K_L \to \pi^+\pi^-\pi^0$ or $\pi^0\pi^0\pi^0$
$\Sigma^+ \to p\mu^\pm e^\mp$	10^{-5}	$\Sigma^+ \to p\pi^0$

or a spontaneously broken global symmetry.* What follows is a brief ramble (whose course depends little on detailed assumptions) through the salient features and implications of these two possibilities.

Family Gauge Symmetry

All of the unseen decays listed in Table 2 would be strictly forbidden if the family gauge symmetry were an exact gauge symmetry as those of quantum electrodynamics and quantum chromodynamics are widely believed to be. Here, however, we do not expect exactness because that would imply the existence, contrary to experience, of an additional fundamental force mediated by a massless vector boson (such as a long-range force like that of the photon or a strong force like that of the gluons but extending to leptons as well as quarks). But we can, as in the standard model, assume a *broken* gauge symmetry.

We begin by placing one or more families in a representation of some family gauge symmetry group. (The correct group might be inferred from ideas such as grand unification or compositeness of fermions. However, it is much more likely that, as in the case of the standard model, this decision will best be guided by hints from experimental observations.) Together, the group and the representation determine currents that describe interactions between members of the representation. (These currents would be conserved if the family symmetry were exact.) For example, if the first and the second families are placed in the representation, an electrically neutral current describes the transformation $e^- \leftrightarrow \mu^-$, just as the charged weak current of the electroweak theory describes the transformation $e^- \leftrightarrow \nu_e$. Since the other family

In principle, we should also consider the possibilities of a discrete symmetry or an explicit breaking of family symmetry (probably caused by some dynamics of a fermion substructure). However, these ideas would be radical departures from the gauge symmetries that have proved so successful to date. We will not pursue them here.

members necessarily fall into the same representation, the $e^- \rightarrow \mu^-$ current includes contributions from interactions between these other members ($d \rightarrow s$, for example), just as the charged weak current for $e^- \rightarrow \nu_e$ includes contributions from $\mu^- \rightarrow \nu_\mu$ and $\tau^- \rightarrow \nu_\tau$.

If we now allow the family symmetry to be a local gauge symmetry, we find a "family vector boson," F, that couples to these currents (Fig. 2) and mediates the family-changing interactions. As in the standard model, the coupled currents can be combined to yield dynamical predictions such as scattering amplitudes, decay rates, and relations between different processes.

Scale of Family Gauge Symmetry Breaking. Weak interactions occur relatively infrequently compared to electromagnetic and strong interactions because of the large dynamical scale (approximately 100 GeV) set by the masses of the $W^\pm$ and Z^0 bosons that break the electroweak symmetry. We can interpret the extremely low rate of family-changing interactions as being due to an analogous but even larger dynamical scale associated with the breaking of a local family gauge symmetry, that is, to a large value for the mass M_F of the family vector boson. The branching-ratio limit listed in Table 2 for the reaction $K_L \rightarrow \mu^\pm + e^\mp$ allows us to estimate a lower bound for M_F as follows.

Like the weak decay of muons, the $K_L \rightarrow \mu e$ decay proceeds through formation of a virtual family vector boson (Fig. 3). The rate for the decay, Γ, is given by

$$\Gamma \cong \frac{g^4_{\rm family}}{M_F^4}\, m_K^5 \,. \qquad (1)$$

Note that the fourth power of M_F appears in Eq. 1 just as the fourth power of M_W does (hiding in the square of the Fermi constant) in the rate equation for muon decay. (Certain chirality properties of the family interaction could require that two of the five powers of the kaon mass m_K in Eq. 1 be replaced by the muon mass. However, since the inferred value of M_F varies as the fourth root of this term, the change would make little numerical difference.) It is usual to assume that $g_{\rm family}$, the family coupling constant, is comparable in magnitude to those for the weak and electromagnetic interactions. This assumption reflects our prejudice that family-changing interactions may eventually be unified with those interactions. Using Eq. 1 and the branching-ratio limit from Table 2, we obtain

$$M_F \gtrsim 10^5\ {\rm GeV}/c^2 \,. \qquad (2)$$

Such a large lower bound on M_F implies that the breaking of a local family gauge symmetry produces interactions much weaker than the weak interactions.

Alternatively, processes like $K_L \rightarrow \mu e$ may be the result of family-conserving grand unified interactions in which quarks are turned into leptons. However, the experimental limit on the rate of proton decay implies that such interactions occur far less frequently than the family-violating interactions considered here.

Experiments with neutrinos, also, indicate a similarly large dynamical scale for the breaking of a local family gauge symmetry. A search for the radiative decay $\nu_\mu \rightarrow \nu_e + \gamma$ has yielded a lower bound on the ν_μ lifetime of 10^5 (m_ν/MeV) seconds. If the mass of the muon neutrino is near its experimentally observed upper bound of 0.5 MeV/c^2, this lower bound on the lifetime is greater than the standard-model prediction of approximately $10^3\,({\rm MeV}/m_\nu)^5$ seconds. Thus, some family-conservation principle may be suppressing the decay.

More definitive information is available from neutrino-scattering experiments. Positive pions decay overwhelmingly (10^4 to 1) into positive muons and muon neutrinos. In the absence of family-changing interactions, scattering of these neutrinos on nuclear targets should produce only negative muons. This has been accurately confirmed: neither positrons nor electrons appear more frequently than permitted by the present systematic experimental uncertainty of 0.1 per-

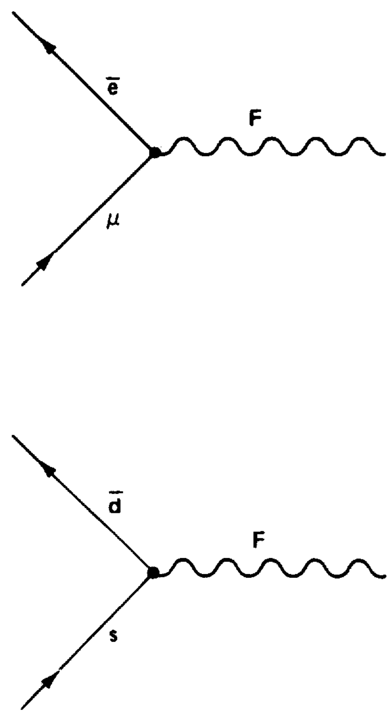

Fig. 2. Examples of neutral family-changing currents coupled to a family vector boson* (F)*. Such couplings follow from the assumption of a local gauge symmetry for the family symmetry.

cent. An investigation of the neutrinos from muon decay has yielded similar results. The decay of a positive muon produces, in addition to a positron, an electron neutrino and a muon antineutrino. Again, in the absence of family-changing interactions, scattering of these neutrinos should produce only electrons and positive muons, respectively. A LAMPF experiment (E-31) has shown, with an uncertainty of about 5 percent, that no negative muons or positrons are produced.

The energy scale of Eq. 2 will not be directly accessible with accelerators in the

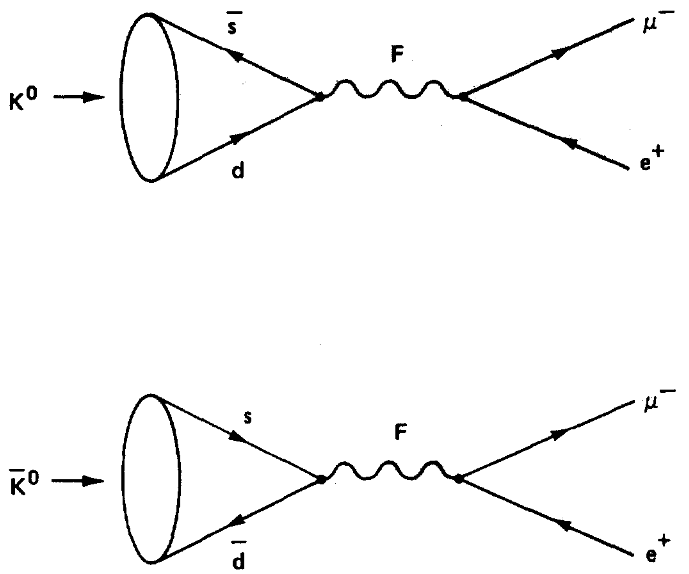

Fig. 3. Feynman diagram for the family-changing decay $K_L \rightarrow \mu^- + e^+$***, which is assumed to occur through formation of a virtual family vector boson*** (F)***. The*** K_L ***meson is the longer lived of two possible mixtures of the neutral kaon*** (K^0) ***and its antiparticle*** ($\bar{K}^0$)***. Neither this decay nor the equally probable decay*** $K_L \rightarrow \mu^+ + e^-$ ***has been observed experimentally; the current upper bound on the branching ratio is*** 10^{-8}.

foreseeable future. The Superconducting Super Collider, which is currently being considered for construction next decade, is conceived of as reaching 40,000 GeV but is estimated to cost several billion dollars. We cannot expect something yet an order of magnitude more ambitious for a very long time. Thus, further information about the breaking of a local family gauge symmetry will not arise from a brute force approach but rather, as it has till now, from discriminating searches for the needle of a rare event among a haystack of ordinary ones. Clearly, the larger the total number of events examined, the more definitive is the information obtained about the rate of the rare ones. For this reason the availability of high-intensity beams of the reacting particles is a very important factor in the experiments that need to be undertaken or refined, given that they are to be carried out by creatures with finite lifetimes!

For example, consider again the decay $K_L \rightarrow \mu e$. Since the rate of this decay varies inversely as the fourth power of the mass of the family vector boson, a value of M_F in the million-GeV range implies a branching ratio lower by four orders of magnitude than the present limit. A search for so rare a decay would be quite feasible at a high-intensity, medium-energy accelerator such as the proposed LAMPF II, which is expected to produce kaon fluxes on the order of 10^8 per second. (Currently available kaon fluxes are on the order of 10^6 per second.) A typical solid angle times efficiency factor for an in-flight decay experiment is on the order of 10 percent. Thus, 10^7 kaons per second could be examined for the decay mode of interest. A branching ratio larger than 10^{-12} could be found in a one-day search, and a year-long experiment would be sensitive down to the 10^{-14} level. Of course, we do not know with absolute certainty whether a positive signal will be found at any level. Nonetheless, the need for such an observation to elucidate family dynamics impels us to make the attempt.

Positive Evidence for Family Symmetry Breaking

Thus, despite expectations to the contrary, we have at present no positive evidence in any neutral process for nonconservation of a family quantum number, that is, for family-changing interactions mediated by exchange of an electrically neutral vector boson such as the F of Figs. 2 and 3. Is it possible that our expectations are wrong—that this quantum number is exactly conserved as are electric charge and angular momentum? The answer is an unequivocal NO! We have—for quarks—positive evidence that family is a *broken* symmetry. To see this, we must examine the effect of the electroweak interaction on the quark mass eigenstates defined by the strong interaction.

We know, for instance, that a K^+ ($= u + \bar{s}$) decays by the weak interaction into a μ^+ and a ν_μ and also decays into a π^+ and a π^0 (Fig. 4). In quark terms this means that the u quark and the $\bar{s}$ quark in the kaon are coupled through a W^+ boson. The two families (up-down and charmed-strange) defined by the quark mass eigenstates under the strong interaction are mixed by the weak interaction. Since the kaon decays occur in both purely leptonic and purely hadronic channels, they are not likely to be due to peculiar quark-lepton couplings. Similar evidence for family violation is found in the decays of D mesons, which contain charmed quarks.

Weak-interaction eigenstates d' and s' may be defined in terms of the strong-interaction mass eigenstates d and s by

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}, \qquad (3)$$

where θ_C, the Cabibbo mixing angle, is experimentally found to be the angle whose sine is 0.23 ± 0.01. (The usual convention, which entails no loss of generality, is to assign all the mixing effects of the weak interaction to the down and strange quarks, leaving unchanged the up and charmed quarks.) The fact that the mass and weak-interaction eigenstates are different implies that a conserved family quantum number cannot be defined in the presence of both the strong and the weak interactions. We can easily show, however, that this conclusion does not contradict the observed absence of *neutral* family-violating interactions.

The weak charged-current interaction describing, say, the transformation of a d' quark into a u quark by absorption of a W^+ boson has the form

$$(\bar{u}d' + \bar{c}s')W^+ = (\bar{u}, \bar{c}) \begin{pmatrix} W^+ & 0 \\ 0 & W^+ \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix}, \qquad (4)$$

which, after substitution of Eq. 3, becomes

$$(\bar{u}d' + \bar{c}s')W^+ = \bar{u}(d\cos\theta_C + s\sin\theta_C)W^+ + \bar{c}(-d\sin\theta_C + s\cos\theta_C)W^+ . \qquad (5)$$

(Here we suppress details of the Lorentz algebra.)

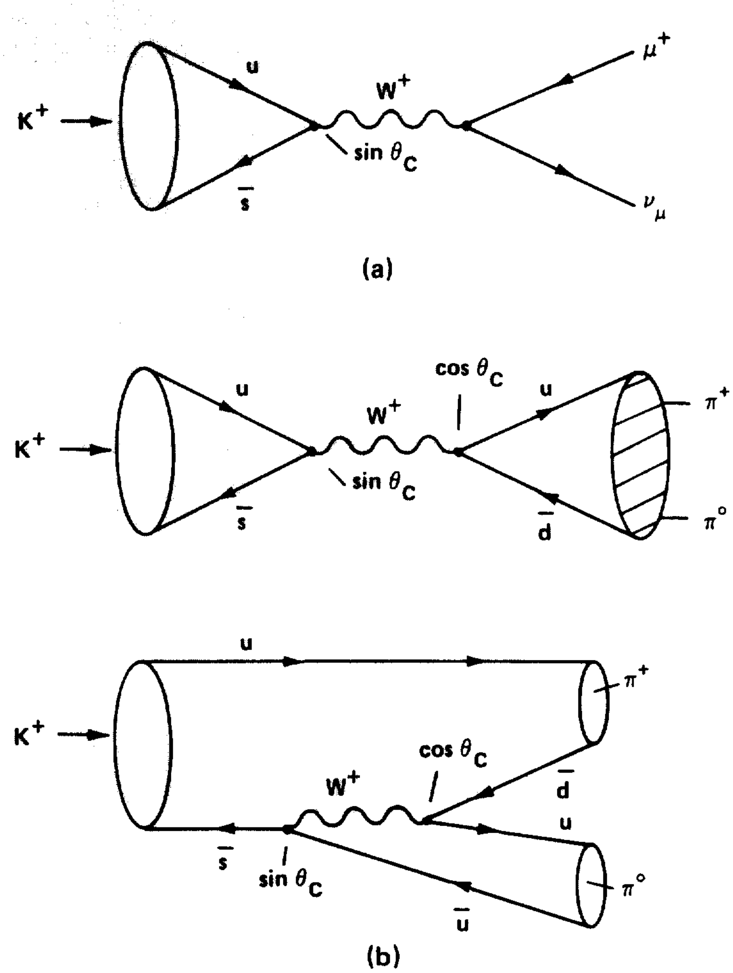

Fig. 4. Feynman diagrams for the decays of a positive kaon into (a) a positive muon and a muon neutrino and (b) a positive and a neutral pion. The ellipse with diagonal lines represents any one of several possible pathways for production of a positive and a neutral pion from an up quark and an antidown quark. These decays, in which the up-down and charmed-strange quark families are mixed by the weak interaction (as indicated by $\sin\theta_C$ *and* $\cos\theta_C$*), are evidence that the family symmetry of quarks is a broken symmmetry.*

Because of the mixing given by Eq. 3, the statement we made near the beginning of this article, that no family-changing decays have been observed, must be sharpened. True, no $s' \to u$ decay has been seen, but, of course, the $s \to u$ decay implied by Eq. 5 does occur. Thus, "No family-changing decays of weak-interaction family eigenstates have been observed" is the more precise statement.

The weak neutral-current interaction describing the scattering of a d' quark when it absorbs a Z^0 has a form like that of Eq. 4:

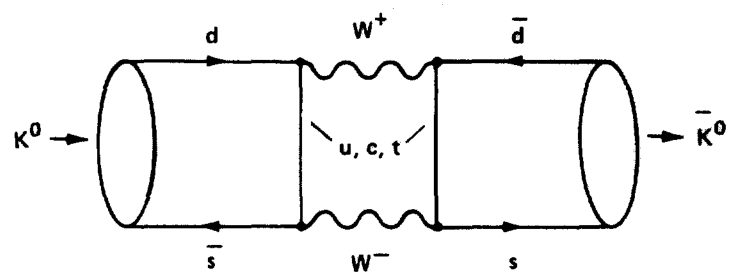

Fig. 5. Feynman diagram for a CP-violating reaction that transforms the neutral kaon into its antiparticle. This second-order weak interaction occurs through formation of virtual intermediate states including either a* u, c, *or* t *quark.

$$(\bar d' d' + \bar s' s')Z^0 = (\bar d', \bar s')\begin{pmatrix} Z^0 & 0 \\ 0 & Z^0 \end{pmatrix}\begin{pmatrix} d' \\ s' \end{pmatrix}. \tag{6}$$

Since the Cabibbo matrix in Eq. 3 is unitary, Eq. 6 is unchanged (except for the disappearance of primes on the quarks) by substitution of Eq. 3:

$$(\bar d' d' + \bar s' s')Z^0 = (\bar d d + \bar s s)Z^0 . \tag{7}$$

Thus, the weak neutral-current interaction does not change d quarks into s quarks any more than it changes d' quarks into s' quarks. It is only the presumed family vector boson of mass greater than 10^5 GeV that may effect such a change.

Family Symmetry Violation and CP Violation

The combined operation of charge conjugation and parity reversal (CP) is, like parity reversal alone, now known not to be an exact symmetry of the world. An understanding of CP violation and proton decay would be of universal importance to explain "big-bang" cosmology and the observed excess of matter over antimatter.

The generalization by Kobayashi and Maskawa of Eq. 3 to the three-family case is introduced in "Particle Physics and the Standard Model"; it yields a relation between family symmetry violation and CP violation. Although other sources of CP violation may exist outside the standard model, this relation permits extraction of information about violation of family symmetry from studies of CP violation.

The phenomenon of CP violation has, so far, been observed only in the K^0- $\bar K^0$ system. The CP eigenstates of this system are the sum and the difference of the K^0 and $\bar K^0$ states. The violation is exhibited as a small tendency for the long-lived state, K_L, which normally decays into three pions, to decay into two pions (the normal decay mode of the short-lived state, K_S) with a branching ratio of approximately 10^{-3}. This tendency can be described by saying that the K_S and K_L states differ from the sum and difference states by a mixing of order ε:

$$|K_S\rangle \cong |K^0\rangle + (1-\varepsilon)\,|\bar K^0\rangle$$

and (8)

$$|K_L\rangle \cong |K^0\rangle - (1-\varepsilon)\,|\bar K^0\rangle .$$

The quark-model analysis based on the work of Kobayashi and Maskawa and the second-order weak interaction shown in Fig. 5 predict an additional CP-violating effect not describable in terms of the mixing in Eq. 8; that is, it would occur even if ε were zero. The effect, which is predicted to be of order ε', where ε'/ε is about 10^{-2}, has not yet been observed, but experiments sufficiently sensitive are being mounted.

Both ε and ε' are related to the Kobayashi-Maskawa parameters that describe family symmetry violation. This guarantees that if the value of ε' is found to be in the expected range, higher precision experiments will be needed to determine its exact value. If no positive result is obtained in the present round of experiments, it will be even more important to search for still smaller values. In either case intense kaon beams are highly desirable since the durations of such experiments are approaching the upper limit of reasonability.

Of course, in principle, CP violation can be studied in other quark systems involving the heavier c, b, and t quarks. However, these are produced roughly 10^8 times less copiously than are kaons, and the CP-violating effects are not expected to be as large as in the case of kaons.

Global Family Symmetry

In our discussion of family-violating processes like $K \to \mu e$, we have, so far, assumed the existence of a massive gauge vector boson reflecting family dynamics. The general theorem, due to Goldstone, offers two mutually exclusive possibilities for the realization of a broken symmetry in field theory. One is the development of just such a massive vector boson from a massless one; the other is the absence of any vector boson and the appearance of a massless scalar boson, or Goldstone boson. The possible Goldstone boson associated with family symmetry has been called the familon and is denoted by f. As is generally true for such scalar bosons, the strength of its coupling falls inversely with the mass scale of the symmetry breaking. Cosmological arguments suggest a lower bound on the coupling of approximately 10^{-12} GeV^{-1}, a value very near (within three orders of magnitude) the upper bound determined from particle-physics experiments.

The familon would appear in the two-body decays $\mu \rightarrow e + f$ and $s \rightarrow d + f$. The latter can be observed in the decay K^+ $(= u + \bar{s}) \rightarrow \pi^+$ $(= u + \bar{d})$ + *nothing else seen*. The familon would not be seen because it is about as weakly interacting as a neutrino. The only signal that the decay had occurred would be the appearance of a positive pion at the kinematically determined momentum of 227 MeV/c.

Such a search for evidence of the familon would encounter an unavoidable background of positive pions from the reaction $K^+ \rightarrow \pi^+ + \nu_i + \bar{\nu}_i$, where the index i covers all neutrino types light enough to appear in the reaction. This decay mode occurs through a one-loop quantum-field correction to the electroweak theory (Fig. 6) and is interesting in itself for two reasons. First, it depends on a different combination of the parameters involved in CP violation and on the number N_ν of light neutrino types. Since N_ν is expected to be determined in studies of Z^0 decay, an uncertainty in the value of a matrix element in the standard-model prediction of the $K^+ \rightarrow \pi^+\nu_i\bar{\nu}_i$ branching ratio can be eliminated. Present estimates place the branching ratio in the range between 10^{-9} and 10^{-10} times N_ν. Second, a discrepancy with the N_ν value determined from decay of the Z^0, which is heavier than the kaon, would be evidence for the existence of at least one neutrino with a mass greater than about 200 MeV/c^2.

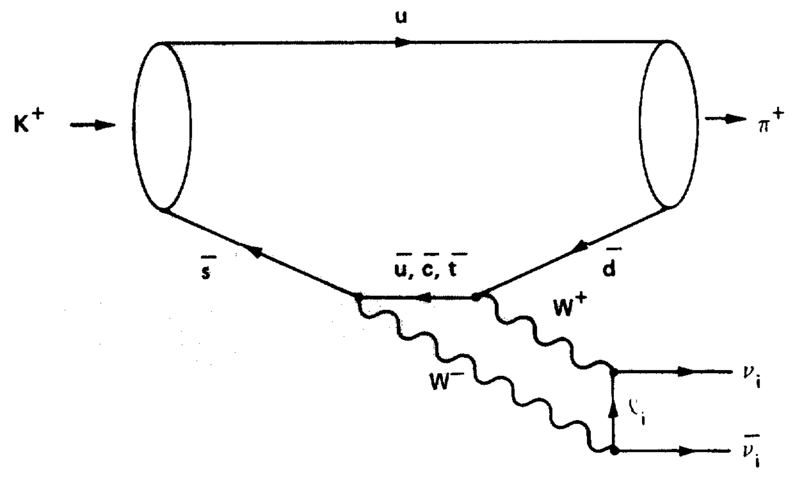

Fig. 6. Feynman diagram for the decay $K^+ \rightarrow \pi^+ + \nu_i + \bar{\nu}_i$, ***where the index*** i ***covers all neutrino types light enough to appear in the reaction. The symbol*** ℓ_i ***stands for the charged lepton associated with*** ν_i ***and*** $\bar{\nu}_i$.

Fermion Masses and Family Symmetry Breaking

The mass spectrum of the fermions is itself unequivocal evidence that family symmetry is broken. These masses, which are listed in Table 1, should be compared to the $W^\pm$ and Z^0 masses of 83 and 92 GeV/c^2, respectively, which set the dynamical scale of electroweak interactions. (The masses quoted are the theoretical values, which agree well with the recently measured experimental values.) The very existence of the fermion masses violates electroweak symmetry by connecting doublet and singlet representations, and the variations in the pattern of mass splittings within each family show that family symmetry is broken. But since we neither know the mass scale nor understand the pattern of the family symmetry breaking, we do not really know the relation between the mass scale of electroweak symmetry breaking and the fermion mass spectrum. It is possible to devise models in which the first family is light because the family symmetry breaking suppresses the electroweak symmetry breaking. Thus, the "natural" scale of electroweak symmetry breaking among the fermions could remain approximately 100 GeV/c^2, despite the small masses (a few MeV/c^2) of some fermions.

Experiments to establish the masses of the neutrinos are of great interest to the family problem and to particle physics in general. Being electrically neutral, neutrinos are unique among the fermions in possibly being endowed with a so-called Majorana mass* in addition to the usual Dirac mass. One approach to determining these masses is by applying kinematics to suitable reactions. For example, one can measure the end-point energy of the electron in the beta decay $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$ or of the muon in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$.

Another quite different approach is to search for "neutrino oscillations." If the neutrino masses are nonzero, weak interactions can be expected to mix neutrinos from different families just as they do the quarks. This mixing would cause a beam of, say, essentially muon neutrinos to be transformed into a mixture (varying in space and in time) of electron, muon, and tau neutrinos. Detection of these oscillations would not only settle the question of whether or not neutrinos have nonzero masses but would also provide information about the differences between the masses of neutrinos from different families. Experiments are in progress, but, since neutrino interactions are infamously rare, high-intensity beams are required to detect any neutrinos at all, let alone possible small oscillations in their family identity. (For details about the tritium beta decay and neutrino oscillation experiments in progress at Los Alamos, see "Experiments To Test Unification Schemes.")

Conclusion

The family symmetry problem is a fundamental one in particle physics, apparently without sufficient information available at present to resolve it. Yet it is as crucial and important a problem as grand unification.

****Majorana mass terms are not allowed for electrically charged particles. Such terms induce transformations of particles into antiparticles and so would be inconsistent with conservation of electric charge.***

and it may well be a completely independent one. The known bound of 10^5 GeV on the scale of family dynamics is an order of magnitude beyond the direct reach of any present or proposed accelerator, including the Superconducting Super Collider. These dynamics may, however, be accessible in studies of rare decays of kaons and other mesons, of CP violation, and of neutrino oscillations. To undertake these experiments at the necessary sensitivity requires intense fluxes of particles from the second or later families. A high-intensity, medium-energy accelerator is a highly cost-effective means of approaching these experimental needs. Unlike the questions on the high-energy frontier, those on the high-intensity frontier are clearly defined. Now we need to answer them. ■

Further Reading

Howard Georgi. "A Unified Theory of Elementary Particles and Forces." Scientific American, April 1981, p. 48.

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Goldman and Nieto have co-authored several papers, including a recent one on tests of the gravitational properties of antimatter. The similarity of their other interests has also led them to work on related topics, although not always at the same time. For example, in 1972 Nieto authored a survey of important experiments in particle physics that could be done at the then new LAMPF. A decade later Goldman organized the Theoretical Symposium on Intense Medium-Energy Sources of Strangeness at the University of California, Santa Cruz, to study those new experiments that would be feasible at LAMPF II. The most recent manifestation of their similar interests is joint editorship of the proceedings of the annual meeting, held in Santa Fe, New Mexico from October 31 to November 3, 1984, of the Division of Particles and Fields of the American Physical Society.