Realizing a Noiseless Subsystem in an NMR Quantum Information Processor

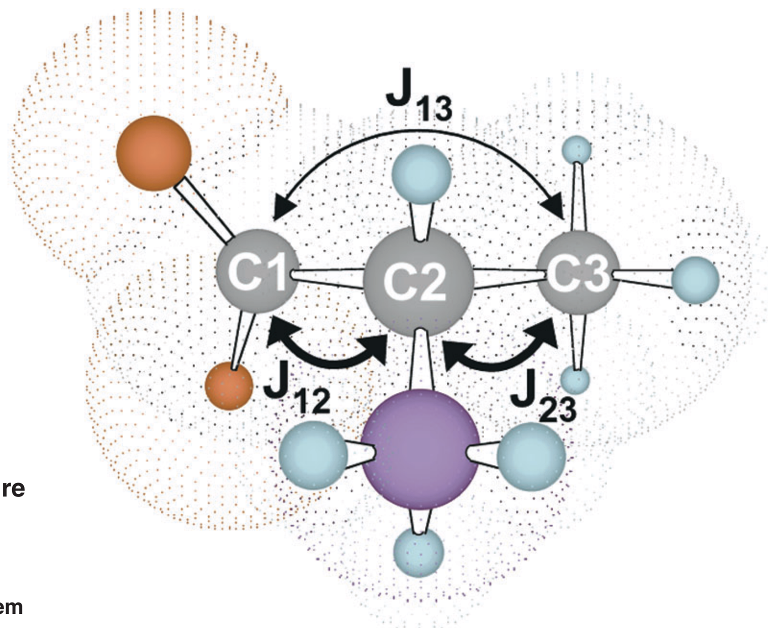

Figure 1. Molecular Structure of ^{13}C-Labeled Alanine
The diagram shows the three carbon-13 (^{13}C) spins used as qubits in the noiseless subsystem experiment as well as the relevant *J*-couplings between those qubits.

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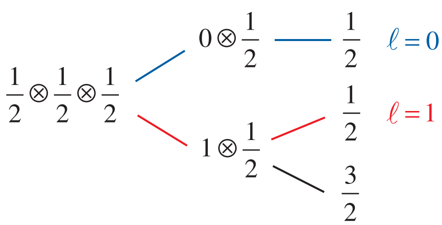
The fact that the occurrence of symmetries in a physical system generally implies the existence of conserved quantities and that these symmetries can be exploited to ease the understanding of the system's behavior is a well-known lesson in physics. The notion of a noiseless subsystem (NS) (Knill et al. 2000) captures this lesson in the context of quantum information processing (QIP), where the challenge is to protect information against the detrimental effects of noise. The link between symmetries, conserved quantities, and NS was discussed at length on page 216 of the article "Introduction to Error Correction". The essential message is that, by encoding information into an abstract subsystem that corresponds to a preserved degree of freedom, noiselessness is guaranteed even if errors still evolve the overall system's state.

Here, we focus on the NS of three spin-1/2 particles introduced in the above-mentioned article (see page 201), along with a discussion of the error-correcting properties of this NS. The physical system is composed of three qubits, subjected to a "far-field" interaction with the environment, whereby the latter couples to the qubits without distinguishing among them. The resulting collective-noise model involves all possible error operators that are symmetric under permutation of the three particles and is specified in terms of the error generators $J_u = (\sigma_u^{(1)} + \sigma_u^{(2)} + \sigma_u^{(3)})/2$, where $u = x, y, z$. By recalling the meaning of the single-spin Pauli operators σ_u^k, the observable J_u represents the projection of the total spin angular momentum $\boldsymbol{J}$ along the u-axis. Because the total-spin observable $J^2 = \boldsymbol{J} \cdot \boldsymbol{J}$ commutes with the error generators and z defines the quantization axis, the eigenvalues j and j_z of J^2 and J_z, respectively, provide useful quantum numbers to label basis states for the three particles.

The NS of interest resides in the four-dimensional subspace $\mathcal{H}_{1/2}$ of the states carrying total angular momentum $j = 1/2$ and having a total z-component $j_z = \pm 1/2$. However, specifying j and j_z does not suffice for completely labeling the states in $\mathcal{H}_{1/2}$: An additional quantum number is needed for removing the two-dimensional degeneracy that remains. Physically, this degeneracy simply means that there are two distinct paths for obtaining a total angular momentum $j = 1/2$ out of three elementary 1/2 angular momenta:



Let the additional quantum number $\ell = 0, 1$ label the two possible routes in the above diagram. Because collective noise does not distinguish among the individual spins and the final eigenvalue j is the same for both paths, the noise can neither distinguish the realized value of ℓ nor change that value. This conserved quantum number can be directly related to the eigenvalues $s_z = \pm 1$ of the $\sigma_z^{(\mathrm{NS})}$ observable of a noiseless qubit, $s_z = 2\ell - 1$. In general, noiseless qubit operators will remain invariant under rotations. The simplest scalars under the rotations are the dot products $s_{12} = \sigma^{(1)} \cdot \sigma^{(2)}$, $s_{23} = \sigma^{(2)} \cdot \sigma^{(3)}$, and $s_{31} = \sigma^{(3)} \cdot \sigma^{(1)}$.

Thus, $\sigma_u^{(\mathrm{NS})}$ observables for the noiseless qubit, where $u = x, y, z$, can be constructed by combining s_{12}, s_{23}, s_{31}, and the identity into three operators that "behave like" the Pauli matrices (Viola et al. 2001a). A good choice is given by $\sigma_x^{(\mathrm{NS})} = 1/2(\mathbb{1} + s_{23})$, $\sigma_y^{(\mathrm{NS})} = \sqrt{3}/6(s_{31} - s_{12})$, and $\sigma_z^{(\mathrm{NS})} = i\sigma_y^{(\mathrm{NS})}\sigma_x^{(\mathrm{NS})}$, where projection onto the relevant $\mathcal{H}_{1/2}$ subspace is understood. Note that the action corresponding to $\sigma_x^{(\mathrm{NS})}$ is simply a permutation exchanging the last two spins. (For an alternative construction of the NS observables, see the article "Introduction to Error Correction," page 216.) Identifying the NS through its observables is equivalent to identifying it through the explicit state space correspondence given in Equation (28) of the above-mentioned article.

The experimental implementation of the three-qubit NS (Viola et al. 2001b) was performed with liquid-state NMR techniques. The three spin-1/2 carbon nuclei of carbon-13-labeled alanine were used as qubits (Figure 1). The information to protect is an arbitrary one-qubit state, $|\psi\rangle = a|0\rangle + b|1\rangle$, where a and b are arbitrary complex amplitudes, and $\langle\psi|\psi\rangle = 1$. This information is initially stored in spin 3, meaning that the three carbon spins are initialized in a pseudopure state corresponding to $|0\rangle_1|0\rangle_2|\psi\rangle_3 = |00\psi\rangle = a|000\rangle + b|001\rangle$. A unitary transformation U_{enc} encodes this input state into a superposition of the two basis states in $\mathcal{H}_{1/2}$ with $j = 1/2$ and $j_z = -1/2$. That is,

$$U_{\mathrm{enc}}|00\psi\rangle \leftrightarrow a|\downarrow\rangle \cdot |0\rangle + b|\downarrow\rangle \cdot |1\rangle = |\downarrow\rangle \cdot |\psi\rangle \ , \qquad (1)$$

where the subsystem representation of Equation (28) has been used.

The three qubits remain stored in the NS memory for a fixed evolution period t_{ev}, during which errors can occur. In a given set of experiments, these errors are designed to implement a desired collective-noise process $\mathcal{E}_{\mathrm{coll}}$ described by a set of error operators $\{E_a\}$. Because of their collective nature, these errors affect only the syndrome subsystem in the pair. Finally, following the evolution period, the unitary transformation U_{dec} decodes a generic noisy state $E_a(|\downarrow\rangle \cdot |\psi\rangle)$ in $\mathcal{H}_{1/2}$ back to the computational basis.

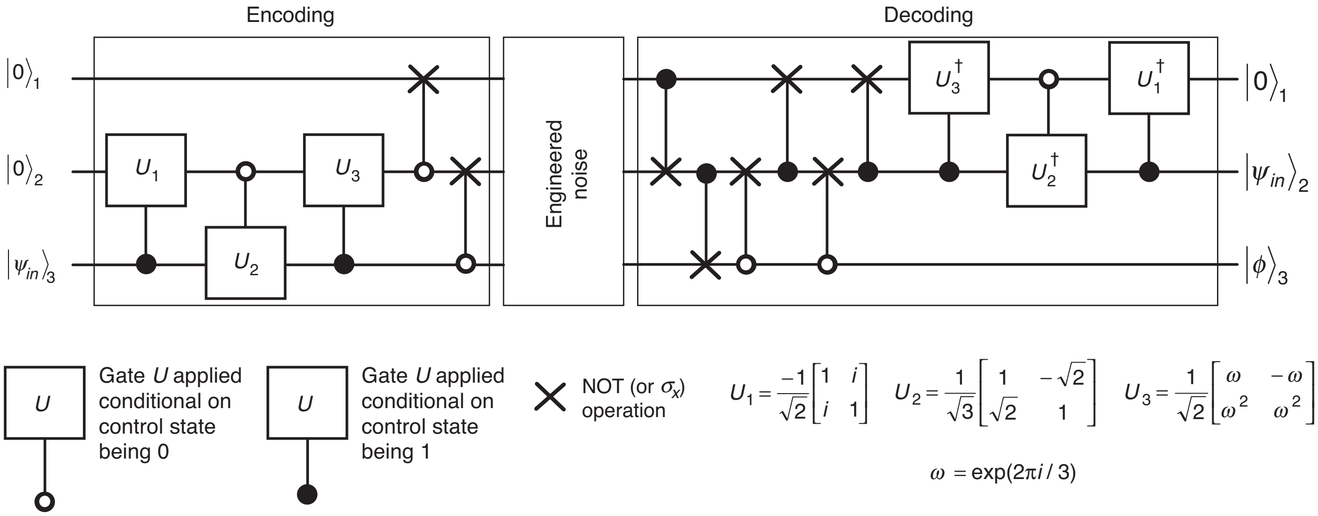


Figure 2. Logical Quantum Network

The diagram shows a logical quantum network for the three-qubit NS experiment. The logical manipulations were translated into sequences of radio-frequency pulses and delays, and complete pulse programs for U_{enc} and U_{dec} resulted from the compilation of the partial pulse programs for individual gates. The pulses were designed to ensure self-refocusing of all the unwanted *J*-coupling and chemical-shift evolutions.

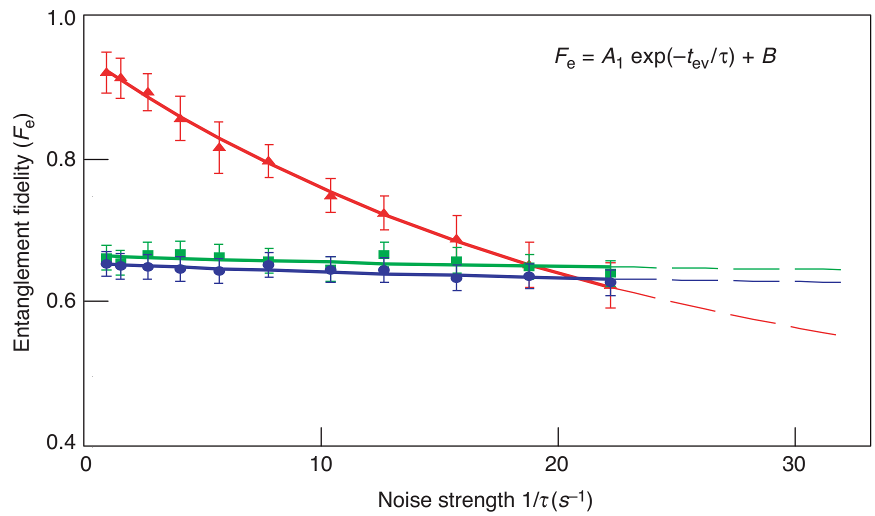
This procedure has the effect of returning the quantum state $|\psi\rangle$ onto qubit 2 upon discarding ("tracing over") spins 1 and 3,

$$\mathrm{Tr}_{1,3}\{U_{\mathrm{dec}}[\mathcal{E}_{\mathrm{coll}}(|\downarrow\rangle\langle\downarrow| \cdot |\psi\rangle\langle\psi|)]U^{-1}_{\mathrm{dec}}\} = |\psi\rangle_2\langle\psi| \ . \tag{2}$$

Figure 2 is a sketch of the quantum network for the experiment.

During the delay period between encoding and decoding, we use gradient diffusion techniques to engineer a desired collective-noise process. In order to fully explore the robustness properties of information encoded in the NS, we applied various error models corresponding to noise along a single axis (see Figure 3), as well as more complicated double- and triple-axis noise processes obtained by "cascading" the action of error models along different spatial directions, in sequence, within a single evolution period (see Table I). To quantify the accuracy of the implemented NS in preserving the quantum data $|\psi\rangle$, we experimentally extracted the entanglement fidelity F_e of the overall process (including encoding, decoding, and engineered noise during storage), where $F_e = 1$ implies perfect preservation.

Our results in Figure 3 and Table I indicate that, as expected, the effects of the applied noise increase exponentially as a function of noise strength for unencoded (UN) information but are largely independent of noise strength for information encoded in the NS. That independence demonstrates that the NS functions as an "infinite-distance" quantum error-correcting code for arbitrary collective errors. On the other hand, the F_e is always about the same and less than 1 in all the NS experiments. The constant reduction in fidelity is suggestive of errors introduced during encoding and decoding manipulations, as well as of noise due to natural noncollective relaxation processes during the whole experiment. ■



$F_e = A_1 \exp(-t_{ev}/t) + B$

A_1	B	Quantum Process
0.51 ± 0.04	0.43 ± 0.03	▲ UN, *y*-axis noise
0.03 ± 0.03	0.64 ± 0.02	■ NS-encoded, *y*-axis noise
0.03 ± 0.03	0.62 ± 0.02	● NS-encoded, *z*-axis noise

Figure 3. Entanglement Fidelities for Engineered Collective Noise along a Single Axis

The fidelity of UN information subjected to engineered collective noise along the *y*-axis (red) decreases exponentially with noise strength τ^{-1} whereas the fidelity of NS-encoded information subjected to collective noise along either the *y*-axis (green) or the *z*-axis (black) remains almost constant independent of noise strength.

In each case, noise was applied for a fixed evolution period t_{ev} of approximately 44 ms.

The flatness of the curve interpolating the NS data demonstrates the behavior of the NS as an infinite-distance quantum error-correcting code for single-axis collective errors of arbitrary strength. The smooth fits to the data are derived from the exponential and parameters displayed under the figure.

Table I. Entanglement Fidelities for Engineered Collective Noise along Two and Three Axes

Quantum Process	Entanglement Fidelity (F_e)
Q_{zx}^{UN}	0.24
Q_{00}^{NS}	0.70
Q_{zx}^{NS}	0.70
Q_{zy}^{NS}	0.70
Q_{000}^{NS}	0.67
Q_{yzx}^{NS}	0.66

Q stands for the one-qubit processes implemented during each run.

Superscripts tell whether the system has been encoded or not.

Subscripts *zx*, *zy*, and *yzx* are for the axes along which noise processes with maximum achievable strength were applied in cascade. Subscripts 00 and 000 indicate that no noise was applied.

Two subscripts indicate shorter delay periods than three subscripts.

Statistical uncertainties in all F_e values are approximately 2%.

Lorenza Viola obtained a Ph.D. degree in physics from the University of Padova (Italy) in 1996. After being a postdoctoral fellow at the D'Arbeloff Laboratory at the Massachusetts Institute of Technology, Lorenza became a Director-funded and then a J. R. Oppenheimer postdoctoral fellow at Los Alamos. Her recent research has been focused on quantum information science, with emphasis on devising schemes for controlling noisy quantum devices and performing robust quantum computation. Lorenza has been a key contributor to the development of quantum error-suppression techniques based on dynamical decoupling and to the theoretical characterization and experimental verification of the notion of a noiseless subsystem as the most general approach to noise-free information storage.

Further Reading

Knill, E., R. Laflamme, and L. Viola. 2000. Theory of Quantum Error Correction for General Noise. *Phys. Rev. Lett.* **84**: 2525.

Viola, L., E. Knill, and R. Laflamme. 2001a. Constructing Qubits in Physical Systems. *J. Phys. A* **34** (35): 7067.

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