Reynolds Number Revisited

As discussed in the earlier sidebar, the Reynolds number is a convenient and physically sound basis for comparing similar flows under different circumstances. For instance, as flow speed through a pipe increases, the drag on the fluid increases and, consequently, so also does the required applied pressure; these increases are reflected in a corresponding increase in Reynolds number. Total friction experienced by the fluid undergoing laminar flow is usually expressed in terms of the Reynolds number R, allowing easy comparison between widely varying tests.

Once the Reynolds number reaches a critical value, however, laminar flow in the pipe becomes turbulent, and further increases in Reynolds number no longer reflect significant changes in measured drag. At this point, the effective turbulence Reynolds number $R_{\rm eff}$ becomes a more appropriate gauge, reflecting the ratio of inertial to turbulence momentum-dissipation effects (rather than inertial to viscous-dissipation effects).

Although the Reynolds number can, in theory, be increased without bound, the turbulence Reynolds number cannot. The value of $R_{\rm eff}$ is not directly and uniquely set by readily measured properties and flow geometry but rather depends on eddy generation and the resulting eddy sizes within the flow field. A limiting value of $R_{\rm eff}$ is observed in turbulent-flow experiments.

To demonstrate this behavior quantitatively, it is convenient to make some simplifying assumptions. Typically, the turbulence viscosity ν_t, which is much larger than the molecular viscosity ν_m, is taken to be equal to the product of eddy size s (the turbulence length scale), an appropriate turbulence velocity (here taken as $K^{1/2}$, where K is the specific turbulence kinetic energy), and a universal constant $1/C_\nu$. Thus

$$R_{\rm eff} \approx C_\nu \frac{u_0 L}{s K^{1/2}}, \qquad (1)$$

where L is a characteristic length for the mean flow.

If, as is usually the case, the turbulence kinetic energy is some fraction of the mean-flow kinetic energy ($K \cong \frac{1}{2} f_K u_0^2$), then

$$R_{\rm eff} \cong C_\nu \sqrt{\frac{2}{f_K}} \left(\frac{L}{s}\right); \qquad (2)$$

that is, $R_{\rm eff}$ is proportional to the ratio of the length scales. As turbulence gains in intensity, its *average* length scale usually decreases slightly, but not without limit. In fact, the largest eddies, those that contain the major fraction of the turbulence kinetic energy, will be some portion of the mean-flow length scale (such as pipe diameter). Therefore, since C_ν is usually about 10 and an upper bound on L/s is typically 20, $R_{\rm eff}$ will seldom exceed several hundred, even in the most intensely turbulent flows. On the other hand, R can be several million or more. ■